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GEOMETRICAL MODELING OF A SLEEVE GALAXY FORM BY METHODS OF DESCRIPTIVE GEOMETRY

Розглянуто графічне моделювання рукавів галактик спіралями. Показано, що рукави галактик мають форму логарифмічних спіралей, причому кут між радіус-вектором та дотичною залишається сталим для однієї і тієї ж галактики, але різним для інших галактик.
Ключові слова: *галактики; логарифмічна спіраль; рукави.*

Graphical modeling of the sleeves of galaxies by spirals is considered. It is shown that the sleeves of the galaxies have the form of logarithmic spirals, with the angle between the radius vector and the tangent remains constant for one and the same galaxy, but different for other galaxies.
Keywords: *galaxies; logarithmic spiral; sleeves.*

Formulation of the problem

There are many material objects in our universe, ranging from microscopic particles of matter to large clusters such as galaxies, nebulae, and more. Of these objects, galaxies stand out the most, in that most of them have a distinct helix shape. Each galaxy has two or more arms that diverge from the center. The clear shape of the sleeves allows them to be approximated by smooth geometric curves, in particular helices, which will allow them to understand more the nature of their formation.

Analysis of recent research and publications

In our time, the shape of the Universe has stabilized and the process of accumulation of matter in and around cells is underway [8]. These cells are galaxies in our universe. Many theories of galaxy formation have been cited in the literature [1, 2], but there is no generally accepted theory [3]. In particular, galaxy arms are formed by the emission of matter from the nucleus, with the motion of matter consisting of rotation around the center and radial motion. These two movements affect the matter and it is formed in the form of sleeves [2]. The formation of the sleeves causes the galaxies to have a disco-shaped shape and all the sleeves are in a plane.

With the development of the science of the universe and the mathematics of the theory of formation of the universe increasingly relied on mathematics, which led to its development, but these works [5, 6] increasingly rely on mathematics that requires deep knowledge.

Spiral curves refer to transcendental curves and have many types [7], and the appearance of a spiral type will allow simple methods to get closer to understanding the nature and formation of galaxies.

Formulation of the study purpose

The purpose of the research is to approximate the sleeves of galaxies with transcendental curves - spiral rallies - and to identify the spiral type by the methods of descriptive geometry, in particular graphic constructions on a plane.

Presenting main material

The spiral as a curve line is constructed in the polar coordinate system. For this purpose, a polar axis *P* is drawn from the polar center *O*, from which a radius vector *r* is deposited at a polar angle φ. Thus, each point *A* of the helix is defined by two coordinates a polar angle φ, which is determined in radians, and a radius vector *r*, which is a function of the polar angle, that is, *A (φ, r)*. The shape of the spiral depends entirely on the function of the radius vector $r = r_0(\varphi)$, but regardless of the type of spiral, the initial radius vector r_0 cannot be zero $r_0 \neq 0$, since in this case the existence of the spiral will be impossible.

The method of galaxy research was carried out on the image of a galaxy taken from the Internet, and the image of the galaxy was chosen such that its plane was perpendicular to the line of sight. Then the sleeves were mid-lines. In the nucleus of the galaxy was located the polar center *O* from which the polar axis of *P*. was determined. From the polar center *O* at a constant angle $\Delta\varphi$ several rays (Fig. 1). These rays give points when crossing the midline of the sleeve *A*, *B*, *C*, *D*, …

Points *A*, *B*, *C*, *D*, … united by chords AB, BC, CD, … . Together with the rays, these chords form triangles OAB, OBC, OCD, … . In these triangles are angles $\angle AOB$, $\angle BOC$, $\angle COB$, … equal to each other and equal to the polar angle $\Delta\varphi$, that is $\angle AOB = \angle BOC = \angle COB = ... = \Delta\varphi$. angle $\angle OAB$, $\angle CBO$, $\angle DCO$, … Their average value was measured and calculated. It was found that for the same galaxy these angles are close in magnitude, their deviation from the mean is $\pm 3{,}7^{o}$. The calculations found that the angles $\angle OAB = \angle CBO = \angle DCO...$ are equal. Since the angles $\angle AOC = \angle BOC$, $\angle BOC = \angle COD$, and the corners $\angle OAB = \angle OBC$, $\angle OBC = \angle OCD$ then triangles ΔOAB, ΔBOC, ΔOCD are similar to each other, $\Delta OAB \sim \Delta BOC \sim \Delta DCO$ … . This means that the differences are the radius vectors $\Delta A = OB - OA$, $\Delta B = OC - OB$, $\Delta C = OD - OC$, … has become, that is $\Delta A = \Delta B = \Delta C = ...$. So the point is A, that moves along a radius vector, for example, OA which rotates evenly at a speed proportional to the distance traveled. That is, the polar angle φ is proportional to the logarithm of the distance *OA*, *OB*, *OC*, … that is, logarithms of radius vectors [7]. Thus, the radius vector rotates at an polar angle φ exponentially. This means that as a result, we have a logarithmic spiral.

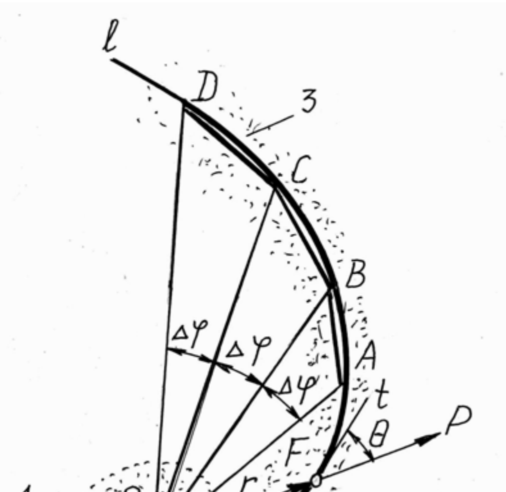


Fig. 1. Scheme of approximation of the galaxy sleeve: 1 — bulge; 2 — core; 3 — sleeve; l is the middle line of the sleeve

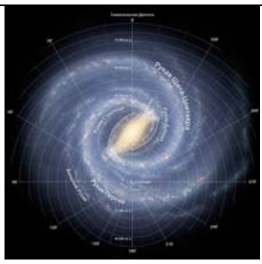
The logarithmic spiral in the polar coordinate system is described by the following equation

$$r = r_0 \cdot e^{w \cdot \varphi}, \tag{1}$$

where r_0 — initial radius vector; w — helix shape factor; φ - the polar angle measured in radians. Coefficient w completely affects the shape of the logarithmic spiral. If $w = 0$, then the spiral turns into a circle, $r = r_0$, if the coefficient is infinite, then the spiral turns into a straight line. In general, the coefficient w is equal to the cotangent of the angle θ between the tangent t at an arbitrary midpoint F and radius vector r, $w = ctg\theta$, (Fig. 1). For the same spiral this angle is constant and does not change at any value of the polar angle φ [4, 7].

To determine the angle θ mid-lines were smoothed in the galaxy's sleeves, smoothed by curves. The angle was measured at five points and then the mean was calculated. Separate data are given in Table. 1.

Table 1. The average value of the angle between the tangent and the radius vector of some galaxies

# S / n	Image galaxies	Name	Direction twisting sleeves	Angle between tangent and radius vector, deg	Coefficient of spiral shape, *w*
1		MN1	Counter-clockwise movement	75º35′	0,257
2		M51	In clockwise direction	72º28′	0,315
3		**NGC 3344**	In clockwise direction	73º45′	0,291
4		M 74	In clockwise direction	68º35′	0,392
5		NGC 1365	Counter-clockwise movement	76º15′	0,243
6		NGC1566	Counter-clockwise movement	69º28′	0,374

Continue of the table 1.

7		M137	In clockwise direction	73º37′	0,294
8		M101	Counter-clockwise movement	77º20′	0,225
9		NGC1753	Counter-clockwise movement	76º50′	0,245
10		M77	Counter-clockwise movement	67º45′	0,408

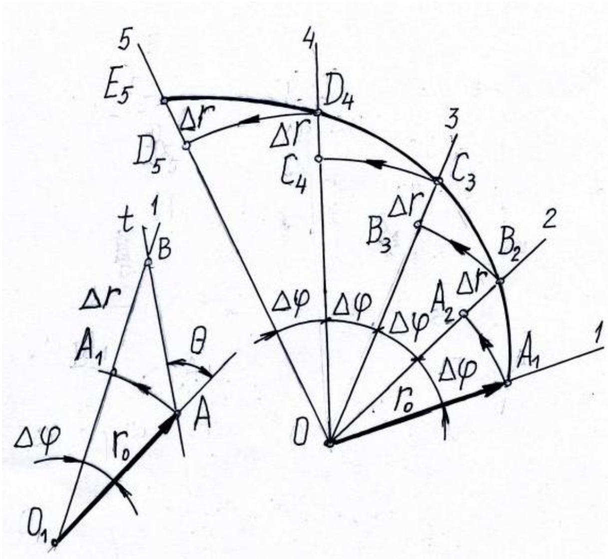
The table shows that the smallest value of the helix parameter w = 0.225 is in the M101 galaxy, which corresponds to the angle θ = 77º20 ′, and the highest w = 0.392 in the M74 galaxy, which corresponds to the angle θ = 68º35 ′. The difference between them is 0.067, or 3º45 ′, which corresponds to 27.3%. Given that the middle lines of the sleeves were constructed graphically, the accuracy can be considered satisfactory.

It should be noted that the number of sleeves ranged from two MN1s, NGC1566 to five NGC1365. The table also shows the direction of spin of the galaxy's sleeves.

The conducted researches have allowed to develop algorithm of construction of logarithmic spirals which has the following points:

1. We define the values of the initial radius vector r_0 and the angle θ between tangent to helix and radius vector.

2. Determine the increment of the radius vector Δr by increasing the polar angle to increase $\Delta\varphi$. To do this, we draw from the polar center O_1 the initial radius vector r_0 (Fig. 2a). From the polar center O_1 under the increase of the polar angle $\Delta\varphi$ we draw a beam $O_1 1$. From the end of the initial radius vector r_0 (point A) at an angle θ we draw a tangent t which intersects with a ray $O_1 1$ giving a point B. Drawing an arc from the polar center O_1 of the circle radius $O_1 A$ determine the point on the ray A_1. The segment $A_1 B$ will be an increment of the radius vector $A_1 B = \Delta r$ when rotated by an angle $\Delta\varphi$.



а *б*

Fig. 2. The scheme to the graphical model of the logarithmic spiral: *a* — the graphical increment of the radius vector; *b* — pobudova spirali

3. To construct a spiral from the polar center O with the interval of the polar angle of $\Delta\varphi$ right-smoke rays, $O1$, $O2$, $O3$, … , $O5$ (Fig. 2b).

4. On the first ray $O1$ we postpone the initial radius vector r_0. From its end (point A_1), the arc of the circle transfers this point to the ray $O2$, and obtains a point A_2. From this point we delay the segment Δr and get the point B_2. The segment OB_2 will be equal to the radius of the vector r_2.

5. We transfer the point B_2 by the arc of the circle to the ray $O3$ and obtain the point B_3 from which we defer the radius vector Δr increment and get the point C_3. The segment OC_3 will be a radius vector r_3.

6. Build until the rays are over. By connecting the points obtained A_1, B_2, C_3, D_4 and E_5 smooth curve we obtain the required logarithmic spiral.

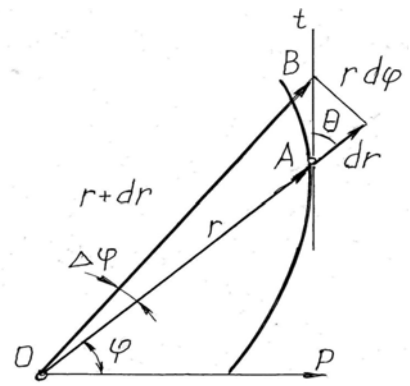


Fig. 3. Scheme for the analytical determination of the angle betweenradius vector and tangent

For analytical verification, we refer to the scheme in Fig. 3. The tangent of the angle θ between the radius vector r and the tangent t will be equal

$$tg\theta = \frac{rd\varphi}{dr}. \qquad (2)$$

From the expression of the logarithmic helix (1) we find the first derivative

$$\frac{dr}{d\varphi} = r_0 we^{w\varphi}.$$

Substituting the resulting expression into the formula for determining the tangent of the angle θ (2) we obtain

$$tg\theta = \frac{r_0 e^{w\varphi}}{r_0 we^{w\varphi}} = \frac{1}{w}.$$

Where will we get the expression for angle θ:

$$\theta = arctg\left(\frac{1}{w}\right).$$

The angle θ values were calculated for all galaxies in Table. 1. The results of the calculations are presented in table. 2.

As can be seen from the table, the calculations obtained the same results as the graphical constructions. This indicates the high accuracy of geometric constructions.

Table 2. Determination of the angle between the radius vector and the tangent

The galaxy number in Table 1	Form factor w	$\frac{1}{w}$	$arctg\left(\frac{1}{w}\right)$	Angle θ, degr
1	0.257	3,891051	1,31924	75,5869
2	0,315	3,174603	1,265636	72,5156
3	0,291	3,436426	1,287617	73,775
4	0,392	2,55102	1,197205	68,5948
5	0,243	4,115226	1,332417	76,3419
6	0,374	2,673797	1,212903	69,4942
7	0,294	3,401361	1,284853	73,6167
8	0.225	4,444444	1,349482	77,3196
9	0,245	4,081633	1,330529	76,2337
10	0,408	2,45098	1,183412	67,8045

Conclusions and prospects for further research

Based on the research we can draw the following conclusions.

1. The development of galaxy arms is subject to the most widespread law in nature — exponential, when growth of some magnitude occurs in proportion to the logarithm.

2. The slight deviation of the angle between the radius vector and the tangent for different spirals indicates that the same elements are present at the base of the galaxies.

3. Since the initial radius vector of the spiral can not be zero, galaxies could not be formed "from nothing", they were formed from the original protomatter.

4. A graphical algorithm for modeling galaxies sleeves by logarithmic spirals is proposed, which can be applied in any field.

The following studies should be conducted in the study of galaxies in which the sleeves break into two or more parts.

References

[1] Ahekian, T. A. (1981). Zviazdy halaktiki, metahalaktika *[Stars of the galaxy, metagalaxy]*. Moscow: Nauka [in Russia].

[2] Borshch, V. L. (2016). Gidrodynamika, kosmogonia I kosmologia v rabotah V. I. Perehresta: ot planetarnuh vihrei do galaktik I temnoi energii *[Hydrodynamics, cosmogony and cosmology in the works of V.I. Perekhresta: from planetary vortexes to galaxies and dark energy]*. Visnuk DNU, Seria "Modeluvannia". – *Bulletin DNU, Seriya "Modeliuvnya", Tom 8(№ 8),* 118-191.

[3] Naginer, D. I. (2001). Elementu kosmologii *[Elements of cosmology]*. Sankt-Pitepbyrg. [in Russia].

[4] Antonovuch, E. A. (Eds). (2004). Narusna geometria. Praktukum *[Descriptive geometry. Workshop]*. Lviv: Svit [inUkrainian].

[5] Perehrest, V. I. (2017) Zastosuvannia vuhrovoi teorii do doslidjennia tvolutsii shvudkostei ta form galaktuk *[Application of the vortex theory to the study of evolution of velocities and forms of galaxies]* Visnuk DNU, Modeluvannia − *Bulletin of DNU, Modeling*, Tom 25 (Vup. 9, № 8), 178-192 [in Ukrainian].

[6] Perehrest, V. I. (2016) Kosmologichni naslidku vuhrovoi gidrodunamiku: vuhrova I temna energiia u kosmosi *[Cosmological effects of vortex hydrodynamics: vortex and dark energy in space]* Visnuk DNU, Modeluvannia − *Bulletin of DNU, Modeling, Tom 24 (Vup. 8, № 8),* 204-221 [in Ukrainian].

[7] Savelov A. A. (1960) Ploskie krivue. Sistematika, svoistva, primeneniia *[Flat curves. Systematics, properties, applications]*. Moscow: Gosudarstvennoe izdatelstvo fiziko-matematicheskoi literature [in Russia].

8. Kormendy J. Kennicutt, Robert C. Secular Evolution and the Formation of Pseudobulges in Disk Galaxies. / J. Kormendy, R. Kennicutt. // Annual Review of Astronomy &Astrophysics. – 2005. – vol. 42. №1. – P. 603–683.

ГЕОМЕТРИЧНЕ МОДЕЛЮВАННЯ ФОРМИ РУКАВІВ ГАЛАКТИК МЕТОДАМИ НАРИСНОЇ ГЕОМЕТРІЇ

Тищенко С.С., Богданов Д.Ю.

Реферат

У статті розглянуто моделювання рукавів галактики логарифмічними спіралями. Для цього центр полярної системи координат був розміщений в центрі галактичного ядра. Рукав галактики був графічно випрямлений середньою вигнутою лінією. Від полярного центру полярні промені проводились під рівними кутами до перетину із середньою лінією рукава. Отримані точки з'єднувалися відрізками. Вимірювання кутів між хордами та полярними променями показали, що ці кути рівні між собою. Оскільки кути між хордами та полярними променями рівні, а полярні кути рівні, трикутники, утворені суміжними полярними променями та хордами, подібні. Це означає, що вектор полярного радіуса пропорційний логарифму полярного кута і підпорядковується найпоширенішому закону в природному експоненціальному. Отже, логарифмічна спіраль знаходиться в основі рукавів галактики. Розглянуто вплив основних параметрів логарифмічної спіралі - початкового радіусного вектора та тангенсу кута між радіусним вектором та дотичною на форму спіралі від кола до прямої.

Встановлено, що кількість галактик, рукави яких закручуються за годинниковою стрілкою, дорівнює кількості галактик, рукави яких закручуються проти годинникової стрілки, що вказує на випадкове скручування рукавів під час формування. Оскільки кут між радіусним вектором і дотичною незначно відрізняється між галактиками, можна зробити висновок, що галактики складаються з одних і тих же елементів.

Запропоновано графічний метод моделювання логарифмічних спіралей, згідно з яким спочатку визначається приріст радіусного вектора, а потім цей приріст послідовно відкладається від радіусного вектора, коли він обертається на постійне збільшення полярного кута. Зростаючі кінці радіусного вектора утворюють логарифмічну спіраль. Оскільки алгоритм заснований на основних параметрах логарифмічної спіралі, він може бути застосований і в інших галузях.

Література

1. Агекян Т. А. Звезды галактики, метагалактика. Москва: Главная редакция физико-математической литературы, 1981. 415 с.
2. Борщ В. Л. Гидродинамика, космогония и космология в работах В.И. Перехреста: от планетарных вихрей до галактик и темной энергии. *Вісник Дніпровського національного університету. Сер. Моделювання.* 2016. Вип. 8. №8. С. 118–191.
3. Нагирнер Д. И. Элементы космологии. Санкт-Петербург, 2001. 55 с.
4. Антонович Є. А. Нарисна геометрія. Практикум. Львів : Світ, 2004. 528 с.
5. Перехрест В. І. Застосування вихрової теорії до дослідження еволюції швидкостей та форм галактик . *Вісник Дніпровського національного університету. Сер. Моделювання.* 2017. Вип. 9. Том 24. №8. С. 178–192.
6. Перехрест В. І. Космологічні наслідки вихрової гідродинаміки: вихрова і темна енергія у космосі / В. І. Перехрест. *Вісник Дніпровського національного університету. Сер. Моделювання.* 2016. Вип. 8. Том 24. №8. С. 204–221.
7. Савелов А. А. Плоские кривые. Систематика, свойства, применения. Москва: Государственное издательство физико-математической литературы, 1960. 293 с.
8. Kormendy J. Kennicutt, Robert C. Secular Evolution and the Formation of Pseudobulges in Disk Galaxies. / J. Kormendy, R. Kennicutt. // Annual Review of Astronomy &Astrophysics. – 2005. – vol. 42. №1. – P. 603–683.