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THEORETICAL FOUNDATIONS OF EXPANSION OF THE NUMERICAL SPACE

In this paper, the theoretical foundations of the expansion of a numerical space are introduced due to the introduction of the concept of a spatially indeterminate unit whose product by an imaginary unit is equal to zero, which ensures that all algebraic operations on "three-dimensional" numbers are called volumetric ones. For volume numbers, all the properties of addition and multiplication are fulfilled, except for the associative property in multiplication, but it is true for a certain set of numbers that form hyperbolic surfaces, and they also have the inverse mirror image property.
***Keywords**: volumetric numbers, imaginary unit, spatially indeterminate unit, real numbers, mirror numbers.*

В даній роботі наведені теоретичні основи розширення числового простору внаслідок введення поняття просторово невизначеної одиниці, добуток якої на уявну одиницю дорівнює нулю, що забезпечує виконання всіх алгебраїчних операцій над «трьохмірними» числами названими об'ємними. Для об'ємних чисел виконуються усі властивості складання та множення за винятком властивості асоціативності при множені, яка все таки виконується для відповідної множини чисел які утворюють гіперболічні поверхні, а також їм притаманна властивість взаємозворотнього дзеркального відображення.
***Ключові слова**: об'ємні числа, уявна одиниця, просторово невизначена одиниця, дійсні числа, дзеркальні числа.*

Formulation of the problem

Analysis of the history of development of numbers leads to the following logical conclusions. The real numbers are represented by points of the numerical line from $-\infty$ to $+\infty$, that is, they are "one-dimensional". Complex numbers, under the condition of introducing an imaginary unit i, are represented by the points of the plane, that is, they are "two-dimensional". Thus, it should be possible to expand the set of numbers that will be represented by points of space, that is, "three-dimensional". Many scientists worked on solving this problem. The development of vector algebra and work with spatial vectors led to the idea of introducing "three-dimensional" numbers. As in the case of complex numbers that can be considered as "two-dimensional", the permissible operations on "three-dimensional" numbers would have to include addition, subtraction, multiplication and division. And that these numbers can be easily and effectively produced algebraic operations, they must have the usual properties of real and complex numbers.

Analysis of recent research and publications

By the middle of the XIX century complex numbers are widely used to represent vectors on the plane and perform operations on them. In this connection attempts have been made to introduce "three-dimensional" numbers to represent spatial vectors and perform operations on them. As in the case of complex numbers, admissible operations on "three-dimensional" numbers would have to include addition, subtraction, multiplication and division [1]. Unfortunately positive results have not been achieved in this direction. The most close to the solution of this problem came Hamilton.
Analogues of the corresponding numbers were proposed by William Hamilton in 1843, which he called quaternions [2]. However the Hamilton quaternions were "four-dimensional" and expanded the number space according to certain rules of multiplication, as well as by not performing the commutativity property in multiplication.

Formulation of the research objective

Thus, the question of expanding the numeric space using "three-dimensional" numbers remained open. To solve the problem, taking into account the analysis of [3,4,5] and the analysis of the theory of complex number systems [6], a generalization was made about the need to revise the notions of imaginary units in number theory with the purpose of expanding the numerical space. According to the analysis performed, the parameters i and j, which make it possible to compile a mathematical notation for a three-dimensional number, must play the role of certain transformation operators, and not only carry the meaning of the corresponding units. And the corresponding actions with these operators must meet certain rules.

Statement of the main material

The authors of this paper introduce the notion of a spatially indeterminate unit j to solve the problem. The introduction of a spatially indeterminate unit leads to an expansion of the concept of number-to volume numbers [7].

The general algebraic formula for the volume number has the form

$$V = a + bi + cj\ ,$$

where: $a, b, c -$ real numbers; $i -$ imaginary unit; $j -$ spatially indefinite unit.

The number a is called the real part of the volume number; $bi -$ its imaginary part ($b -$ the coefficient of the imaginary part); $cj -$ its spatial part ($c -$ coefficient with the spatial part). Giving all possible real values, we get all possible volumetric numbers.

The geometric interpretation of the volume number (Fig. 1) is as follows.

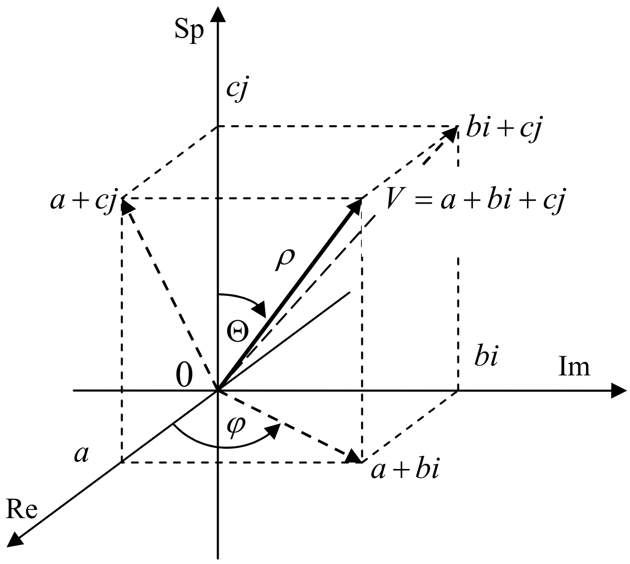


Fig. 1. Geometric interpretation of a volume number.
Re – is the real axis, Im – is the imaginary axis, Sp – is the spatial axis.

Since each point of space is completely determined by the radius vector of this point, then to each volume number there corresponds a definite vector emerging from the pole to the corresponding point. Thus, the volume numbers can be represented by both space points and vectors. According to the geometric interpretation, the volume modulus is a sphere.

Proceeding from the definition, two volume numbers are considered equal if their real parts and coefficients are equal separately for imaginary and spatial parts. Geometrically, the volume numbers are equal if their imaging vectors are equal. The concept of "more" and "less" for volume numbers does not exist.

If we enter instead of the Cartesian coordinates of a point (representing the volume number) its spherical coordinates (Fig. 1), then we obtain a trigonometric formula for writing the volume number

$$V = \rho(\sin\Theta\cos\varphi + i\sin\Theta\sin\varphi + j\cos\Theta),$$

where: $\rho -$ length of the radius vector of the corresponding point; $\varphi -$ longitude; $\Theta -$ polar distance.

The relationship between the Cartesian and spherical coordinates of the point:

$$a = \rho\sin\Theta\cos\varphi;\ b = \rho\sin\Theta\sin\varphi;\ c = \rho\cos\Theta;$$

$$\rho = \sqrt{a^2 + b^2 + c^2};$$

$$\varphi = Arctg\frac{b}{a};\ \Theta = Arctg\frac{\sqrt{a^2 + b^2}}{c}.$$

Positive directions are shown in Fig. 1.

Given that $a + bi$ there is a complex number z, then the volume number can be written in the form

$$V = z + cj.$$

A particular case of a volume number is a complex number provided that $c = 0$,

$$V = a + bi,$$

or a real number, provided that $b = c = 0$,

$$V = a.$$

Axioms of volume numbers.

1. $j \times j = j^2 = -1$.
2. $i \times j = j \times i = 0$.

The logical explanation of the second axiom is as follows. In relation to imaginary numbers, Leibniz wrote: "The Spirit of God found the finest vent in this miracle of analysis, the freak of the world of ideas, the dual essence that lies between being and non-being, which we call the imaginary root of the negative unit." A spatially indefinite unit contains, at the moment, an even more contradictory and incomprehensible meaning. Therefore, the product of an imaginary unit and a spatially indeterminate unit must be zero. A physical analogue in this case may be the interaction of matter with antimatter, although it is possible to only compare the substance with the imaginary unit conditionally.

The mathematical justification of the second axiom is as follows. The concept of an imaginary unit i implies an expression $1i$ since $i^2 = i \times i = -1$. A purely imaginary number has the form bi, where is a $b -$ real number. If we assume that i is some imaginary transformation operator (matrix) that makes it possible to extract the square root of a negative number, then in the general form a purely imaginary number can be written $(b)i$, where $b \in \{-\infty, +\infty\}$. If we consider the option when the parameter b is absent (but not equal to zero), then, accordingly, we obtain the following expression $(\ )i$. Then we will have the condition

$$(\ )i \times (\ )i = [(\ )i]^2 = -(\ ).$$

Since for a spatially indeterminate unit the condition $j \times j = j^2 = -1$ is true, by analogy with the imaginary number, the spatial number can be written in the form $(c)j$, where $c \in \{-\infty, +\infty\}$. If we consider the option when the parameter c is absent (but not equal to zero), then, accordingly, we obtain the following expression $(\ )j$. Consequently, we become the condition

$$(\ )j \times (\ )j = [(\ )j]^2 = -(\ ).$$

Taking into account the characteristicof the introduced transformation operators (matrices), their product equals zero

$$(\ )i \times (\ )j = (\ )j \times (\ )i = 0(\ ).$$

It is possible that a spatially indefinite unit can have a dual character and have a deeper meaning than it appears at the moment.

Algebraic operations on volume numbers are performed in the same way as over complex numbers or over ordinary trinomials, assuming that $i^2 = -1$, $j^2 = -1$, $i \times j = j \times i = 0$.

The addition of two volume numbers is determined by the formula

$$(a_1 + b_1 i + c_1 j) + (a_2 + b_2 i + c_2 j) = (a_1 + a_2) + (b_1 + b_2)i + (c_1 + c_2) j .$$

The subtraction of two volume numbers is given by

$$(a_1 + b_1 i + c_1 j) - (a_2 + b_2 i + c_2 j) = (a_1 - a_2) + (b_1 - b_2)i + (c_1 - c_2) j .$$

The multiplication of two volume numbers is given by

$$(a_1 + b_1 i + c_1 j) \times (a_2 + b_2 i + c_2 j) = (a_1 a_2 - b_1 b_2 - c_1 c_2) + (a_1 b_2 + a_2 b_1)i + (a_1 c_2 + a_2 c_1) j .$$

Two volume numbers are called mutually conjugate if their real parts are equal, and imaginary and spatial ones differ only in sign. The product of mutually conjugate numbers is a real number

$$(a + bi + cj) \times (a - bi - cj) = a^2 + b^2 + c^2 .$$

The division of two volume numbers is given by

$$\frac{a_1 + b_1 i + c_1 j}{a_2 + b_2 i + c_2 j} = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{a_2^2 + b_2^2 + c_2^2} + \frac{a_2 b_1 - a_1 b_2}{a_2^2 + b_2^2 + c_2^2} i + \frac{a_1 c_2 - a_2 c_1}{a_2^2 + b_2^2 + c_2^2} j .$$

Basic properties of addition for any volume numbers.

1. Commutativity $V_1 + V_2 = V_2 + V_1$.
2. Associativity $V_1 + (V_2 + V_3) = (V_1 + V_2) + V_3$.
3. The zero property $V + 0 = V$.
4. The property of the opposite element $V + (-V) = 0$.
5. Perform subtraction through addition $V_1 - V_2 = V_1 + (-V_2)$.

Basic properties of multiplication for any volume numbers.

1. Commutativity $V_1 \times V_2 = V_2 \times V_1$.
2. The property of units $V \times 1 = V$.
3. The zero property $V \times 0 = 0$.
4. Distributivity $V_1 \times (V_2 + V_3) = V_1 \times V_2 + V_1 \times V_3$.

The associativity property for multiplication

$$V_1 \times (V_2 \times V_3) = (V_1 \times V_2) \times V_3$$

for arbitrary volume numbers does not hold. However, this property is valid when the condition that the product of the opposite real coefficients for the imaginary and spatial parts of the first and third factors are equal,

$$b_1 \times c_3 = b_3 \times c_1 .$$

That is, for the set of real numbers b_1, b_3, c_1, c_3, that form hyperbolic curves in the imaginary-spatial plane, and the corresponding volume numbers, that form hyperbolic surfaces.

Particular cases of the value of the numbers V_1 and V_3 and for which the associative property in multiplication is satisfied are given in Table 1. The volumetric number V_2 is arbitrary.

Table 1. Condition for the performance of the associativity property in multiplication

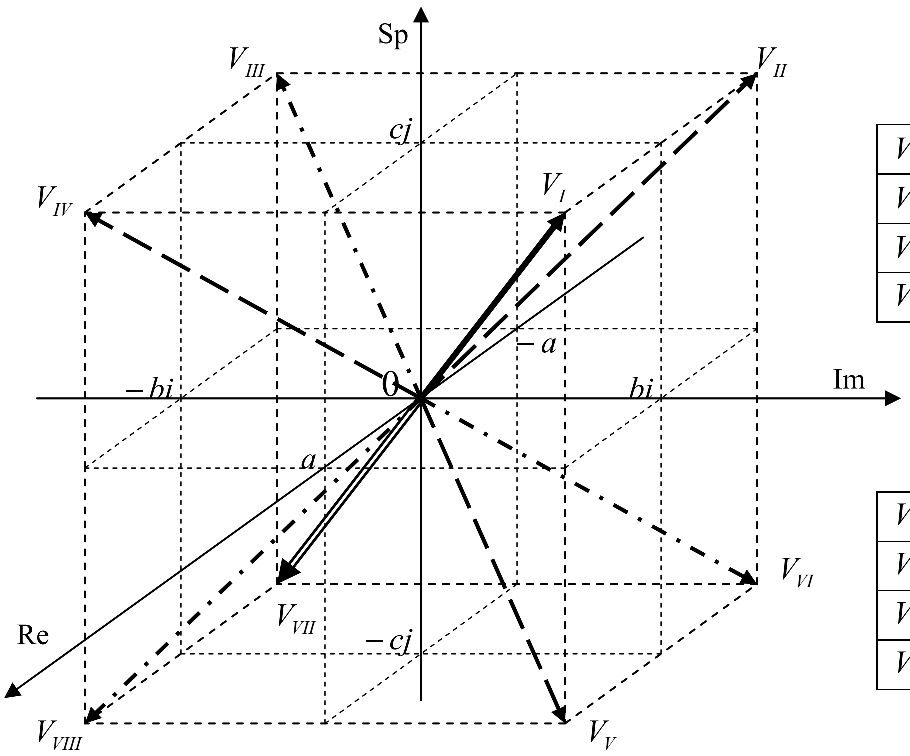
$b_1 \times c_3 = b_3 \times c_1$		V_1	V_3
$b_1 = 0$	$c_1 = 0$	a_1	$a_3 + b_3 i + c_3 j$
	$b_3 = 0$	$a_1 + c_1 j$	$a_3 + c_3 j$
$c_3 = 0$	$b_3 = 0$	$a_1 + b_1 i + c_1 j$	a_3
	$c_1 = 0$	$a_1 + b_1 i$	$a_3 + b_3 i$
$b_1 = b_3 = c_1 = c_3 = 0$		a_1	a_3

Special properties of volume numbers.

According to the geometric interpretation, the volume numbers of the form

$$V = a + bi + cj$$

fill in the entire eight octant of the numeric space (Fig. 1), provided that $a, b, c -$ take any values from $-\infty$ to $+\infty$. The numbers in which the coefficients a, b, c modulo are pairwise equal are called mirror ones. An arbitrary volume number of any octant corresponds to seven mirror numbers, which can be considered as its mirror image (Fig. 2).



$V_I = a + bi + cj$
$V_{II} = -a + bi + cj$
$V_{III} = -a - bi + cj$
$V_{IV} = a - bi + cj$

$V_V = a + bi - cj$
$V_{VI} = -a + bi - cj$
$V_{VII} = -a - bi - cj$
$V_{VIII} = a - bi - cj$

Fig. 2. Distribution of mirror numbers in octans.

If we consider the volume number of the first octant V_I, then the following mirror images correspond to it. The numbers V_{II}, V_{IV}, V_V are a mirror image of the V_I relative to the planes SpIm, ReSp, ReIm. The numbers $V_{III}, V_{VI}, V_{VIII}$ are a mirror image of the V_I relative to the the Sp, Im, Re axes, as well as the mirror image of the numbers V_{II}, V_{IV}, V_V relative to the corresponding planes. The number V_{VII} is a mirror image of the V_I relative to the origin of coordinates, as well as a mirror image of the numbers $V_{III}, V_{VI}, V_{VIII}$ relative to the corresponding planes.

Numbers V_{II}, V_{IV}, V_V are called the first-order mirror image of a number V_I. The numbers $V_{III}, V_{VI}, V_{VIII}$ are called the second-order mirror image of a number V_I. The number V_{VII} is called the opposite number to V_I or a third-order mirror image.

Taking into account the distribution of mirror numbers in octants (Fig. 2), the inverse of the mirror image also takes place. Thus, we can draw up the following scheme of a system of reciprocal mirror image of volume numbers.

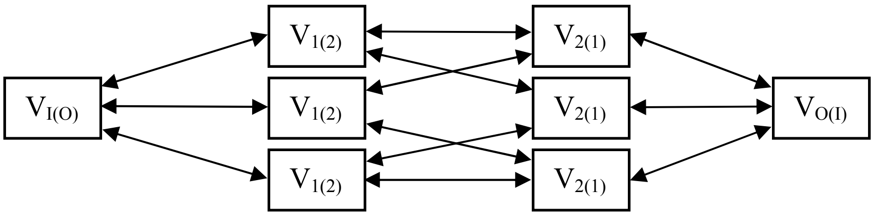


Fig. 3. The reciprocal mirror image of volumetric numbers.

In the scheme of reciprocal mirror image of volume numbers (Fig. 3) the following designations are accepted: V_I – initial volume number; V_1 – mirror image of the first order of the number V_I; V_2 – mirror image of the second order of the number V_I; V_O – the opposite volume number or a mirror image of the third order.

Proceeding from the corresponding scheme for the entire set of volumetric numbers, there are four variants of the inverse mirror image:

$$V_I \Leftrightarrow \begin{Bmatrix} V_{II} \\ V_{IV} \\ V_V \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} V_{III} \\ V_{VI} \\ V_{VIII} \end{Bmatrix} \Leftrightarrow V_{VII}; \quad V_{II} \Leftrightarrow \begin{Bmatrix} V_I \\ V_{III} \\ V_{VI} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} V_{IV} \\ V_V \\ V_{VII} \end{Bmatrix} \Leftrightarrow V_{VIII};$$

$$V_{III} \Leftrightarrow \begin{Bmatrix} V_{II} \\ V_{IV} \\ V_{VII} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} V_I \\ V_{VI} \\ V_{VIII} \end{Bmatrix} \Leftrightarrow V_V; \quad V_{IV} \Leftrightarrow \begin{Bmatrix} V_I \\ V_{III} \\ V_{VIII} \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} V_{II} \\ V_V \\ V_{VII} \end{Bmatrix} \Leftrightarrow V_{VI}.$$

Consider the addition of an arbitrary volumetric number of the first octant V_I with its corresponding mirror numbers:

1) with a first-order mirror image

$$V_I + V_{II} = (a + bi + cj) + (-a + bi + cj) = 2(bi + cj),$$
$$V_I + V_{IV} = (a + bi + cj) + (a - bi + cj) = 2(a + cj),$$
$$V_I + V_V = (a + bi + cj) + (a + bi - cj) = 2(a + bi);$$

2) with a mirror image of the second order

$$V_I + V_{III} = (a + bi + cj) + (-a - bi + cj) = 2cj,$$
$$V_I + V_{VI} = (a + bi + cj) + (-a + bi - cj) = 2bi,$$
$$V_I + V_{VIII} = (a + bi + cj) + (a - bi - cj) = 2a;$$

3) with a mirror image of the third order

$$V_I + V_{VII} = (a + bi + cj) + (-a - bi - cj) = 0.$$

The corresponding results are valid for an arbitrary volumetric number of any octant.

Thus, taking into account the result obtained and the geometrical interpretation of the volume number, the following conclusions can be drawn:

1. The sum of an arbitrary volume number with its first-order mirror image is equal to twice the projection of the radius-vector of a given number by the corresponding coordinate plane.

2. The sum of an arbitrary volume number with its second-order mirror image is equal to twice the projection of the radius-vector of a given number on the corresponding coordinate axis.

3. The sum of the opposite numbers is zero.

Consider the product of an arbitrary specular volume number of any octant with its corresponding mirror numbers.

The results of multiplying the mirror numbers of eight octant are presented in Table 2.

Table 2. The product of mirror numbers

№	I	II	III	IV
I	$(a^2-b^2-c^2)+ +2abi+2acj$	$-a^2-b^2-c^2$	$(-a^2+b^2-c^2)- -2abi$	$(a^2+b^2-c^2)+ +2acj$
II	$-a^2-b^2-c^2$	$(a^2-b^2-c^2)- -2abi-2acj$	$(a^2+b^2-c^2)- -2acj$	$(-a^2+b^2-c^2)+ +2abi$
III	$(-a^2+b^2-c^2)- -2abi$	$(a^2+b^2-c^2)- -2acj$	$(a^2-b^2-c^2)+ +2abi-2acj$	$-a^2-b^2-c^2$
IV	$(a^2+b^2-c^2)+ +2acj$	$(-a^2+b^2-c^2)+ +2abi$	$-a^2-b^2-c^2$	$(a^2-b^2-c^2)- -2abi+2acj$
V	$(a^2-b^2+c^2)+ +2abi$	$(-a^2-b^2+c^2)+ +2acj$	$(-a^2+b^2+c^2)- -2abi+2acj$	$a^2+b^2+c^2$
VI	$(-a^2-b^2+c^2)- -2acj$	$(a^2-b^2+c^2)- -2abi$	$a^2+b^2+c^2$	$(-a^2+b^2+c^2)+ +2abi-2acj$
VII	$(-a^2+b^2+c^2)- -2abi-2acj$	$a^2+b^2+c^2$	$(a^2-b^2+c^2)+ +2abi$	$(-a^2-b^2+c^2)- -2acj$
VIII	$a^2+b^2+c^2$	$(-a^2+b^2+c^2)+ +2abi+2acj$	$(-a^2-b^2+c^2)+ +2acj$	$(a^2-b^2+c^2)- -2abi$

№	V	VI	VII	VIII
I	$(a^2-b^2+c^2)+ +2abi$	$(-a^2-b^2+c^2)- -2acj$	$(-a^2+b^2+c^2)- -2abi-2acj$	$a^2+b^2+c^2$
II	$(-a^2-b^2+c^2)+ +2acj$	$(a^2-b^2+c^2)- -2abi$	$a^2+b^2+c^2$	$(-a^2+b^2+c^2)+ +2abi+2acj$
III	$(-a^2+b^2+c^2)- -2abi+2acj$	$a^2+b^2+c^2$	$(a^2-b^2+c^2)+ +2abi$	$(-a^2-b^2+c^2)+ +2acj$
IV	$a^2+b^2+c^2$	$(-a^2+b^2+c^2)+ +2abi-2acj$	$(-a^2-b^2+c^2)- -2acj$	$(a^2-b^2+c^2)- -2abi$
V	$(a^2-b^2-c^2)+ +2abi-2acj$	$-a^2-b^2-c^2$	$(-a^2+b^2-c^2)- -2abi$	$(a^2+b^2-c^2)- -2acj$
VI	$-a^2-b^2-c^2$	$(a^2-b^2-c^2)- -2abi+2acj$	$(a^2+b^2-c^2)+ +2acj$	$(-a^2+b^2-c^2)+ +2abi$
VII	$(-a^2+b^2-c^2)- -2abi$	$(a^2+b^2-c^2)+ +2acj$	$(a^2-b^2-c^2)+ +2abi+2acj$	$-a^2-b^2-c^2$
VIII	$(a^2+b^2-c^2)- -2acj$	$(-a^2+b^2-c^2)+ +2abi$	$-a^2-b^2-c^2$	$(a^2-b^2-c^2)- -2abi-2acj$

According to the results of Table 2, the following conclusions can be drawn:

1. The product of a mirror volume number on itself or on the opposite is always a volume number.

2. The product of a volume number with its mirror image of the first and second order is not a volume number.

3. The product of the opposite numbers themselves is equivalent to each other.

Two mirror numbers are called mutually conjugate if their real parts are equal, and imaginary and spatial ones differ only in sign. The product of mutually conjugate mirror numbers is a real number of the form $a^2 + b^2 + c^2$.

Two mirror numbers are called oppositely conjugate if their imaginary and spatial parts are equal, and the real parts differ only in sign. The product of the oppositely conjugate mirror numbers is the real number of the form $-a^2 - b^2 - c^2$.

The sum of products of mutually conjugate and oppositely conjugate numbers forms corresponding cross zero loops

$$(V_I \times V_{II}) + (V_{II} \times V_{VII}) + (V_{VII} \times V_{VIII}) + (V_{VIII} \times V_I) = 0,$$

$$(V_{III} \times V_{IV}) + (V_{IV} \times V_V) + (V_V \times V_{VI}) + (V_{VI} \times V_{III}) = 0.$$

Conclusions

The theoretical foundations of the expansion of a numerical space due to the introduction of the concept of a spatially indeterminate unit whose product by an imaginary unit equals zero ensures the implementation of theall algebraic operations on "three-dimensional" numbers, which are called volumetric ones.

The general algebraic and trigonometric formula of a volume number provides, as in the case of complex numbers, the admissible operations of addition, subtraction, multiplication and division.

For all volume numbers, all the properties of addition and multiplication are satisfied. An exception is the associative property in multiplication. However this property is valid when the condition that the product of the opposite real coefficients are equal for the imaginary and spatial parts of the first and third factors. That is, for the set of real numbers b_1, b_3, c_1, c_3 that form hyperbolic curves in the imaginary-spatial plane, and the corresponding volume numbers formhyperbolic surfaces.

An arbitrary volume number of any octant corresponds to seven mirror numbers, which can be considered as its mirror image. Mirror numbers are those in which the coefficients a, b, c modulo are pairwise equal. According to the geometric interpretation of volume numbers, the mirror image relative to the planes corresponds to the first-order image, and the second-order image relative to the axes.

Mirror numbers have the following basic properties:

- the sum of an arbitrary volume number with its first-order and second-order mirror image is equal to twice the projection of the radius-vector of a given number by the corresponding plane or the coordinate axis;

- the product of a mirror volume number on itself or on the opposite is always a volume number, and with its mirror image of the first and second order are not a volume number.

These numbers can play a big role in algebra and mathematical analysis, providing concrete interpretations in the study of contradictory, and in some cases dual, physico-mathematical aspects of the world. Such studies using volume numbers and their properties are conducted by the authors of this work on the development of mathematical models of the evolutionary processes of the world [8].

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ТЕОРЕТИЧНІ ОСНОВИ РОЗШИРЕННЯ ЧИСЛОВОГО ПРОСТОРУ

Романюк О.Д., Романюк Р.О.

Реферат

Логічний аналіз історії розвитку теорії чисел і особливо векторної алгебри, наштовхував на ідею введення «тримірних» чисел, над якими можна було б безперешкодно і ефективно проводити операції алгебри при дотриманні основних властивостей дійсних і комплексних чисел.

Ближче всіх до вирішення даного завдання підійшов Гамільтон, назвавши відповідні числа кватерніонами, які були «чотирьохмірні» і розширювали числовий простір за умови відповідних правил множення, а також не виконанням властивості комутативності при множенні.

Таким чином, питання про розширення числового простору з використанням «трьохмірних» чисел залишалося відкритим. Для вирішення поставленого завдання було прийнято припущення, що уявні одиниці в теорії чисел які дають можливість скласти математичний запис «тримірного» числа, повинні грати роль деяких операторів перетворення, а не тільки нести сенс відповідних одиниць. І відповідні дії з цими операторами повинні відповідати певним правилам.

Авторами даної роботи для вирішення поставленого завдання вводиться поняття просторово невизначеної одиниці, що дало можливість скласти алгебраїчну і тригонометричну формулу «тримірного» так названого об'ємного числа. Добуток просторово невизначеної одиниці на уявну одиницю дорівнює нулю, що забезпечує виконання алгебраїчних операцій над об'ємними числами з дотриманням всіх властивостей додавання і множення за винятком властивості асоціативності при множенні, проте воно справедливо для певної множини чисел, які утворюють гіперболічні поверхні. Згідно геометричній інтерпретації об'ємні числа, в яких дійсні коефіцієнти по модулю попарно рівні, утворюють безліч дзеркальних чисел, з особливими властивостями.

Представлені теоретичні основи розширення числового простору внаслідок введення поняття просторово невизначеної одиниці, добуток якої на уявну одиницю дорівнює нулю забезпечує виконання всіх операцій алгебри над «тримірними» числами, названими об'ємними, для яких виконано розширений аналіз їх основних властивостей. Дані числа можуть зіграти

велику роль в алгебрі та математичному аналізі, забезпечуючи конкретні інтерпретації при дослідженні суперечливих, а в деяких випадках і двоїстих, фізико-математичних аспектів світу. Такого роду дослідження, з використанням об'ємних чисел і їх властивостей, проводяться авторами даної роботи по розробці математичних моделей еволюційних процесів світу.

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