DOI: 10.31319/2519-8106.2(47)2022.268411
УДК 629.114.2.02.001.5
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# MATHEMATICAL MODEL OF MOVEMENT OF A WHEELED TRACTOR WITH ITS VERTICAL AND HORIZONTAL OSCILLATIONS INCLUDED

*This article provides a mathematical model of the operation of a wheeled tractor with a steady uneven load on the hook when driving over bumps. At the same time, the machine-tractor unit is presented as a dynamic system with two input actions determined by the load on the hook and terrain irregularities, and one output coordinate determined by the amplitude of the oscillations of the engine crankshaft speed. In this case, the dynamic and structural diagrams serve as the basis for compiling a mathematical model.*
***Keywords****: dynamics, wheeled tractor, vibrations, unevenness of the supporting surface, crankshaft speed.*

*В настоящей статье приводится математическая модель процесса работы колесного трактора с установившейся неравномерной нагрузкой на крюке при езде по неровностям. При этом машинно-тракторный агрегат представлен в виде динамической системы с двумя входными воздействиями, определяемыми нагрузкой на крюке и неровностями рельефа, и одной выходной координатой, определяемой амплитудой колебаний частоты вращения коленчатого вала двигателя. В этом случае основой для составления математической модели служит динамическая и структурная схемы.*
***Ключевые слова****: динамика, колесный трактор, колебания, неровности опорной поверхности, частота вращения коленчатого вала.*

## Formulation of the problem

The operation of the machine-tractor unit during various agricultural operations is accompanied by continuous changes in the load, which causes a change in the crankshaft speed and, consequently, the engine output. In addition, the random polyharmonic nature of the load leads to the occurrence of local resonant oscillations in such a multi-link dynamic system as a machine-tractor unit, which further increases the amplitude of oscillations of the crankshaft speed, and, as a result, leads to an increase in power losses, potentially available power plant of the tractor.

## Analysis of recent achievements and publications

The problems of improving the performance properties of tractors are one of the main ones in mechanical engineering. Their solution is carried out according to various corrections: an increase in productivity [1, 2], an increase in technical and economic indicators and an improvement in environmental performance and reliability [3, 4], the use of new fuels and types of engines, a decrease in energy losses in various units (rolling resistance of wheels, transmission losses, etc.) [5—7], improvement of automation and optimization of the control of the working processes of units [8, 9], and a number of other areas that require both theoretical and experimental research.

The tractor is part of the "car-driver-environment" system (here, "environment" also means the supporting surface along which the machine moves), and its properties are manifested in interaction with the elements of this system. Therefore, the significance of a certain operational property in assessing the efficiency of using a tractor depends on the conditions under which this property manifests itself, i.e., on the operating conditions.

All operational properties are closely related to each other, and a change in the design parameters of the tractor, undertaken to improve one of the properties, inevitably affects the rest [1—8]. Traction and speed properties determine the maximum traction force on the tractor hook, but it can also be limited due to insufficient ride smoothness. Therefore, the final assessment of the tractor is made taking into account the entire range of operational properties.

The smoothness of the tractor during its movement on roads with uneven surfaces depends on the layout characteristics, suspension parameters and tires [2, 3]. At the same time, it should be noted that these parameters, in addition to running smoothness, also significantly affect the operation of the tractor engine, exercising a polyharmonic effect on its crankshaft, and reducing the energy performance of the entire tractor against the background of increased fuel consumption [1, 5, 6].

**Research goal statement**

The machine-tractor unit is usually considered as an automatic control system, where the fluctuation of the moment of resistance on the motor shaft ( $\Delta M_{\varphi}$ ).is taken as a single input signal. It takes into account all external disturbances applied to the tractor, as well as the characteristics of the systems of the tractor itself, and determines the speed mode of the engine (and hence the tractor). However, studies of the mathematical model created on this basis cannot explain how individual components and parameters of various tractor systems affect the engine dynamics. Therefore, the aim of the work is to synthesize a dynamic system of a machine-tractor unit with two input actions, determined by the load on the hook and uneven terrain, and one output coordinate, determined by the amplitude of oscillations of the engine crankshaft speed.

**Presentation of the main material**

Let us consider the dynamic (Fig. 1) and structural (Fig. 2) schemes that describe the movement of the tractor under the influence of previously defined perturbations**.**

The schemes are compiled under the following basic assumptions: only the steady-state operation mode with a variable random load is considered; the movement of the tractor is assumed to be rectilinear; the effect of turbocharging on engine performance is not taken into account; engine intermittency is not taken into account; transmission stiffness and damping are not taken into account.

Taking as input $P_{кр} = P_{кр}(t)$ and $P_q = P_q(t)$ , the tractor can be considered as a four-mass dynamic system with two inputs (Fig. 1), where 1 is the engine and transmission of the tractor, which are replaced by a conditional “shaft”; 2 — translational movement of the tractor, replaced by the conditional "tractor shaft"; 3 — vertical movement of the tractor frame, replaced by the conditional "frame shaft"; 4 — vertical movement of the front axle, which is replaced by the conditional "front axle shaft". The coupling between the engine and the rest of the masses schematically depicts the grip of the tractor with the soil, and the perturbations in the form of moments of resistance of the implement $M_c$ and the moment of resistance that occurs when moving over bumps $M_q$ and damping of the suspension and tires [10, 11].

On the basis of this dynamic system, a functional structural diagram has been developed, which consists of the following main elements: an engine and a tractor, which includes the oscillating masses of the tractor (let's call them the "skeleton"), suspension and chassis system. The block diagram for the engine was developed in [2]. For the engine and the tractor, it is correspondingly more complicated: the oscillating masses (skeleton), the suspension and running system of the tractor constitute an additional circuit (Fig. 2), where fluctuations in the traction force on the hook are taken as input actions ( $\Delta P_{кр}$ )and fluctuations in soil reactions affecting the tractor chassis ( $\Delta P_q$ ),and the output parameter is the oscillation of the moment of resistance on the crankshaft of the engine $\Delta M_{\varphi}$ . This parameter, in turn, determines the speed of the engine and, therefore (taking into account the traction properties), the actual speed of the tractor itself ( $\Delta v_д$ ).And fluctuations $\Delta v_д$ influence the inputs $\Delta P_{кр}$ and $\Delta P_q$ , i.e. we can say that the system is closed.

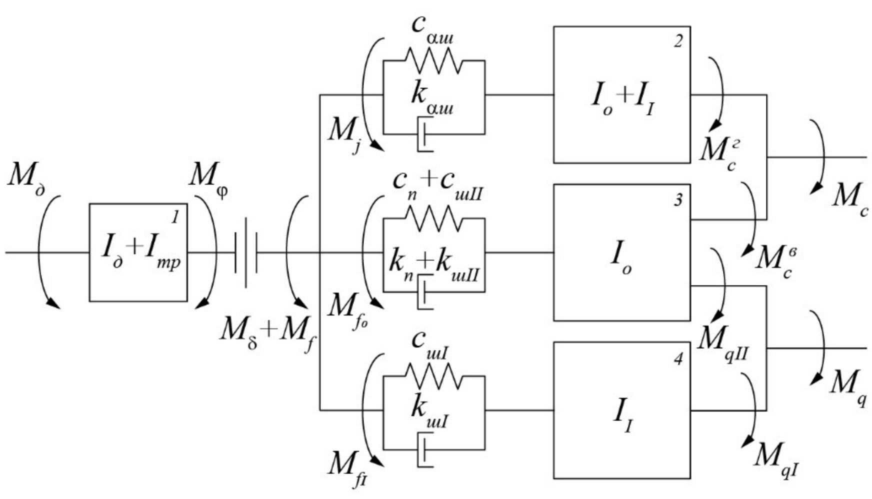


*Fig. 1.* Dynamic scheme of the tractor operation process

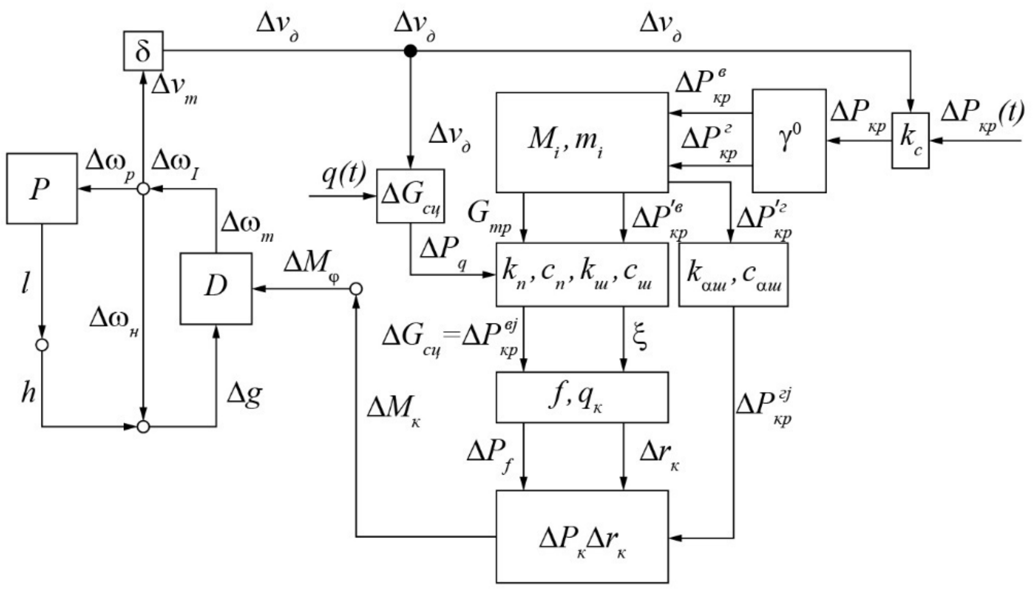


*Fig. 2.* Structural diagram of the tractor operation process

Let us dwell on the relationship between the engine and the tractor. The equation of an engine with a free intake and a regulator is known [2]. Itlookslike:

$$\begin{cases} -I_1 \dfrac{d\omega_1}{dt} + A_1\Delta\omega_1 + A_2\Delta h = \Delta M_\varphi; \\ m\dfrac{d^2 z}{dt^2} + N\dfrac{dz}{dt} + F_p z = B\Delta\omega_1; \\ \Delta h = -b_1 z \text{ при } z \ge 0; \\ \Delta h = -b_2 z \text{ при } z < 0, \end{cases} \tag{1}$$

where $I_1$ — the moment of inertia of the front axle of the tractor reduced to the corresponding conditional shaft; $A_1 = \partial M_\partial / \partial\omega_1$; $A_2 = \partial M_\partial / \partial h$; $h$ — displacement of the trailer point relative to the longitudinal plane of symmetry of the tractor; $m$ — the mass of the moving parts of the regulator reduced to the clutch; $N$ — regulator damping factor; $F_p = \frac{\partial E}{\partial t} - \frac{\partial A}{\partial t}\omega_{10}^2$ — regulator stability factor; $E$ — restoring force of the regulator springs; $l$ — governor clutch coordinate ($z = \Delta l$); $A$ — load inertia coefficient of the regulator; $B = 2\omega_{10}A(l_0)$ — amplification coefficient; $b_1$ and $b_2$ — coefficients that determine the steepness of the dependence curve $\Delta h(z)$ on the regulatory and correctional branches of the characteristic.

Consider the relationships within the tractor. The tractor is an oscillatory system consisting of several masses — the frame, the front axle and the mass of the entire tractor, interconnected, as well as with the applied disturbances through elastic links and dampers. The number of possible displacements of the tractor masses is very large. We restrict ourselves to six degrees of freedom: $z$ — vertical oscillation of the center of gravity of the tractor frame; $\alpha$ — longitudinal oscillation of the tractor frame; $\beta$ — angular transverse oscillation of the tractor frame; $x$ — longitudinal twitching of the center of gravity of the tractor; $\xi_1$ — vertical oscillation of the center of gravity of the front axle; $\beta_1$ — lateral oscillation of the front axle.

Let us consider equivalent systems corresponding to vibrations in the longitudinal and transverse planes, as well as along the longitudinal axis of the tractor (Fig. 3—6). Let's use the equations of dynamics. To do this, we apply all the forces acting on the masses of the tractor. We will count the deformations of elastic elements and tires from the position of static equilibrium, when the static load is balanced by the elastic force from the static deflection.

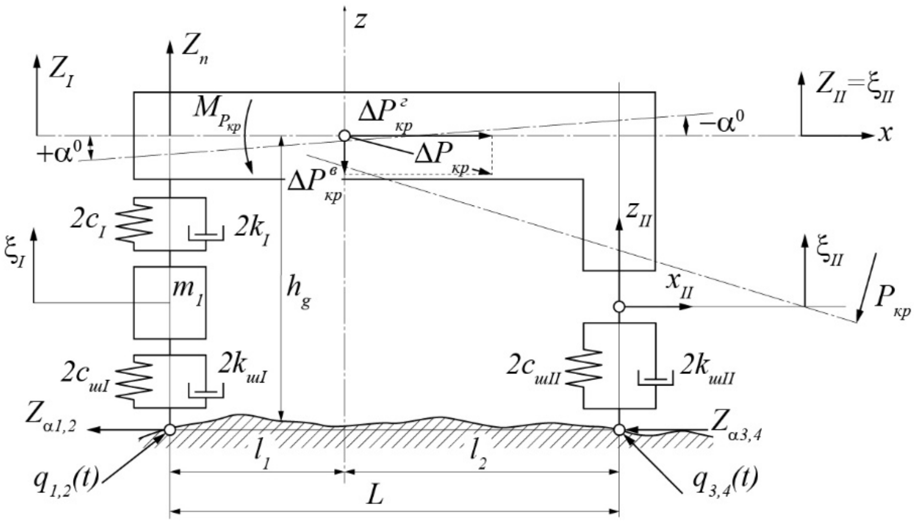


*Fig. 3.* Equivalent system used to describe the vertical and horizontal longitudinal oscillations of the tractor

Force transmitted through suspension:

$$Z_n = 2c_1(z + l_1\alpha - \xi_1) + 2k_1(\dot{z} + l_1\dot{\alpha} - \dot{\xi}_1), \tag{2}$$

where $c_1$ — suspension stiffness; $l_1$ — longitudinal coordinate of the center of gravity of the tractor frame; $k_1$ — suspension drag coefficient.

Force transmitted through the rear tires:

$$Z_{II} = Z_3 + Z_4 = c_{шII}(\xi_3 - q_3) + k_{шII}(\dot{\xi}_3 - \dot{q}_3) + c_{шII}(\xi_4 - q_4) + k_{шII}(\dot{\xi}_4 - \dot{q}_4), \tag{3}$$

where $c_{шII}$ and $k_{шII}$ — stiffness and drag coefficient of the rear tires, respectively.

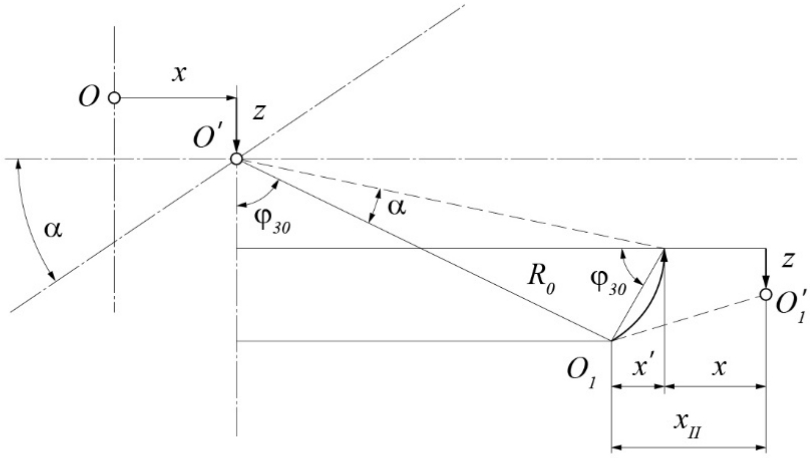


*Fig. 4.* Scheme for taking into account the influence of longitudinal angular oscillations of the tractor frame on its longitudinal twitching ($OO_1$ — initial position of the center of gravity and axis of the rear axle; $O'O_1'$ — final position of the center of gravity and the axis of the rear axle)

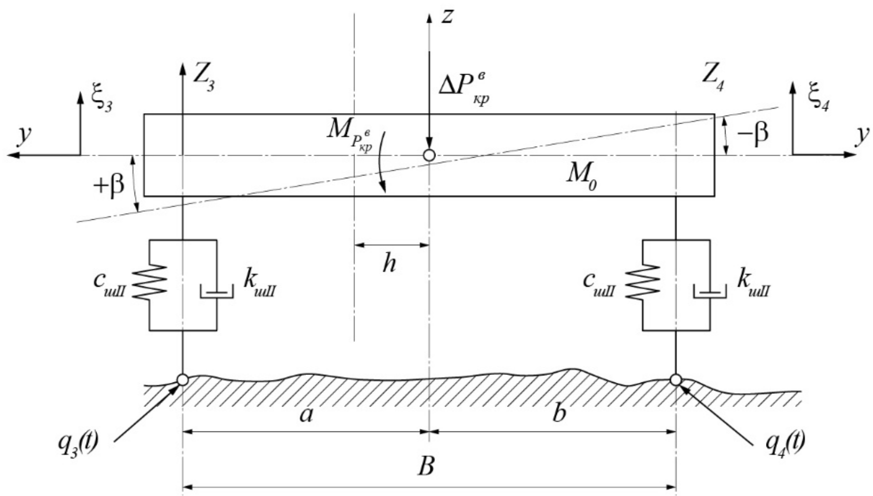


*Fig. 5.* Equivalent system used to describe the vertical oscillations of the tractor frame in the transverse plane

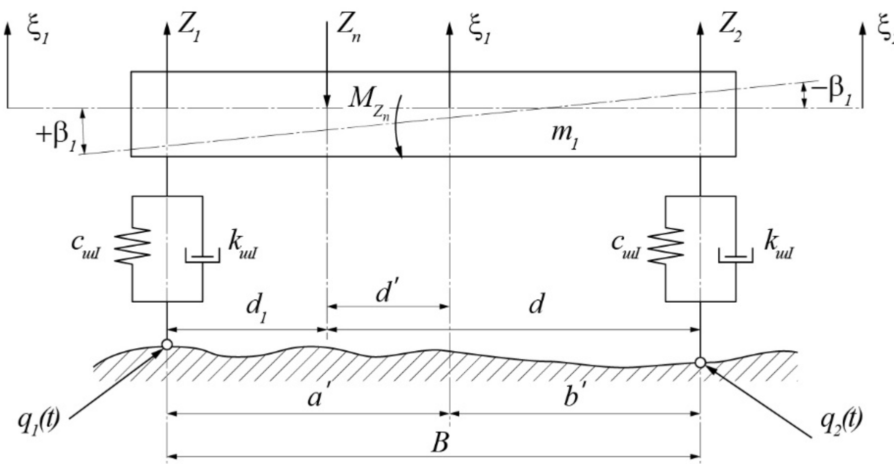


*Fig. 6.* Equivalent system used to describe the vertical oscillations of the front axle of the tractor in the transverse plane

Forces transmitted through the front tires:

right $$Z_1 = c_{шI}(\xi_1 - q_1) + k_{шI}(\dot{\xi}_1 - \dot{q}_1)\,; \quad (4)$$

left $$Z_2 = c_{шI}(\xi_2 - q_2) + k_{шI}(\dot{\xi}_2 - \dot{q}_2)\,. \quad (5)$$

Tangential forces transmitted through tires:

for front wheels:

$$X_{\alpha_{1,2}} = 2c_{\alpha ш_I}x + 2k_{\alpha ш_I}x\,; \quad (6)$$

for rear wheels:

$$X_{\alpha_{3,4}} = 2c_{\alpha ш_{II}}x + 2k_{\alpha ш_{II}}x\,, \quad (7)$$

where $c_{\alpha ш_I}$ and $c_{\alpha ш_I}$ — tangential stiffness of the front and rear tires, respectively; $k_{\alpha ш_I}$ and $k_{\alpha ш_I}$ — tangential drag coefficients of the front and rear tires, respectively.

For the core mass $M_o$ and masses of the front axle of the tractor $m_1$, as well as masses $M_o + m_1$ write the following equilibrium equations (the notation of quantities is given in Fig. 3—6):

$$\begin{cases} M_o\ddot{z} + Z_n + Z_{II} - \Delta P_{кр}^{в} = 0; \\ M_o\rho_y^2\ddot{\alpha} + Z_n l_1 - Z_3 l_2 - Z_4 l_2 + X_{\alpha 1,2}h_g + X_{\alpha 3,4}h_g - \Delta P_{кр} r = 0; \\ M_o\rho_x^2\ddot{\beta} + Z_3 a - Z_4 b - \Delta P_{кр}^{в} h = 0; \\ (M_o + m_1)\ddot{x} + X_{\alpha 1,2} + X_{\alpha 3,4} - \Delta P_{кр}^{г} = 0; \\ m_1\ddot{\xi}_1 + Z_1 + Z_2 - Z_n = 0; \\ m_1\rho_1^2\ddot{\beta}_1 + Z_1 a' - Z_2 b' - Z_n d' = 0, \end{cases} \quad (8)$$

where $P_{кр}^{в}$ and $P_{кр}^{г}$ — respectively vertical and horizontal components of the traction force on the hook; $a'$ and $b'$ — transverse coordinates of the center of gravity of the front axle of the tractor; $d'$ — displacement of the line of action of a force $Z_{II}$ from the center of gravity of the front axle.

In our case, it is advisable to move from vertical and horizontal displacements of the center of gravity of the skeleton, as well as vertical and angular displacements of the center of gravity of the front axle, to displacements of the front and rear points of the skeleton and the axles of the driving wheels of the tractor: $z_1$ —vertical movements of the frame over the front axle; $\xi_{II}$ — vertical displacement of the center of gravity of the rear axle; $\xi_1$, $\xi_2$, $\xi_3$, $\xi_4$—vertical movements of the axles of the driving wheels; $\alpha$ — angular longitudinal displacement of the core; $x_{II}$—longitudinal twitching of the rear axle axle.

To switch to a new coordinate system, we use the following dependencies (see fig. 3—6) [12, 13]:

$$\begin{cases} z = \frac{l_2}{L}z_1 + \frac{l_1}{L}\frac{b}{B}\xi_3 + \frac{l_1}{L}\frac{a}{B}\xi_4 = \frac{l_2 z_1 + l_1\xi_{II}}{L}; \\ x = x_{II} - x' = x_{II} - R_0\alpha\cos\varphi_{30}; \\ \alpha = \frac{1}{L}z_1 - \frac{1}{L}\frac{b}{B}\xi_3 - \frac{1}{L}\frac{a}{B}\xi_4 = \frac{z_1 - \xi_{II}}{L}; \\ \xi_{II} = \frac{b\xi_3 + a\xi_4}{B}; \xi_I = \frac{b'\xi_1 + a'\xi_2}{B}; \\ \beta = \frac{\xi_3 - \xi_4}{B}; \beta_{II} = \frac{\xi_1 - \xi_2}{B}, \end{cases} \quad (9)$$

where $x'$ — movements of the rear axle, corresponding to the rotation of the frame around its center of gravity (see fig. 4); $L$ and $B$ — base and width of the tractor, respectively; $\varphi_{30}$ — angle between vertical and $R_0$ — distance from the center of gravity of the frame to the rear axle of the tractor.

To obtain the equations of motion for the second coordinate system, we use the formulas (2)—(7) and (9). Substituting them into the system of equations (8) and carrying out the appropriate transformations, we obtain a system (10) of eight differential equations describing the oscillations of the tractor both in the vertical and horizontal directions:

$$\begin{cases} R_0 M_1 \ddot{\alpha} - M_2 \ddot{x}_{II} + k_{zI}\dot{z}_I + c_{zI}z_I - \dfrac{l_1}{R_0}(k_{\xi 1}\dot{\xi}_1 + c_{\xi 1}\xi_1 + k_{\xi 2}\dot{\xi}_2 + c_{\xi 2}\xi_2) - \dfrac{l_2}{R_0}(k_{шII}\dot{\xi}_3 + c_{шII}\xi_3 + k_{шII}\dot{\xi}_4 + c_{шII}\xi_4) = \\ = \dfrac{1}{R_0}(r\Delta P_{кр} - h_g \Delta P_{кр}^{г}) - \dfrac{l_2}{R_0}(\Delta P_{q3} + \Delta P_{q4}); \\ M_3 \ddot{x}_{II} + k_{xII}\dot{x}_{II} + c_{xII}x_{II} + R_0 M_4 \ddot{\alpha} - (\rho_y + h_g)k_\alpha \dot{\alpha} - (\rho_y + h_g)c_\alpha \alpha + k_{zI}\dot{z}_I + c_{zI}z_I - \\ - \dfrac{l_1}{R_0}(k_{\xi 1}\dot{\xi}_1 + c_{\xi 1}\xi_1 + k_{\xi 2}\dot{\xi}_2 + c_{\xi 2}\xi_2) - \dfrac{l_2}{R_0}(k_{шII}\dot{\xi}_3 - c_{шII}\xi_3 - k_{шII}\dot{\xi}_4 - c_{шII}\xi_4) = \\ = \dfrac{1}{R_0}(r\Delta P_{кр} + \rho_y \Delta P_{кр}^{г}) - \dfrac{l_2}{R_0}(\Delta P_{q3} + \Delta P_{q4}); \\ M_5 \ddot{z}_I + k'_{zI}\dot{z}_I + c'_{zI}z_I + M_6 \ddot{\xi}_{II} + k'_{\xi II}\dot{\xi}_{II} + c'_{\xi II}\xi_{II} + k'_{xII}\dot{x}_{II} + c'_{xII}\dot{x}_{II} - k_{\xi 1}\dot{\xi}_1 - \\ - c_{\xi 1}\xi_1 - k_{\xi 2}\dot{\xi}_2 - c_{\xi 2}\xi_2 = \dfrac{1}{L}(r\Delta P_{кр} + l_2 \Delta P_{кр}^{в}); \\ M_7 \ddot{\xi}_{II} - k'_{\xi II}\dot{\xi}_{II} - c'_{\xi II}\xi_{II} + M_6 \ddot{z}_I + k''_{zI}\dot{z}_I + c''_{zI}z_I - k'_{xII}\dot{x}_{II} - c'_{xII}x_{II} + k_{шII}\dot{\xi}_3 + \\ + c_{шII}\xi_3 + k_{шII}\dot{\xi}_4 + c_{шII}\xi_4 = \dfrac{1}{L}(l_1 \Delta P_{кр}^{в} - r\Delta P_{кр}) + \Delta P_{q3} + \Delta P_{q4}; \\ M_8 \ddot{\xi}_3 + k_{шII}\ddot{\xi}_3 + c_{шII}\xi_3 + \dfrac{b}{B}(M_9 \ddot{z}_I + 2k_I \dot{z}_I + 2c_I z_I) + M_{10}\ddot{\xi}_4 - \\ - \dfrac{b}{B}(k'_{\xi 1}\dot{\xi}_1 + c'_{\xi 1}\xi_1 + k'_{\xi 2}\dot{\xi}_2 + c'_{\xi 2}\xi_2) = \dfrac{b+h}{B}\Delta P_{кр}^{в} + \Delta P_{q3}; \\ M_{11}\ddot{\xi}_4 + k_{шII}\dot{\xi}_4 + c_{шII}\xi_4 + \dfrac{a}{B}(M_9 \ddot{z}_I + 2k_I \dot{z}_I + 2c_I z_I) + M_{10}\ddot{\xi}_3 - \\ - \dfrac{a}{B}(k'_{\xi 1}\dot{\xi}_1 + c'_{\xi 1}\xi_1 + k'_{\xi 2}\dot{\xi}_2 + c'_{\xi 2}\xi_2) = \dfrac{a-h}{B}\Delta P_{кр}^{в} + \Delta P_{q4}; \\ m_1 \ddot{\xi}_1 + k''_{\xi 1}\dot{\xi}_1 + c''_{\xi 1}\xi_1 + m_2 \ddot{\xi}_2 + \dfrac{d}{B}(k'_{\xi 2}\dot{\xi}_2 + c'_{\xi 2}\xi_2 - 2k_1 \dot{z}_1 - 2c_1 z_1) = \Delta P_{q1}; \\ m_3 \ddot{\xi}_2 + k''_{\xi 2}\dot{\xi}_2 + c''_{\xi 2}\xi_2 + m_2 \ddot{\xi}_1 + \dfrac{d_1}{B}(k'_{\xi 1}\dot{\xi}_1 + c'_{\xi 1}\xi_1 - 2k_1 \dot{z}_1 - 2c_1 z_1) = \Delta P_{q2}; \end{cases} \quad (10)$$

In this case, the following designations are introduced:

$M_1$, $M_2$, $M_3$, $M_4$ — reduced tractor weights:

$$M_1 = M_o\left(\frac{\rho_y^2}{R_0^2} + \frac{h_g}{R_0}\cos\varphi_{30}\right) + m_1 \frac{h_g}{R_0}\cos\varphi_{30}; \quad (11)$$

$$M_2 = (M_o + m_1)\frac{h_g}{R_0}; \quad (12)$$

$$M_3 = (M_o + m_1)\frac{\rho_y}{R_0}; \quad (13)$$

$$M_4 = M_o\left(\frac{\rho_y^2}{R_0^2} - \frac{\rho_y}{R_0}\cos\varphi_{30}\right) - m_1\frac{\rho_y}{R_0}\cos\varphi_{30}\,; \tag{14}$$

$M_5$, $M_6$, $M_7$, $M_8$, $M_9$, $M_{10}$, $M_{11}$ — reduced masses of the tractor frame:

$$M_5 = M_o\frac{l_2^2 + \rho_y^2}{L^2}\,; \tag{15}$$

$$M_6 = M_o\frac{l_1 l_2 - \rho_y^2}{L^2}\,; \tag{16}$$

$$M_7 = M_o\frac{l_1^2 + \rho_y^2}{L^2}\,; \tag{17}$$

$$M_8 = M_o\frac{\frac{l_1}{L}b^2 + \rho_x^2}{B^2}\,; \tag{18}$$

$$M_9 = M_o\frac{l_2}{L}\,; \tag{19}$$

$$M_{10} = M_o\frac{\frac{l_1}{L}ab - \rho_x^2}{B^2}\,; \tag{20}$$

$$M_{11} = M_o\frac{\frac{l_1}{L}a^2 + \rho_x^2}{B^2}\,; \tag{21}$$

$m_1$, $m_2$, $m_3$ — reduced masses of the front axle of the tractor:

$$m_2 = m_1\frac{a'b' - \rho_1^2}{B^2}\,; \tag{22}$$

$$m_3 = m_1\frac{a'^2 + \rho_1^2}{B^2}\,; \tag{23}$$

$q_i$, $\dot{q}_i$ — ordinates of irregularities under each driving wheel and their derivatives ($i = \overline{1,4}$); $c_{zI}$, $c_{\xi 1}$, $c_{\xi 2}$, $c_\alpha$, $c'_{zI}$, $c'_{\xi II}$, $c'_{xII}$, $c''_{zI}$, $c'_{\xi 1}$, $c'_{\xi 2}$, $c''_{\xi 1}$, $c''_{\xi 2}$ — reduced vertical and horizontal stiffness of the suspension and tires of the tractor:

$$c_{zI} = 2\frac{l_1}{R_0}c_I\,; \tag{24}$$

$$c_{\xi 1} = 2\frac{b'}{B}c_I\,; \tag{25}$$

$$c_{\xi 2} = 2\frac{a'}{B}c_I\,; \tag{26}$$

$$c_\alpha = 2\cos\varphi_{30}(c_{\alpha шI} + c_{\alpha шII})\,; \tag{27}$$

$$c_{xII} = 2\frac{\rho_y + h_g}{R_0}(c_{\alpha шI} + c_{\alpha шII})\,; \tag{28}$$

$$c'_{zI} = 2\left[c_I - \frac{h_g R_0}{L^2}\cos\varphi_{30}(c_{\alpha шI} + c_{\alpha шII})\right]; \tag{29}$$

$$c'_{\xi II} = 2h_g(c_{\alpha шI} + c_{\alpha шII})\frac{R_0}{L^2}\cos\varphi_{30}\,; \tag{30}$$

$$c'_{xII} = 2\frac{h_g}{L}(c_{\alpha шI} + c_{\alpha шII})\,; \tag{31}$$

$$c''_{zI} = 2\frac{a}{B}c_I\,; \tag{32}$$

$$c'_{\xi 1} = 2\frac{b'}{B}c_I\,; \tag{33}$$

$$c'_{\xi 2} = 2\frac{a'}{B}c_I\,; \tag{34}$$

$$c''_{\xi 1} = c_{шI} + 2\frac{db'}{B^2}c_I\,; \tag{35}$$

$$c''_{\xi 2} = c_{шI} + 2\frac{d_1 a'}{B^2}c_I\,; \tag{36}$$

$k_{zI}$, $k_{\xi 1}$, $k_{\xi 2}$, $k_{xII}$, $k_{\alpha}$, $k'_{zI}$, $k'_{\xi II}$, $k'_{xII}$, $k''_{zI}$, $k'_{\xi 1}$, $k'_{\xi 2}$, $k''_{\xi 1}$, $k''_{\xi 2}$ — reduced vertical and horizontal drag coefficients of the tractor suspension and tires:

$$k_{zI} = 2\frac{l_1}{R_0}k_1\,; \tag{37}$$

$$k_{\xi 1} = 2\frac{b'}{B}k_1\,; \tag{38}$$

$$k_{\xi 2} = 2\frac{a'}{B}k_1\,; \tag{39}$$

$$k_{xII} = 2\frac{\rho_y + h_g}{R_0}(k_{\alpha шI} + k_{\alpha шII})\,; \tag{40}$$

$$k_{\alpha} = 2\cos\varphi_{30}(k_{\alpha шI} + k_{\alpha шII})\,; \tag{41}$$

$$k'_{zI} = 2\left[k_1 - \frac{h_g R_0}{L^2}\cos\varphi_{30}(k_{\alpha шI} + k_{\alpha шII})\right]; \tag{42}$$

$$k'_{\xi II} = 2h_g(k_{\alpha шI} + k_{\alpha шII})\frac{R_0}{L^2}\cos\varphi_{30}\,; \tag{43}$$

$$k'_{xII} = 2\frac{h_g}{L}(k_{\alpha шI} + k_{\alpha шII})\,; \tag{44}$$

$$k''_{zI} = 2\frac{a}{B}k_1\,; \tag{45}$$

$$k'_{\xi 1} = 2\frac{b'}{B}k_1\,; \tag{46}$$

$$k'_{\xi 2} = 2\frac{a'}{B}k_1\,; \tag{47}$$

$$k''_{\xi 1} = k_{шI} + 2\frac{db'}{B^2}k_1\,; \tag{48}$$

$$k''_{\xi 2} = 2\frac{d_1 a'}{B^2}k_1 + k_{шI}\,. \tag{49}$$

Fluctuations in soil reactions on the driving wheels of the tractor when moving over bumps [4]:

$$\Delta P_{qi} = k_{шI,II}\dot{q}_i + c_{шI,II}q_i\,,\ i = \overline{1,4}\,,\ q_i = q(\Delta v_д, t)\,. \tag{50}$$

We agree that the movement of the frame and wheels below the static line is positive, above — negative. Longitudinal movements of the tractor backwards are positive, forwards are negative. Considering the foregoing and solving the system of equations (10) regarding the oscillations of the tractor wheels, we write expressions for determining the oscillations of the rolling radii of all the driving wheels of the tractor:

$$\Delta r_{кi} = \xi_i + q_i\,,\ i = \overline{1,4}\,. \tag{51}$$

To determine the oscillations of the moment of resistance on the driving wheels, it is necessary to know the oscillations of the tangential traction force on each wheel:

$$\Delta P_{кi} = \Delta P_{fi} + \Delta P_{кр}^{г\, ji},\ i = \overline{1,4}, \tag{52}$$

where $\Delta P_{fi} = \Delta G_{сцi} f$ —fluctuations in the rolling resistance force on each wheel; $f$ —rolling resistance coefficient.

Coupling weight fluctuations per wheel:

$$\begin{cases} \Delta G_{сц1} = m_1 \ddot{\xi}_1; \\ \Delta G_{сц2} = m_3 \ddot{\xi}_2; \\ \Delta G_{сц3} = M_8 \ddot{\xi}_3; \\ \Delta G_{сц4} = M_{11} \ddot{\xi}_4. \end{cases} \tag{53}$$

Fluctuations in the horizontal component of the hook resistance on each wheel:

$$\begin{cases} \Delta P_{кр}^{г\, j1} = \lambda M_3 \dfrac{b}{B} \ddot{x}_{II}; \\ \Delta P_{кр}^{г\, j2} = \lambda M_3 \dfrac{a}{B} \ddot{x}_{II}; \\ \Delta P_{кр}^{г\, j3} = (1-\lambda) M_3 \dfrac{b}{B} \ddot{x}_{II}; \\ \Delta P_{кр}^{г\, j4} = (1-\lambda) M_3 \dfrac{a}{B} \ddot{x}_{II}, \end{cases} \tag{54}$$

where $\lambda = G_I / G_{тр}$ — bearing weight distribution coefficient; $G_I$ — weight on the front support; $G_{тр}$ — total tractor weight.

Fluctuations in the moment of resistance on the driving wheels of the tractor:

$$\Delta M_{кi} = \Delta (P_{кi} r_{кi}),\ i = \overline{1,4}. \tag{55}$$

Taking into account the onboard gearbox, interwheel differential and power transmission, we write the expression for the oscillations of the moment of resistance on the crankshaft of the engine:

$$\Delta M_{\varphi} = \frac{\sum_{i=1}^{4} \Delta M_{кi}}{i_{тр} \eta_{тр}}, \tag{56}$$

where $i_{тр}$ — the gear ratio of the transmission from the wheel to the crankshaft of the engine; — efficiency of the entire transmission.

Having solved the system of equations (1), we obtain fluctuations in the angular velocity of the engine crankshaft $\Delta\omega_1$.

To determine the fluctuations in the actual speed of the tractor, it is necessary to know the fluctuations of its theoretical speed. The presence of cross-axle differentials predetermines the possibility of different theoretical speeds of translational movement of the tractor wheels [14, 15] depending on the resistances applied to them. However, based on the condition of straightness of motion adopted by us, fluctuations in the actual speed of the translational motion of all wheels, and hence the tractor ($\Delta v_д$) in general should be the same: $\Delta v_{д1} = \Delta v_{д2} = \Delta v_{д3} = \Delta v_{д4} = \Delta v_д$.

Therefore, in order to determine , it is enough to know the fluctuations in the angular velocity and the slipping of one of the tractor wheels. Let us determine the angular velocity of the wheels of the front axle, taking into account the differential. Let us use the laws of dynamics [16—19]. To do this, we determine the moments acting on the front axle of the tractor, and compose the equilibrium equation at each moment of time:

$$\begin{cases} I_к \left( \frac{d\Delta\omega_п}{dt} + \frac{d\Delta\omega_л}{dt} \right) = \Delta M_{диф} - \Delta M_1 - \Delta M_2; \\ \Delta\omega_п + \Delta\omega_л = 2\Delta\omega_1, \end{cases} \tag{57}$$

where $I_к$ — the moment of inertia of the right or left wheels, reduced to the axis of rotation; $\Delta\omega_п$, $\Delta\omega_л$ — fluctuations in the angular velocity of the right and left semiaxes; $\Delta\omega_I = \frac{\Delta\omega_1}{i_{кпп} i_{раз} i_{гл.п}}$ — oscillation of the angular velocity of the driven gear of the final drive of the front axle ($i_{кпп}$, $i_{раз}$, $i_{гл.п}$ — gear ratio of the gearbox, transfer case and final drive, respectively); $\Delta M_{диф} = \Delta M_\varphi i_{кпп} i_{раз} i_{гл.п} \eta_{кпп} \eta_{раз} \eta_{гл.п}$ — engine torque reduced to the final drive gear; $\Delta M_1 = \frac{\Delta M_{к1}}{i_{ред}\eta_{ред}}$, $\Delta M_2 = \frac{\Delta M_{к2}}{i_{ред}\eta_{ред}}$ —moments of resistance on the right and left axle shafts ($i_{ред}$, $\eta_{ред}$ —gear ratio and efficiency of the wheel gear).

Equation (57) allows you to determine the fluctuations in the theoretical speed of the forward movement of the right and left wheels:

$$\begin{cases} \Delta v_{теор1} = \frac{\Delta\omega_п \Delta r_{к1}}{i_{ред}}; \\ \Delta v_{теор2} = \frac{\Delta\omega_л \Delta r_{к1}}{i_{ред}}. \end{cases} \tag{58}$$

Fluctuations in the actual speed of the translational movement of the wheels (and hence the tractor) are determined from an expression of the form:

$$\Delta v_д = \Delta v_{д1} = \Delta v_{теор1}(1-\delta_1) = \Delta v_{д2} = \Delta v_{теор2}(1-\delta_2)\,. \tag{59}$$

where $\delta_1 = \delta(\varphi_1)$, $\delta_2 = \delta(\varphi_2)$ — slippage of the right and left wheels of the front axle, respectively; $\varphi_1$, $\varphi_2$ —friction coefficients of the right and left wheels of the front axle, respectively.

Similarly, you can represent the expression for the rear axle.

The influence of fluctuations in the actual speed of movement on the traction resistance of the tractor can be expressed through fluctuations in soil resistivity:

$$\Delta P_{кр} = \Delta k_c ab\,, \tag{60}$$

where $a$, $b$ — plowing depth and plow width; $\Delta k_c = k_c(\Delta v_д)$ — fluctuations in soil resistivity depending on fluctuations in the actual speed of the tractor.

## Conclusions

Thus, the mathematical model of the process includes equations (10)—(56), (1), (57)—(60) and takes into account the main factors (traction resistance and terrain irregularities) that cause oscillatory processes inside the tractor.

The possibility of solving this mathematical model using applied mathematical programs of symbolic calculation makes it possible to study the operation of a machine-tractor unit with various combinations of design parameters of tractor elements without manufacturing in metal.

Development of a simulation technique on a PC can have an effect in determining the traction and dynamic qualities of serial tractors, and will also make it possible to predict these qualities for newly created machines.

## Reference


[1] OECD, Code 2, OECD Standard Code for the Ofdcial Testing of Agricultural and Forestry Tractor Performance. *Organization for Economic Cooperation and Development.* Paris. France. 2021.

[2] Kocher M., Adamchuk V., Smith J., Hoy R. (2011) Verifying power claims of high-power agricultural tractors without a PTO to sell in Nebraska. *Applied Engineering in Agriculture*. Vol. 27(5). P. 711–715.

[3] Howard C., Kocher M., Hoy R., Blankenship E. (2013) Testing fuel efficiency of tractors with both continuously variable and standard geared transmissions. *Transactions of the ASABE*. Vol. 56(3). P. 869–879.
[4] Roeber J., Pitla S., Hoy R., Luck J., Kocher M. (2017) Development and validation of a tractor drawbar force measurement and data acquisition system (DAQ). *Applied Engineering in Agriculture*. Vol. 33(6). P. 781–789.
[5] Rasool S., Raheman H. (2018) Improving the tractive performance of walking tractors using rubber tracks, *Biosystem Engineering*. Vol. 167(1). P. 51–62.
[6] Parker M., Stott A., Bodie M., Frankenstein S., Shoop S. (2021) Vehicle mobility on highly organic soils, *Journal of Terramechanics*. Vol. 98(1). P. 16–24.
[7] Cholodowski J., Dudzinski P., Ketting M. (2021) On the energy losses due to tracks vibrations in rubber track crawler vehicles. *Archives of Civil and Mechanical Engineering*. Vol. 21(2). P. 59.
[8] Shafaei S., Loghavi M., Kamgar S. (2021) Profound insight into tractor energy dissipation through inevitable interaction inside wheel-soil interface for the period of plowing works, *Soil and Tillage Research.* Vol.211(1). P 1049–1098.
[9] Bloch A., Reyhanoglu M. and McClamroch N. (1992) Control and stabilization of nonholonomic dynamic systems *IEEE. Transactions on Automatic Control.*Vol. 37.P. 1746–1757.
[10] Bloch A 2003 Nonholonomic Mechanics and Control. Springer Verlag. New York.
[11] Bullo F. and Lewis A. (2004) Geometric Control of Mechanical Systems.*Applied Mathematics.*Vol. 49. P. 354–365.
[12] Bullo F. and Lewis A. (2007) Reduction, linearization, and stability of relative equilibria for mechanical systems on riemannian manifolds.*Acta Applicandae Mathematicae.*Vol. 99(1). P. 53–95.
[13] Bullo F. and Lewis A. (2004) Geometric Control of Mechanical Systems Modeling, Analysis and Design for Simple Mechanical Control Systems. Springer: New York. USA.
[14] Crouch P. (1981) Geometric structures in systems theory.*IEE Proceedings D (Control Theory and Applications).*Vol. 128. P. 242–252. doi: 10.1049/ip-d.1981.0051
[15] Isidori A. (1995) Nonlinear Control Systems. Springer Verlag. 3rd edition.
[16] Khalil H. (2002) Nonlinear Systems. 3rd ed. Prentice Hall: Hoboken. NJ. USA.
[17] Muller P. (1998) Stability and Optimal Control of Nonlinear Descriptor Systems: A Survey. *Applied Mathematics and Computer Science.* Vol.8. P. 269–286.
[18] Nowicki M. and Respondek W. (2021) Mechanical state-space linearization of mechanical control systems and symmetric product of vector fields.*Proceedings of the 7th IFAC LHMNC*October 11-13(Berlin, Germany). P. 365–378.
[19] Oliva W. (2002) Geometric Mechanics. Springer.


## МАТЕМАТИЧНА МОДЕЛЬ РУХУ КОЛІСНОГО ТРАКТОРА З ОБЛІКОМ ЙОГО ВЕРТИКАЛЬНИХ І ГОРИЗОНТАЛЬНИХ КОЛИВАНЬ

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Анотація Робота машинно-тракторного агрегату під час виконання різних сільськогосподарських робіт супроводжується безперервною зміною навантаження, що зумовлює зміну частоти обертання колінчастого вала, а отже, і потужності двигуна. Крім того, випадковий полігармонічний характер навантаження призводить до виникнення локальних резонансних коливань у такій багатоланковій динамічній системі, як машинно-тракторний агрегат, що ще більше збільшує амплітуду коливань частоти обертання колінчастого вала, і, як наслідок, призводить до збільшення втрат потужності, потенційно доступної силової установки трактора. У статті наведено математичну модель роботи колісного трактора зі сталим нерівномірним навантаженням на гак при русі по нерівностях. При цьому машинно-тракторний агрегат представлений як динамічна система з двома вхідними діями, що визначаються навантаженням на гак і нерівностями рельєфу, та однією вихідною координатою, що визначається амплітудою

коливань частоти обертання колінчастого вала двигуна. У цьому випадку динамічна та структурна схеми є основою для складання математичної моделі. Таким чином, математична модель процесу враховує основні чинники (тяговий опір і нерівності рельєфу), що викликають коливальні процеси всередині трактора. Можливість розв'язання даної математичної моделі за допомогою прикладних математичних програм символьного розрахунку дає змогу досліджувати роботу машинно-тракторного агрегату з різними комбінаціями конструктивних параметрів елементів трактора без виготовлення в металі. Розробка методики моделювання на ПК може вплинути на визначення тягово-динамічних якостей серійних тракторів, а також дасть можливість прогнозувати ці якості для новостворених машин.

**Література**

1. OECD, Code 2, OECD Standard Code for the Ofdcial Testing of Agricultural and Forestry Tractor Performance. *Organization for Economic Cooperation and Development.* Paris. France. 2021.
2. Kocher M., Adamchuk V., Smith J., Hoy R. Verifying power claims of high-power agricultural tractors without a PTO to sell in Nebraska. *Applied Engineering in Agriculture*. Vol. 27(5). 2011. P. 711–715.
3. Howard C., Kocher M., Hoy R., Blankenship E. Testing fuel efficiency of tractors with both continuously variable and standard geared transmissions. *Transactions of the ASABE*. Vol. 56(3). 2013. P. 869–879.
4. Roeber J., Pitla S., Hoy R., Luck J., Kocher M. Development and validation of a tractor drawbar force measurement and data acquisition system (DAQ). *Applied Engineering in Agriculture*. Vol. 33(6). 2017. P. 781–789.
5. Rasool S., Raheman H. Improving the tractive performance of walking tractors using rubber tracks, *Biosystem Engineering*. Vol. 167(1). 2018. P. 51–62.
6. Parker M., Stott A., Bodie M., Frankenstein S., Shoop S. Vehicle mobility on highly organic soils, *Journal of Terramechanics*. Vol. 98(1). 2021. P. 16–24.
7. Cholodowski J., Dudzinski P., Ketting M. On the energy losses due to tracks vibrations in rubber track crawler vehicles. *Archives of Civil and Mechanical Engineering*. Vol. 21(2). 2021. P. 59.
8. Shafaei S., Loghavi M., Kamgar S. Profound insight into tractor energy dissipation through inevitable interaction inside wheel-soil interface for the period of plowing works, *Soil and Tillage Research.* Vol.211(1). 2021. P. 1049–1098.
9. Bloch A., Reyhanoglu M. and McClamroch N. Control and stabilization of nonholonomic dynamic systems *IEEE. Transactions on Automatic Control.*Vol. 37. 1992. P. 1746–1757.
10. Bloch A 2003 Nonholonomic Mechanics and Control. Springer Verlag. New York.
11. Bullo F. and Lewis A. Geometric Control of Mechanical Systems.*Applied Mathematics.*Vol. 49. 2004. P. 354–365.
12. Bullo F. and Lewis A. Reduction, linearization, and stability of relative equilibria for mechanical systems on riemannian manifolds.*Acta Applicandae Mathematicae.*Vol. 99(1). 2007. P. 53–95.
13. Bullo F. and Lewis A. Geometric Control of Mechanical Systems Modeling, Analysis and Design for Simple Mechanical Control Systems. Springer: New York. USA. 2004.
14. Crouch P. Geometric structures in systems theory.*IEE Proceedings D (Control Theory and Applications).*Vol. 128. 1981. P. 242–252. doi: 10.1049/ip-d.1981.0051
15. Isidori A. Nonlinear Control Systems. Springer Verlag. 3rd edition. 1995.
16. Khalil H. Nonlinear Systems. 3rd ed. Prentice Hall: Hoboken. NJ. USA. 2002.
17. Muller P. Stability and Optimal Control of Nonlinear Descriptor Systems: A Survey. *Applied Mathematics and Computer Science.* Vol.8. 1998. P. 269–286.
18. Nowicki M. and Respondek W. Mechanical state-space linearization of mechanical control systems and symmetric product of vector fields.*Proceedings of the 7th IFAC LHMNC* October 11-13(Berlin, Germany). 2021. P. 365–378.
19. Oliva W. Geometric Mechanics. Springer. 2002.