MATHEMATICAL MODEL OF THE PROCESS FOR FORMING A PURE CEMENT ZONE AROUND THE HARDENED ELEMENTS USING JET TECHNOLOGY

MATHEMATICHNA МОДЕЛЬ ПРОЦЕСУ ВИНИКНЕННЯ ЗОНИ ЧИСТОГО ЦЕМЕНТА НАВКОЛО ЗАКРИПЛЕНІХ ЕЛЕМЕНТІВ ПРИ ВИКОРИСТАННІ СТРУМІННОЇ ТЕХНОЛОГІЇ

Mathematical modeling and substantiation of the physical processes that occur in rock consolidation processes are very important for the further parameters calculations of these processes. This article examines the phenomenon of the shell appearance of pure cement mix around the hardened rock-soluble elements, which are formed as a result using the jet grouting of dispersed rocks with high-pressure jets of hardening cement mixtures. Physical model and the construction of mathematical model that explain the nature of the appearance of such shells are also considered. The thickening of the jet can be described by an ordinary differential equation if we consider the rock as an isotropic medium. If the values of the first and second order of smallness are neglected, it is possible to estimate the thickness of the zone of pure cement in a first approximation. At the same time, the difference between theoretical and experimental results does not exceed 16 % on average. It is possible either to save hardening materials or to increase the filtering properties of the Anti-Filtration Curtains because the shell of pure cement mixture on the hardened element improves its filtering properties.

Keywords: hardening mixture, rock-soluble elements, jet grouting technology, discharge pressure, thickness of the pure cement zone.
Розділ 2. Моделювання та оптимізація в технології конструкційних матеріалів

Розглянута математична модель, що пояснює виникнення зони чистого цементу при закріпленні дисперсної породи струменями високого тиску. На підставі розглянутої моделі зроблений висновок щодо залежності тиску подачі закріїляющего розчину і товщини зони чистого цементу навколо закріплених елементів.

Вважаємо породу ізотропним середовищем, потовщення струменя можна описати звичайним диференціальним рівнянням. Якщо знехтувати величинами першого і другого порядку малоспів, можна у першому наближенні оцінити величину товщини зони чистого цементу. При цьому різниця між теоретичними та експериментальними результатами в середньому не перевищує 16 %.

Можна зробити висновок, що товщина зони чистого закріплюючого розчину при збільшенні тиску подачі розчину збільшується за лінійним законом і коефіцієнт кореляції між розрахунковими та експериментальними значеннями становить 0,97. Даний ефект цікавий тим, що оболонка з чистого цементного розчину на порідно-розчинному елементі покращує його фільтраційні властивості, що необхідно враховувати при створенні протифільтраційних зав'язок.

Ключові слова: закріпляючі розчини, порідно-розчинні елементи, струменне закріплення порід, тиск нагнітання, товщина зони чистого цементу.

Problem’s Formulation

Mathematical modeling and substantiation of the rock consolidation and hardening physical processes is very important for the further parameters calculations of these processes. Building and reconstruction of large cities in our country requires special methods of conducting construction and hydraulic works. It is often necessary to do this in rocks that require the use of special hardening methods. This article examines the phenomenon of the shell appearance that made of pure cement mixture around hardened rock-soluble elements, which are formed as a result of the use of jet grouting of dispersed rocks with high-pressure jets of hardening cement mixtures, and the creation of the physical and mathematical models that explain the nature of such shell appearance.

At present, the jet grouting is the most effective and economical method. Its essence consists in the simultaneous destruction of the rock by a high-pressure jet of cement mix, the mixing of the crushed rock with it and, after hardening, the formation of a rock-soluble element. The phenomenon of the formation around such an element of a shell made of pure cement mix was discovered during field and industrial tests. Later it was confirmed during many observations. But such a shell must affect the physical and technical properties of the hardened element; they must be taken into account in further calculations. In this article, the mathematical model of the appearance of such a shell is considered and the method of calculating its thickness is given.

Analysis of recent research and publications

We use the installation that described in [1, 2] in the experimental studies. A cement mix of different densities was used as a self-hardening mixture. Fig. 1 and Fig. 2 show photographs of the cross-sections of hardened elements, on which you can see zones of mixing of the rock with pure cement of the hardening mix.

When viewed under a microscope, it can be seen that the jet of the grouting mix, when it penetrates into the rock, moves in the longitudinal and transverse directions, while moving and mixing the rock particles (Fig. 1).

This article is devoted to the explanation of this phenomenon. Jet grouting is considered a relatively recent development; it is used mainly for soil improvement and the effective solution of many geotechnical and geoeological problems. The jet grouting method differs significantly from other grouting methods. A wide range of soil types can be treated using this method. Therefore, the scope of jet grouting is expanding [3—5]. It should be noted that much attention is paid to the strength properties of jet grout columns in jet grouting technique [5].
Many authors notice that jet grouting is an ideal ground modification technique compared to reinforcement and ground treatment method. Then to understand the jet grouting process properly the effects of several parameters on grout mixing’s physical and mechanical properties should be studied carefully. These design parameters are defined on the basis of soil or rock characteristics and content as well as the required degree of quality. During the realization of jet grouting columns parameters have to be measured and recorded [3—7].

**Formulation of the study purpose**

To create the mathematical model of the shell appearance that made of pure cement around the fixing elements, when using jet grouting technology, and to develop a methodology for calculating its thickness.

**Presenting main material**

The penetration of the jet into the rock stops [7—9] if the speed of the jet \( u_2 \) reached

\[
\frac{u_2}{k} = \sqrt{\frac{8\eta}{\pi\rho_c d_0}},
\]

where \( \eta \) is an impact strength of the rock, J/m\(^2\); \( \rho_c \) is a density of the jet substance, kg/m\(^3\); \( d_0 \) is a nozzle diameter, m.

However, the jet grouting has a non-zero velocity, and although the energy of the jet is not sufficient to penetrate the rock, it is sufficient to shift the particles due to the free pore space. The space freed from soil particles is filled with jet substance. As a result, a zone of pure self-hardened cement that unmixed with the rock is formed outside the hardening element. It is shown the penetration of the jet grouting into the rock until and after the velocity reached at \( u_2^k \) (Fig. 3).

If we consider the rock as an isotropic medium, then the following differential equation can be written from the law of the energy conservation [9]

\[
dm \cdot \frac{(u_2^k)^2}{2} = F \cdot dl,
\]

where \( dm \) is the effective weight of the jet material, kg; \( F \) is strength of rock resistance to jet spreading, N; \( dl \) is the depth of jet press into the rock, m.

The mass of the jet is \( dm = Q \cdot dt \), where \( Q \) is the consumption of jet material, m\(^3\)/sec; \( dt \) is the time of the jet activity in one direction, sec.
The mass of shearing rock is \( M = \rho_n \cdot V \), where \( \rho_n \) is the rock density; \( V \) is the volume by which the rock is displaced by the jet due to its porosity, m³.

Time of the jet action in one direction is \( dt = \frac{d_0}{\pi d_1 \omega} \), where \( d_1 \) is a diameter of the working tool, m; \( \omega \) is the rotation velocity of the tool, rad/sec.

The jet thickening occurs according to a linear law and
\[
\frac{db}{dt} = \frac{db}{dx} \cdot \frac{dx}{dt}.
\]
Then \( b = \frac{d_0}{d_1} H \) in boundary case, where \( b \) is half the stream thickness, m; \( H \) is the depth of jet spreading into the rock, m. Let \( b = l \). We neglect the values of the first and second orders of smallness, and then the increase in the depth of penetration of the grout into the rock due to its compaction is

\[
dl \approx \frac{dt \cdot Q \left( \frac{d_0}{d_1} \right)^2}{4 \rho_0 \cdot g \cdot k_1 \left( \frac{d_0}{d_1} H + 2 b^2 \left( \frac{d_0}{d_1} \right) + b^2 \right)},
\]

where \( k_1 \) is rock friction coefficient.

The consumption of jet material is
\[
Q = S \cdot \sqrt{\frac{q \cdot P_0}{\rho_1}},
\]

where \( q \) is an equipment pressure loss indicator [2]; \( P_0 \) is the discharge pressures of the grout mixture, Pa; \( \rho_c \) is the mixture density, kg/m³.

Therefore, we estimate the value \( dl \) in the first approximation. A plot of the dependence of the pure cement zone thickness for the fine-grained sands is plotted at various pressures of the hardening mixture based on the data of laboratory and theoretical studies (Fig. 4). The difference between the theoretical and experimental results does not exceed 16% on average [9].

It can be concluded that the thickness of the pure grout mixture zone with increasing mixture supply pressure increases according to a linear law and the correlation coefficient between calculated and experimental values is 0.97, when we analyze this graph (Fig. 3). This effect is interesting because the shell of pure cement mortar on the rock-soluble element improves its filtering properties, which must be taken into account when creating Anti-Filtration Curtains with high-pressure jets grouting.
**Fig. 4.** Dependence of the thickness of the pure cement zone on the discharge pressure of the grout mixture

**Conclusions**

1. It was established that a zone of pure cement is formed at the boundary of the rock-soluble element with unfixed dispersed rock as a result of the performed experimental studies.

2. It was established that the thickness of the pure cement zone changes according to a linear law and depends on the discharge pressure of the grouting as a result of mathematical modeling of the physical processes taking place.

3. It was established that the correlation coefficient between calculated and experimental values is 0.97, which indicates a high correlation between theoretical calculations and practical phenomena. The considered model describes this phenomenon well.

4. It is possible either to save materials for grout mixture or to increase the filtering properties of Anti-Filtration Curtains as a result of the fact that the shell of pure cement mortar on the hardened element improves its filtering properties.

**References**


Список використаної літератури