DETERMINATION OF REGULARITIES OF CHANGE OF PASSENGER FLOWS DURING THE EXISTING ROUTE NETWORK OF BUSES ON CITY ROUTES

The paper establishes the relationship between the values of passenger traffic, taking into account the non-stationarity of the hours of the day with the trigonometric Fourier series, the coefficients of which have their own values for each city. The change in the values of the demand for transportation by days of the week is approximated in comparison with the polynomial Fourier polynomial of a function of the corresponding degree. The article proves that the change in the value of passenger traffic by hours of the day, days of the week and months of the year is a typical example of a dynamic Fourier time series.

Key words: passenger traffic; function; optimal; transportation; research; statistics; system.

Problem’s Formulation

Surveys are conducted to identify passenger flows, distribute them by directions, and collect data on changes in passenger flows over time.

Work on the survey of passenger flows in any way and regardless of the duration and breadth of coverage should be carried out according to a pre-drawn up and approved plan. The plan is developed taking into account specific conditions and should be realistic in terms of execution, volume of work and number of executors. The plan usually consists of three parts: preparation for the survey; work on the survey and statistical processing of the collected information.

The results of surveys of passenger flows are used both to improve the organization of passenger traffic on existing routes, and to reorganize the transport network as a whole. By means of inspections it is possible to establish also the basic technical and operational indicators of work of buses: volume of transportations, passenger turnover, average distance of travel of passengers, filling of buses and their number on routes, flight time and number of changes of work, speed, intervals and frequency of movement dress time. These data serve as a basis for improving both the route system and the organization of traffic and operation of buses in general.

Analysis of recent research and publications

The most common means of determining passenger traffic at present in the practice of transport organizations are field surveys. According to the means of the survey are divided into continuous and selective. Each of these surveys can be conducted by several methods: tabular, silhouette, questionnaire [1, 2].

Full scale survey methods have high accuracy (error is about 5 % [3]), but have significant drawbacks. First, they require large expenditures of money and human resources to carry them out. Secondly, as a rule, they require a lot of time to process the results, as a result, these survey results appear with a delay and do not carry reliable information about the actual passenger traffic. In addi-
tion, as a result of the survey, you can get passenger traffic only for the existing route network. The entropic approach to solving transport problems was used by Wilson and was often used in modeling the choice when solving transport problems (choice of destination, mode of transport, route of destination) [4]. Zabolotsky G. pays special attention to methods of forecasting passenger flows using extrapolation methods [5].

According to a study by Arrak A. [6] there is a problem of evaluating the work of passenger transport in economic and social aspects and their coordination, as factors of economic and social efficiency change at different rates and in different directions. In other words, solutions that are effective in the economic sense can have a negative impact on social aspects, namely, increasing transport fatigue and reducing the quality of transportation.

**Formulation of the study purpose**

The aim of the work is theoretical statistical studies of the formation of passenger traffic using trigonometric Fourier series.

**Presenting main material**

Uneven distribution of passenger flows over time has a great impact on the organization of passenger traffic and increase the efficiency of passenger transport. Of greatest interest are the fluctuations in the hours of the day, because data on the size and nature of hourly flows are the basis for choosing the effective type of rolling stock and its number; calculation of indicators, characterizing the movement of buses; drawing up a traffic schedule; organization of effective work schedules of bus crews. Fluctuations in passenger traffic by hours of the day are associated with the mode of operation of enterprises and organizations, educational institutions, household organizations. Significant morning and evening passenger tensions are created by the travel of the population between industrial areas and residential areas in this period of time. There are two peak periods on weekdays. The first (morning) is characterized by the greatest duration (1.5—2 hours) and high intensity. The second (evening) is less stressful and longer in time. During peak periods, with insufficient capacity on the route, there is an overcrowding of buses. In this case, the filling factor reaches 1.2, which reduces the quality of passenger traffic.

In the off-peak period there is a significant decline in passenger traffic. At this time, business and cultural trips of the population predominate. The off-peak time without taking appropriate measures causes a decrease in the efficiency of bus use, a significant increase in the intervals of their movement and, as a consequence, an increase in waiting time for the passenger to board and, accordingly, the duration of the trip.

Another situation is observed on weekends and holidays, when there is a gradual increase in passenger traffic to 11—12 o’clock in the afternoon and then a gradual decrease.

The formation of passenger flows is reproduced under the complex influence of many different factors, the degree of influence of which is different [7]. Various economic and mathematical methods are used to identify the degree of influence of both individual factors and their combination on passenger traffic. The main method of studying trends in passenger transport is forecasting. It is the main means of substantiating long-term plans, and the accuracy of forecasts determines the reality of planning decisions that are made. Correlation modeling is best suited for creating multifactor models of passenger traffic formation.

Fluctuations in passenger traffic are random but natural. Changing the amount of passenger traffic by hours of the day, days of the week and months (seasons) of the year is a typical example of a time series. The change in the values of passenger traffic, taking into account the non-stationarity by hours of the day and months of the year, in the general case can be described by a trigonometric Fourier series, the coefficients of which have their own values for each city. The change in the values of the demand for transportation by days of the week is well approximated in comparison with the polynomial Fourier polynomial of a function of the corresponding degree.

For an existing route network, the value of demand for transportation per unit time of 1 hour is described by the following expression, which connects the factor (time) and the dependent variable:

\[
Z(t) = Z_o + Z_o(t) + Z_m(t) + Z_o(t) \tag{1}
\]
where \( Z_o \) — the average annual value of demand for transportation per unit time; \( Z_d(t), Z_w(t), Z_m(t) \) — respectively daily, weekly and seasonal fluctuations in demand values, which are determined by following the equations

\[
Z_d(t) = \sum_{i=1}^{h_2} (\beta_{2,i} \cdot \left(7 \cdot \left(\frac{t}{168}\right)\right)^i), \quad (3)
\]

\[
Z_m(t) = \sum_{i=1}^{h_3} (\beta_{3,i} \cdot \left(2184 \cdot \left(\frac{t}{2184}\right)\right)^i), \quad (4)
\]

Substituting equations (2)—(4) in (1) we obtain the expression

\[
Z(t) = Z_o + \sum_{i=1}^{h_1} (\beta_{1,i} \cdot \sin \left(\frac{2\pi \cdot it}{18}\right) + \alpha_{1,i} \cdot \cos \left(\frac{2\pi \cdot it}{18}\right) + \sum_{i=1}^{h_2} (\beta_{2,i} \cdot \left(7 \cdot \left(\frac{t}{168}\right)\right)^i) + \sum_{i=1}^{h_3} (\beta_{3,i} \cdot \sin \left(\frac{2\pi \cdot it}{2184}\right) + \alpha_{3,i} \cdot \cos \left(\frac{2\pi \cdot it}{2184}\right)).
\]

where \( \beta_{1,i}, \beta_{3,i}, \alpha_{1,i}, \alpha_{3,i} \) — Fourier polynomial coefficients; \( \beta_{2,i} \) — the coefficient of the power polynomial of the \( i \)-th degree; \( h_{1,3} \) — Fourier polynomial order; \( h_2 \) — order of a power polynomial; \( t \) — the current value of the calendar time from the beginning of the year in hours; 24, 168, 2184 — periods of fluctuations in demand for transportation, respectively, daily, weekly and seasonal.

The constant coefficients of the series, which are obtained in statistical analysis, reflect a set of factors and the degree of their influence on the magnitude and nature of changes in passenger traffic at a particular time. Checking the adequacy of the equation with research data is reproduced by Fisher's test. Using the proposed dependence, you can predict the magnitude of passenger traffic at a particular time, which allows you to make an adequate decision.

The parameters (coefficients) of the equations are determined by the following dependencies

\[
a_o = \frac{1}{m} \sum_{i=1}^{m} y_{ei};
\]

\[
a_k = \frac{2}{m} \sum_{i=1}^{m} y_{ei} \cos \left(\frac{2\pi \cdot it}{m}\right);
\]

\[
b_k = \frac{2}{m} \sum_{i=1}^{m} y_{ei} \sin \left(\frac{2\pi \cdot it}{m}\right).
\]

where \( y_{ei} \) — experimental values of the dependent variable at the \( i \)-th calculation points.

Verification of the adequacy of the equation of the polynomial of the Fourier series by research data is performed according to Fisher's test. In this case, when calculating the number of degrees of freedom, the number of factors means the number of used Fourier series harmonics.

The measure of consistency can also be the coefficient of the average linear error of approximation \( E \), which is determined by the equation

\[
E = \sqrt{\frac{1}{m} \sum_{i=1}^{m} \left(\frac{y_{ei} - y_{mi}}{y_{mi}}\right)^2},
\]

where \( y_{mi} \) — theoretical values of the dependent variable at the \( i \)-x calculation points; \( m \) — an array of calculated values.

When calculating the numbers of harmonics included in the equation, it is recommended to take them adaptively according to the maximum value of the Fisher criterion \( F \) statistics or the minimum coefficient of the average linear error of approximation \( E \). Harmonics that cause a decrease in \( F \) or an increase in \( E \) are not taken into account.

The study of statistical dependences is based on correlation-regression analysis, which allows to answer the question of the existence of a relationship between random variables, as well as to assess the degree of density of statistical dependence. The tool of regression analysis is the regression equation. The input data for correlation-regression analysis is statistical information that contains the values of factors and their dependent parameter [8, 9].

One of the possible schemes for correlation-regression analysis is the following:

1) a mutual pairwise correlation analysis between all possible factors and duplicating factors are excluded (from duplicating factors for further calculations, one of them is excluded — dependent);
2) the type of regression equation (communication model) is accepted;
3) the parameters of the regression equation are calculated;
4) the significance of individual factors in the model and the adequacy of the regression equation with the experimental data as a whole are checked. If there are no insignificant factors and the regression equation agrees with the research data — the solution is obtained;

5) insignificant factors are rejected and new calculations are made.

The resulting regression equation is a model of the relationship between the factor space and the dependent parameter. If the relationship is insignificant, the calculations are either repeated with another type of regression equation or stopped.

The statistics that characterize the density of the relationship between the factors and the dependent variable is the correlation coefficient of the set, which shows what part of the variance of the dependent variable is explained by the accepted regression model, and is determined by the equation

\[ R = \sqrt{\frac{s_{\text{obs}}^2}{s_{\text{total}}^2}}, \]  

(10)

where \( s_{\text{obs}}^2 = \sum_{m=1}^{m} (y_{m} - a_0)^2 \) — explanatory sum of squares of deviations from estimation of mathematical expectation (\( m \) — number of experiments); \( s_{\text{total}}^2 = \sum_{i=1}^{m} (y_{ei} - a_0)^2 \) — the total sum of the squares of the deviations from the estimate of mathematical expectation; \( a_0 \) — estimation of mathematical expectation of a random variable.

The difference between the total and the explained sum of squares is the residual (unexplained) sum of deviations from the estimate of mathematical expectation and is determined by the equation

\[ s_{\text{res}}^2 = s_{\text{total}}^2 - s_{\text{obs}}^2 = \sum_{i=1}^{m} (y_{ei} - y_{mi})^2. \]  

(11)

Then through \( s_{\text{res}}^2 \) the value of the correlation coefficient of the set is calculated by the formula:

\[ R = \sqrt{1 - \frac{s_{\text{res}}^2}{s_{\text{total}}^2}}. \]  

(12)

The value of \( R \) may be in the range from 0 to 1.0. At \( R = 0 \) there is no relationship between the factors and the dependent variable, and at \( R = 1.0 \) — there is a functional relationship.

To test the hypothesis of the significance of the correlation coefficient of the set and the consistency of the regression equation with the research data, the statistics of Fisher's criterion for the following equations are used.

\[ F = \frac{s_{\text{obs}}^2/n}{s_{\text{res}}^2/(m-n-1)} = \frac{s_{\text{obs}}^2(m-n-1)}{s_{\text{res}}^2n}. \]  

(13)

or

\[ F = \frac{R^2(m-n-1)}{(1-R^2)n}, \]  

(14)

where \( s_{\text{res}}^2 \) and \( s_{\text{obs}}^2 \) — the residual variance for the dependent parameter is explained accordingly.

In order not to reject the hypothesis that the research data are consistent with the obtained regression equation, the calculated statistics of the Fisher test should be greater than the tabular value (\( F > F_t \)). The tabular value of \( F_t \) is determined depending on the level of significance and the number of degrees of freedom \( k_1 = n \) and \( k_2 = m - n - 1 \) (\( n \) is the number of factors).

If \( F < F_t \), it is taken into account that the regression equation does not agree with the research data.

Fisher's test statistics can be used to assess the significance of individual factors. The factor is insignificant if its exclusion from the model does not cause a significant decrease in the statistics of the Fisher test. Thus exclusion of an insignificant factor can provide increase in statistics \( F \) [10].

For the existing route network, we determine the value of the demand for transportation using the Fourier polynomial. For calculation we choose routes № 2а, 4, 6, 14, 23, and 23а as on them the most significant passenger flows.

The share of passenger traffic by days of the week compared to Wednesday, shown in Fig. 1. Here is an example of the calculation of the Fourier polynomial for the calculation of hourly passenger traffic on the route № 4 "Railway station — Blvd. Builders" in February, the day of the week — Wednesday.
The parameters (coefficients) of the Fourier polynomial are calculated by formulas (6)—(8): when $m = 18$, $k = 9$ we obtain:

$$a_0 = \frac{1}{18} \sum_{i=1}^{18} y_{ei};$$

$$a_k = \frac{2}{18} \sum_{i=1}^{18} (y_{ei} \cdot \cos(\frac{2 \cdot \pi \cdot k \cdot i}{18}));$$

$$b_k = \frac{2}{18} \sum_{i=1}^{18} (y_{ei} \cdot \sin(\frac{2 \pi k \cdot i}{18})).$$

Thus, we obtain the Fourier polynomial for the calculation of hourly passenger flows on the route № 4:

$$y_{mi} = 1003.78 + (-1148.732 \cdot \cos(\frac{2 \pi \cdot 5 \cdot i}{18}) + 310.283 \cdot \sin(\frac{2 \pi \cdot 5 \cdot i}{18})),$$

(15)

where $i$ — ordinal year of age.

The calculation of the parameters and criteria of the Fourier polynomial for other routes under study is performed in a similar way.

The results of calculations of theoretical values of hourly passenger flows on all routes under study are shown in tabl. 1.

**Table 1.** Theoretical values of hourly passenger flows on routes

<table>
<thead>
<tr>
<th>Time of day</th>
<th>Route number</th>
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<td>4</td>
<td>6</td>
<td>14</td>
<td>23</td>
<td>23</td>
<td>2</td>
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<td>6-8</td>
<td>339</td>
<td>546</td>
<td>432</td>
<td>388</td>
<td>412</td>
<td>401</td>
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<td>8-10</td>
<td>732</td>
<td>739</td>
<td>816</td>
<td>954</td>
<td>998</td>
<td>880</td>
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<td>10-12</td>
<td>954</td>
<td>847</td>
<td>365</td>
<td>882</td>
<td>1001</td>
<td>990</td>
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<tr>
<td>12-13</td>
<td>188</td>
<td>692</td>
<td>283</td>
<td>426</td>
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<td>246</td>
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<tr>
<td>13-14</td>
<td>214</td>
<td>189</td>
<td>312</td>
<td>249</td>
<td>406</td>
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<td>14-15</td>
<td>354</td>
<td>489</td>
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<td>247</td>
<td>357</td>
<td>204</td>
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<td>15-16</td>
<td>735</td>
<td>694</td>
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<td>501</td>
<td>892</td>
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<td>16-18</td>
<td>892</td>
<td>687</td>
<td>532</td>
<td>774</td>
<td>899</td>
<td>861</td>
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<td>18-20</td>
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<td>825</td>
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<td>812</td>
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<td>659</td>
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<tr>
<td>20-21</td>
<td>309</td>
<td>441</td>
<td>302</td>
<td>456</td>
<td>609</td>
<td>448</td>
</tr>
</tbody>
</table>
Conclusions

By carrying out simple transformations it is also possible to calculate theoretical values of hourly passenger flows on all routes of the city as fluctuations of passenger flows have casual, but natural character. The change in the value of passenger traffic by hours of the day, days of the week and months (seasons) of the year is a typical example of a dynamic Fourier time series.

References

Автори на підставі оцінювання функціювання системи пасажирського громадського автотранспорту запропонували методику по вдосконаленню провізних можливостей маршрутної мережі, яка існує у місті Кам'янське, міських автобусів загального користування.

Виведені авторами залежності представляють собою зв'язок між факторним простором і декількома змінними і залежними параметрами. Статистикою, яка характеризує щільність зв'язку між факторами і залежною зміною, являється коефіцієнт кореляції множини прийнятої регресійної моделі.

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