DOI: UDC 621.9.06-52 A. Shumeyko, doctor of Technical Sciences, professor V. Korotkov, candidate of Technical Sciences, docent Dnipro State Technical University, Kamianske

CREATION MODEL FOR THE CNC MACHINE OF TECHNOLOGICAL SYSTEM PLIABILITY PLIABILITG FOR THE CNC MACHINE

The article consider using splines to develop a mathematical model of details shaping surfaces in the cutting process on CNC machines. As a result of workload simulations and obtaining data about technological system pliability, expected processing errors are determined, which can be considered at preparation stage of the control program.

Keywords: Mathematical model; polar splines; system of equations; surface modeling.

В статті розглянутий варіант використання сплайнів для побудови математичної моделі формоутворення поверхонь деталей при обробці різанням на верстатах з ЧПК. За результатами імітації робочих навантажень і отримання даних про податливість технологічної системи визначаються очікувані похибки обробки, які можуть бути ураховані на стадії підготовки керуючої програми.

Ключові слова: математична модель; полярні сплайни; система рівнянь; моделювання поверхні.

Problem's Formulation

A characteristic feature of CNC machines is the performing of all the main and auxiliary movements of the working parts according to pre-prepared program on the set distance. If such movements are carried out by the form-building tool, the corresponding sizes on a detail surface turn out. Many factors prevent accurate shaping, even when accurately recording movements in the control program. For example, the elastic and thermal deformations of the CNC machine technological system elements, actually present during metalworking, and each machine has its own "individual" characteristics. As a rule, in order to establish the laws governing the formation of possible size deviations during processing, it is necessary to carry out many tests of each CNC machine, which is very difficult in the production environment.

In this regard, there is a need to create a reliable model of the behavior of the machine in a real technological process to predict the accuracy and quality of processing.

To create a mathematical model, for example, a metalworking process on CNC machines with a high degree of reliability, it is necessary to conduct a variety of tests simulating real processes, which can be carried out using special devices [1, 2]. For contour milling, the scheme of simulation actions on a forming tool, for example, an end mill, is shown in figure 1, where radiuses $r_1, r_2, ... -$ the deviation values of the center of the tool under the influence of simulation efforts, enclosed at an angle $\Delta \mu_1, \Delta \mu_2, ...$. For a specific simulation effort applied to the tool at various angles, we obtain a radial diagram.

The objective of the research is to establish, with the help of a mathematical model of the flexibility of the CNC machine, the probable deviations of the tool from the programmed value in the entire range of workloads and the directions of their action. And, creation of the model is carried out according to limited experimental research for a particular machine. This approach significantly reduces the complexity of testing and allows you to analyze the processes of formation of processing errors.

The results obtained are based on a completely constructive hypothesis that, within the framework of the machine's operating modes, the processing accuracy changes continuously.

Let's pass to formalization of a task.

Let the values received at some duty of the machine be described by a curve $= r(\mathbf{u})$, set by points of the measurements received as a result of carrying out an experiment (\mathbf{u}_i, r_i) (i = 0, 1, ..., n), at that

$$0 = \mathbf{u}_0 < \mathbf{u}_1 < \mathbf{u}_2 \dots < \mathbf{u}_n < 2\mathbf{p}$$

 $r_i \ge 0$ (i = 0, 1, ..., n), $r_0 = r_n$

and

and

$$\Delta u_i = u_i - u_{i-1}$$
 (*i* = 1,2,...,*n*)



Fig. 1. Diagram of the formation of deviations of the position of the tool under the action of simulation efforts at various angles

Further we will consider a case when

and we assume that for all

$$u_i = \frac{pi}{n} \quad (i = 0, 1, \dots m2n)$$
$$i \in Z$$
$$r_i = r_i + 2n.$$

First, we introduce some notation needed later on.

Analysis of recent research and publications Splines on a uniform grid

Splines are called piecewise polynomial functions with a homogeneous structure [3, 4, 5]. The most common and historically earliest example of a spline is a broken line. Splines have a number of great features that have made them successful in a variety of applications. So, in comparison with the classical apparatus of approximation of functions, splines have better approximative properties.

Spline of minimum defect of an order r by given partition

$$\Delta h = \{ih\} \infty i = -\infty \Delta h = \{ih\}$$

called r - 1 times continuously differentiable function sr(t), which at each interval (ih, (i + 1)h) $(i = 0, \pm 1, \pm 2, ...)$ is a polynomial of degree not higher rr.

Note that on the set of splines of order $r (\ge 0)$ there is a spline identically equal to zero and a spline identically equal to one.

From here and from the fact that the work of a polynomial of degree of r on number σ is a polynomial of the same degree if $\sigma \neq 0$ and the identity zero for $\sigma = 0$, as well as the fact that the sum of polynomials of degree n < m < r u m < rm < r is a polynomial of degree no higher than r, we see that the set of all splines of degree r over the partition Δh is a linear space.

So if $Sr(\Delta h)$ the set of all splines of the minimal defect by partition Δh , and

$$y_{r,i} \in S_r(\Delta h) \ i = 0, \pm 1, \pm 2, ...,$$

then a linear combination of functions $y_{r,i}$ will also lie in the set $S_r(\Delta h)$, i.e.

$$\sum i C_i i y_{r,i} \in S_r(\Delta h). \tag{1}$$

B-splines

B-spline represents a spline with the minimum carrier for the fixed degree of a spline and an order of smoothness. The value of B-splines consists that any spline of the given degree and smoothness can be written down in the form of the linear combination of the corresponding B-splines. That is, the B-spline represents "brick" by means of which it is possible to construct a spline with the given properties [3, 4, 5]. Parabolic B-spline is as follows

$$B_{2,h}(t) = \begin{cases} 0,125\left(3-\frac{2t}{h}\right)^2 \left(t \in \left[\frac{h}{2},\frac{3h}{2}\right]\right) \\ 0,75-0,25\left(\frac{2t}{h}\right)^2 \left(t \in \left[-\frac{h}{2},\frac{h}{2}\right]\right) \\ 0,125\left(3+\frac{2t}{h}\right)^2 \left(t \in \left[-\frac{3h}{2},-\frac{h}{2}\right]\right) \\ 0 (|t| \ge 3h/2) \end{cases}$$

and function graph $B_{2,h}(t)$ has the form



Fig. 2. View of the second-order B-spline

and, accordingly, the parabolic spline of the minimal defect in the partition Δ_h can be written as $S_{2,h}(t) = \sum_{-\infty}^{\infty} c_i B_{2,h}(t - (2i - 1)h/2).$ (2)

It is clear, that for any numbers $\#_i$ narrowing splines (2) per segment [0,T] (where T = nh) there is a spline, in addition, for any particular $t \in (ih, (i_1)/h)$ representation (2) uses only three terms. This allows you to write splines (2) as

$$S_{2,h}(t) = \sum_{-i=-\infty}^{n+1} c_i B_{2,h}(t - (2i - 1)h/2).$$
(3)
And for $t \in (ih, (i + 1)/h)$

$$S_{2,h}(t) = \frac{c_{i+1} + 6c_i + c_{i-1}}{8} + \frac{c_{i+1} - c_{i-1}}{2h} (i - ih) + \frac{c_{i+1} - 2c_i + c_{i-1}}{2h^2} (i - ih)^2.$$
(4)

Let's consider the splines further called — the polar splines offered in work [5]

$$c_{2}(r, \mathbf{u}) = \sum_{i=1}^{2n} \left(r_{i} - \frac{1}{8} \Delta^{2} r_{i} \right) B_{2, \mathbf{p}/n}(\mathbf{u} - \mathbf{u}_{i}).$$
(5)

As already noted, for each value of μ , only three terms in (5) are nonzero. More precisely, if $\mu \in [(i - 0.5)p/n, (i + 0.5)p/n]$, to

$$c_{2}(r, \mathbf{u}) = \left(r_{i-1} - \frac{1}{8}\Delta^{2}r_{i-1}\right)B_{2,p/n}(\mathbf{u} - \mathbf{u}_{i-1}) + \left(r_{i} - \frac{1}{8}\Delta^{2}r_{i}\right)B_{2,p/n}(\mathbf{u} - \mathbf{u}_{i}) + \left(r_{i+1} - \frac{1}{8}\Delta^{2}r_{i+1}\right)B_{2,p/n}(\mathbf{u} - \mathbf{u}_{i+1}).$$

Besides, in work [5], it is shown that for all whole i equalities are carried out $c_2(r, u_i) = r_i - \frac{1}{64}\Delta^4 r_i$.

So splines $c_2(r, \mu)$ almost interpolated (accurate to $O(\Delta^4 r_i)$) input data (μ_i, r_i) $(i \in Z)$. This causes many spline properties $c_2(r, \mu)$. For example, if $r(\mu)$ - three times continuously differentiable 2p - periodic function, then

$$\|r - c_2(r)\|_c \le \frac{7}{96} \left(\frac{p}{n}\right)^3 \|r^{(3)}\|_c$$
$$\|r' - c_2'(r)\|_c \le \frac{1}{12} \left(\frac{p}{n}\right)^3 \|r^{(3)}\|_c$$

Besides,

где
$$||f||_c = max_t |f(t)|.$$

Such results are called a priori or guaranteed accurate estimates for the recovery of smooth curves. They show that splines restore with high accuracy not only the smooth curves themselves, but also their derivatives. From this it is easy to derive asymptotic estimates of the errors in the reconstruction of smooth curves by polar splines.

Let the $r(\mathfrak{u})$ there is three times a periodic function, continuously differentiable on all axis 2p. Then for $\mathfrak{u} \in \left[\frac{(i-0,5)p}{n}, (i+0,5)p/n\right] \operatorname{пpu} n \to \infty$ uniformly across \mathfrak{u} and on *i* ratios are carried out

$$r(\mathbf{u}) - c_2(r, \mathbf{u}) = \frac{1}{24} \left(\frac{p}{n}\right)^3 (1 - \phi^2) \phi r^{(3)}(t) + 0(n^{-4}),$$

$$r'(\mathbf{u}) - c'_2(r, \mathbf{u}) = \frac{1}{24} \left(\frac{p}{n}\right)^2 (1 - 3\phi^2) r^{(3)}(t) + 0(n^{-3}),$$

where $r = (\mathbf{u} - \mathbf{u}_i)/(\mathbf{p}/n)$.

Thus, using the obtained polar almost interpolation spline, it is possible to within $O(n^{-3})$ get any intermediate value of the machine operating mode.

Our further discussion is devoted to the study of those values for which there are no experimental data.

The idea of the proposed design is as follows. We have a data set (radial diagrams) for each experiment. For each set, we construct, using the proposed method, a polar spline. A lot of polar splines are "frames", on which we pull the surface, which, in fact, represents all the possible modes of the CNC machine, lying in the framework of the experiments. To construct this surface, we will use the tensor product of second-order splines — biquadratic splines.

Tensor product of spaces

Let *U*- linear space of functions defined on some set *X* with values in the domain of real numbers and *V*- linear space of functions defined on some set *Y* with values in the space \mathbb{R} . \mathbb{P} For any functions $u \in U \bowtie v \in V$ ratio

$$w(x, y) = u(x)v(y)$$

For all $(o, y) \in X \forall Y$ defines a function on a plane $X \forall Y$, which is called the tensor product of functions u and v and is indicated $u \otimes v$.

The set of all finite linear combinations of functions on the plane $X \Psi Y$ shape $u \otimes v$ called tensor product U and $u \otimes v$ and $u \otimes v$. In this way

$$U \otimes V = \left\{ \sum_{i=1}^{n} c_i (u_i \otimes v_i) : c_i \in R, u_i \in U, v \in V, i = 1, \dots, n \right\}$$

and $U \otimes V$ is a linear space defined on a plane $X \times Y$.

Let $U = P_N$ — space of algebraic polynomials of order N defined on the axis of real numbers $X = R \ H \ U = P_M$ — space of algebraic polynomials of order M on = R. Then $U \otimes V$ this is the linear space $P_{N,M}$, whose elements are polynomials of two variables, of degree not higher than N in the first variable and not higher than M in the second, whose domain is the plane $X \Psi Y = R^2$.

Presenting main material

Main result

Consider the surface description by the tensor product of B-splines defined on a square lattice ih = (ih, jh), where $i \in Z^2$.

The biquadratic spline of the minimal defect s with nodes at points (ih, jh) is called interpolation for functions f with bounded values $f_{i+\frac{1}{2},j+1/2}$ in points ((i + 1/2)h, (j + 1/2)h), if

$$f_{i+\frac{1}{2},j+\frac{1}{2}} - s_{i+\frac{1}{2},j+\frac{1}{2}} = 0.$$

Surface $S_{r,s}(\boldsymbol{u})$, described by the tensor product of B-splines has the form

 $\boldsymbol{S}_{r,s}(\boldsymbol{P},\boldsymbol{u}) = \sum_{i \in Z^2} \boldsymbol{P}_i N_{r,s}(\boldsymbol{u} - \boldsymbol{i}/h),$

where $\boldsymbol{u} = (u, v) \in R^2$ and

 $N_{r,s}(\boldsymbol{u}-i\boldsymbol{h}) \equiv B_r(\boldsymbol{u}-i\boldsymbol{h})B_s(\boldsymbol{v}-j\boldsymbol{h})$

normalized tensor product of B-splines of order rs with a lattice with nodes

$$i = (i, j)$$
.

Consider biquadratic splines $S_{2,2}(P, u)$. Function graph $N_{2,2}(u)$ looks as follows



Fig. 3. View of the biquadratic spline $S_{2,2}(P, u)$

Using the type of parabolic B-spline, for $\mathbf{u} \in [ih, jh] \Psi[(i+1)h, (j+1)h]$ get $S_{2,2}(\mathbf{P}, \mathbf{u}) = \sum_{i \in \mathbb{Z}^2} \mathbf{P}_i N_{2,2}(u-ih) = \mathbf{P}_{i,j} \frac{1+2\frac{u-ih}{h}-2\left(\frac{u-ih}{h}\right)^2}{2} \cdot \frac{1+2\frac{v-jh}{h}-2\left(\frac{v-jh}{h}\right)^2}{2} + \mathbf{P}_{i+1,j} \frac{\left(1-\frac{u-ih}{h}\right)^2}{2} \cdot \frac{1+2\frac{v-jh}{h}-2\left(\frac{v-jh}{h}\right)^2}{2} + \mathbf{P}_{i+1,j} \frac{\left(1-\frac{v-jh}{h}\right)^2}{2} \cdot \frac{1+2\frac{v-jh}{h}-2\left(\frac{v-jh}{h}\right)^2}{2} + \mathbf{P}_{i+1,j-1} \frac{\left(1-\frac{v-jh}{h}\right)^2}{2} + \mathbf{P}_{i+1,j-1} \frac{\left(1-\frac{v-jh}{h}\right)^2}{2} + \mathbf{P}_{i+1,j-1} \frac{\left(1-\frac{v-jh}{h}\right)^2}{2} + \mathbf{P}_{i+1,j+1} \frac{\left(1-\frac{v-jh}{h}\right)^2}{2} + \mathbf{P}_{i+1,j+1} \frac{\left(\frac{u-ih}{h}\right)^2}{2} \cdot \frac{\left(\frac{v-jh}{h}\right)^2}{2} + \mathbf{P}_{i+1,j+1} \frac{\left(\frac{u-ih}{h}\right)^2}{2} + \mathbf{P}_{i+1,j+1} \frac{\left(\frac{u$

In matrix form, it will have the following form:

$$\boldsymbol{S}_{2,2}(\boldsymbol{P},\boldsymbol{u}) = \left[1 \ \frac{u-ih}{h} \ \left(\frac{u-ih}{h}\right)^2\right] \ \boldsymbol{M}_{2,2} \ \boldsymbol{P}_{2,2} \ \boldsymbol{M}_{2,2}^T \left[\frac{v-jh}{h} \\ \left(\frac{v-jh}{h}\right)^2\right],$$

1

where $M_{2,2}$ — matrix size 3×3

$$M_{2,2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 \\ -2 & 2 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

and

$$P_{2,2} = \begin{bmatrix} \boldsymbol{P}_{i-1,j-1} & \boldsymbol{P}_{i,j-1} & \boldsymbol{P}_{i+1,j-1} \\ \boldsymbol{P}_{i-1,j} & \boldsymbol{P}_{i,j} & \boldsymbol{P}_{i+1,j} \\ \boldsymbol{P}_{i-1,j+1} & \boldsymbol{P}_{i,j+1} & \boldsymbol{P}_{i+1,j+1} \end{bmatrix}.$$

From here we immediately get the values at the point
$$\left(\left(i+\frac{1}{2}\right)h, \left(j+\frac{1}{2}\right)h\right)$$
:
 $S_{2,2}(P, ih) = \frac{9}{16}P_{i,j} + \frac{3}{32}\left(P_{i,j+1} + P_{i,j-1} + P_{i+1,j} + P_{i-1,j}\right) + \frac{1}{64}\left(P_{i-1,j+1} + P_{i-1,j-1} + P_{i+1,j+1} + P_{i+1,j-1}\right) = P_{i,j} - \frac{1}{64}\Delta P_{i,j}$, (6)

where

 $\Delta P_{i,j} = 28P_{i,j} - 6(P_{i,j+1} + P_{i,j} + P_{i+1,j} + P_{i-1,j}) - (P_{i+1,j+1} + P_{i+1,j-1} + P_{i-1,j-1} + P_{i-1,j+1}).$ Let the surface **f** be given by the points

$$f_{i+1/2} = f\left(\left(i+\frac{1}{2}\right)h, \left(j+\frac{1}{2}\right)h\right), \quad (i,j=0,\pm 1,\pm 2,\dots)$$

and

$$\boldsymbol{P}_{i}^{1} = \boldsymbol{P}_{i}^{1}(\boldsymbol{f}) - \boldsymbol{f}_{i+\frac{1}{2}} + \frac{1}{64}\Delta\boldsymbol{f}_{i+\frac{1}{2'}} = \frac{23}{16}\boldsymbol{f}_{i+\frac{1}{2'}j+\frac{1}{2}} - \frac{3}{32}\left(\boldsymbol{f}_{i+\frac{1}{2'}j+\frac{3}{2}} + \boldsymbol{f}_{i+\frac{1}{2}j-\frac{1}{2}} + \boldsymbol{f}_{i+\frac{3}{2'}j+\frac{1}{2}} + \boldsymbol{f}_{i-\frac{1}{2'}j+\frac{1}{2}}\right) - \frac{1}{64}\left(\boldsymbol{f}_{i-\frac{1}{2'}j+3/2} + \boldsymbol{f}_{i-\frac{1}{2'}j-1/2} + \boldsymbol{f}_{i+\frac{3}{2'}j-1/2} + \boldsymbol{f}_{i+\frac{3}{2'}j+3/2}\right), \tag{7}$$

The corresponding spline is denoted by

$$S_{2,2}^{1}(\boldsymbol{P}, \boldsymbol{u}) = \sum_{i \in Z^{2}} \boldsymbol{P}_{i}^{1} N_{2,2}(\boldsymbol{u} - ih).$$
(8)

when

$$S_{2,2}^{1}\left(f,\left(i+\frac{1}{2}\right)h\right) - f_{i+\frac{1}{2}} + \frac{1}{4096}\Delta^{2}f_{i+1/2}$$

If the function f(u) has all continuous derivatives up to fourth order inclusive, then there exists a point $\mathbf{u}_0 \in [ih, jh] \mathbb{Y}[(i+1)h, (j+1)h]$ such that

$$S_{2,2}^{1}\left(\boldsymbol{f}, \left(\boldsymbol{i} + \frac{1}{2}\right)\boldsymbol{h}\right) = f_{\boldsymbol{i}+1/2} + \frac{3}{8}\boldsymbol{h}^{4}\left(\frac{\partial^{4}f}{\partial u^{4}}\Big|_{u_{0}} + 2\frac{\partial^{4}f}{\partial u^{2}\partial v^{2}}\Big|_{u_{0}} + \frac{\partial^{4}f}{\partial v^{4}}\Big|_{u_{0}}\right) \quad . \tag{9}$$

In work [6] it is shown that if $f(x, y) \in C^{6}(\mathbb{R}^{2})$, what

$$f_{i,j} - S_{i,j} = \frac{h^4}{128} \left(\frac{\partial^4 f}{\partial x^4} \Big|_{(ih,jh)} + \frac{\partial^4 f}{\partial y^4} \Big|_{(ih,jh)} \right) + O(h^6), h \to 0.$$
⁽¹⁰⁾

That is, the spline $S_{2,2}^1$ asymptotically coincides with the interpolation biquadratic spline S.

We examined biquadratic splines defined on the entire plane R^2 . Let $\Omega = [0, Hh] \Psi[0, Wh]$ rectangle with square step grid h.

Then, to obtain an interpolation biquadratic spline, it is necessary to solve a system of HYW linear equations. From relations (9) μ (10) it is clear, that splines $S_{2,2}^1$ asymptotically coincide with interpolation and without significant loss of accuracy, instead of interpolation splines, we can use splines $S_{2,2}^1$.



Fig. 4. Spline-surface, simulating these CNC machine operating modes

Conclusions

Using the obtained simulation results at the stage of preparing control programs for CNC machines allows you to compare possible processing options to determine the most efficient. Modeling of processes is possible when using CAM systems using on PC with average productivity.

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ПОБУДОВА МОДЕЛІ ПОДАТЛИВОСТІ ТЕХНОЛОГІЧНОЇ СИСТЕМИ ВЕРСТАТА З ЧПК Шумейко А.А., Коротков В.С.

Реферат

При виготовленні деталей на верстатах з ЧПК виникають пружні деформації технологічної системи, які впливають на точність обробки. Величина похибок, що виникають при цьому, залежить від режимів обробки і технічного стану технологічної системи верстата. Визначення можливих відхилень розмірів від дії сил різання створює можливість вносити в керуючу програму обробки корекції для компенсації похибок.

Метою роботи є створення методики розрахунку очікуваних похибок руху формоутворюючого інструменту на верстатах з ЧПК при дії сил різання у всьому діапазоні робочих навантажень.

Для досягнення поставленої мети необхідно створити математичну модель процесу формоутворення при обробці на верстатах з ЧПК, яка дозволяє отримати необхідну інформацію для точного розрахунку траєкторії руху інструментів при заданих режимах обробки.

Для здійснення прогнозу точності обробки при використанні сплайнів запропонована математична модель формування розмірів деталей при обробці різанням. За результатами імітації робочих навантажень на верстаті з ЧПК і отримання даних про податливість технологічної системи проводиться моделювання обробки з визначенням величини очікуваних похибок.

Використання отриманих результатів моделювання на етапі підготовки керуючих програм для верстатів з ЧПУ дозволяє порівнювати можливі варіанти процесу обробки і визначати більш ефективний. Моделювання процесів можливо при використанні систем автоматизованого проектування з комп'ютерами середньої продуктивності.

В умовах реального виробництва застосування розроблених методів і моделей дозволить підвищити точність обробки на верстатах з ЧПК і підвищити ефективність виробництва в цілому.

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