DOI: 10.31319/2519-8106.1(50)2024.305490
UDC 519.6:519.642:621.3.013

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AN EFFICIENT ALGORITHM FOR SOLUTION OF THE MAGNETOSTATICS PROBLEM FOR QUASIVISESYMMETRIC SYSTEMS

ЕФЕКТИВНИЙ АЛГОРИТМ РІШЕННЯ ЗАДАЧІ МАГНІТОСТАТИКИ ДЛЯ КВАЗІВІСЕСИМЕТРИЧНИХ СИСТЕМ

As you know, in the practice of magnetostatic calculations, three classes of magnetic systems (MS) are distinguished: flat, axisymmetric and three-dimensional. But there is a subclass of MS that naturally combines the features of axisymmetric and three-dimensional systems. This class is of great practical importance. Such MC are widely used, first of all, in electronic engineering: they are a number of electron-optical devices, primarily klystrons and magnetrons.

Such systems have an almost axisymmetric geometry, but, what is very important, have a three-dimensional part, as the working area of the device. This introduces three-dimensional disturbances into the Pochtak axisymmetric MS, and these inclusions significantly distort the axisymmetric nature of the field precisely in the working part of the MS. First of all, it can be holes or slits in multi-beam electronic optical systems, filter matrices in magnetic separators, and the like. Moreover, the volume of this three-dimensional part is quite insignificant. According to the condition of the task, the calculation of the field strength must be performed precisely in the working, three-dimensional part of the MC, and, as a rule, the calculation must be performed very accurately.

The calculation of axisymmetric as a three-dimensional MS is unsatisfactory, first of all, as it is very inefficient. In addition, it is necessary to find the field strength in the immediate vicinity of the working area of the QAS. Therefore, a high level of discretization is required in the working area of the QAS. Even if you try to maintain approximately the same level in the working part and the axisymmetric part of the MC, the number of elements will be unacceptably large, and this will lead to poor convergence of the iterative process. If the discratization is performed unevenly, it will also lead to even worse convergence of the iterative process of the decision to determine the magnetization vector of the MC due to the significant variation in the sizes and shapes of these elements. Because the system of linear algebraic equations (LAL) in both cases will be ill-conditioned. The non-linear nature of the solution process and the possible saturation of the magnetic field further complicate the situation.

Keywords: *magnetic systems; integrated equalization; electron-optical devices; approximation; numerical modeling.*

Розглянуто підклас магнітостатичних систем, який поєднує в собі особливості вісесиметричних та тривимірних систем, — квазівісесиметричні системи. Показано, що розрахунок таких магнітних систем має істотні особливості. Запропоновано ефективний цих, що складається з двох етапів. Наведено практичний розрахунок такої магнітної системи.

Ключові слова: *магнітні системи; інтегральні рівняння; електронно-оптичні прилади; апроксимація; чисельне моделювання.*

Problem's Formulation

Modeling and optimizing the operation of magnetostatic systems (MS) is highly relevant, as such systems make up a significant part of modern high-tech devices. To model them, it is necessary to create efficient and versatile algorithms that allow for a detailed study of MS operation in various modes. The requirements for an algorithm for high-precision calculation and optimization of such MSs should be described. A prerequisite is the ability to calculate MSs for a nonlinear magnetic environment, complex, almost arbitrary geometry of the magnetic system, and arbitrary primary fields. In particular, the saturation of the magnetic circuit must be modeled efficiently. Also, such algorithms should allow modeling MS for arbitrary ferromagnetic materials.

In the practice of magnetostatic calculations, three classes of MS are distinguished: planar, axisymmetric, and three-dimensional, and the calculation of three-dimensional systems is much more complicated than axisymmetric, and even more so than planar ones. But there is a subclass of MS that naturally combines the features of axisymmetric and three-dimensional systems. It is of great practical importance. These MSs are widely used primarily in electronic optics: a number of electron-optical devices [1—2], primarily klystrons and magnetrons. MS of this structure are also widely used in magnetic separation.

Such systems have an almost axisymmetric geometry, but, crucially, they have a three-dimensional part containing the working area of the device. This introduces three-dimensional perturbations in the initial axisymmetric MS, and such inclusions significantly distort the axisymmetric character of the field in the working part of the MS. These can be holes or slits in multibeam electron-optical systems, filter matrices in magnetic separators, and so on. Moreover, the volume of such a three-dimensional part is insignificant, less than 2—5% of the entire MS. However, the developer is interested in calculating the field strength in the working, three-dimensional part of the MS, and, as a rule, the calculation must be performed very accurately.

Therefore, formally, the formulation requires considering such an MS as a three-dimensional one, since it is the three-dimensional nature of the field that plays a decisive role in the operation of the device, for example, when focusing electron beams in the pole tip aperture zone in multipath electron-optical systems [1—2]. On the other hand, the rapid attenuation of three-dimensional disturbances at a distance from the working part makes it unreasonable to calculate the entire MS in a three-dimensional approximation, although almost all known programs are based on such principles. As already mentioned, the decisive role is played by the fact that it is in the working area that almost precise field calculation is required, and in the main part the calculation is not essential or can be performed with less accuracy. This allows us to consider such systems as a separate subclass of both axisymmetric and three-dimensional magnetic systems. Logically to call them quasi-axisymmetric systems (QAS). The calculation of these systems has significant features that distinguish them from the class of three-dimensional and axisymmetric systems MS.

It should be noted that the calculation of QAS as a three-dimensional MS is unsatisfactory, first of all, as it is very inefficient. In addition, since it is necessary to find the field strength in the immediate vicinity of the QAS working area, a high level of discretization is required in this area. If you keep approximately the same level of discretization in the working and axisymmetric parts of the MS, the number of discretization elements will be unacceptably large, and this will lead, as practice shows, to poor convergence of the iteration process. If the discretization is performed unevenly, it will lead to even worse convergence of the iterative process of determining the MS magnetization vector. The saturation of the magnetic circuit further complicates the situation. Consequently, for conventional algorithms, the iterative process will converge poorly for such systems and the result will be inaccurate.

Analysis of recent research and publications

The most elementary method for modeling MS is the finite difference method (FDM) [3], but for stationary problems it is currently only of theoretical importance. A more advanced method for calculating MS is the finite element method (FEM) [4—6]. But FEM has a significant drawback in terms of calculating open MS. For this method, it is necessary to set boundary conditions of the first kind, and such boundaries are chosen where the magnetic potential is approximately equal to zero. But this cannot be done precisely, which introduces an error in the calculation.

Another method for calculating MS is the method of integral equations for physical field vectors [7]. For open MS, it has a significant advantage over FEM.

Formulation of the study purpose

The purpose of this paper is to formulate and justify an algorithm for the efficient calculation of QAS based on the method of integral equations [7]. When designing electron-optical systems, a large number of numerical experiments are required to optimize the geometric parameters of the MS, so the algorithm must meet rather stringent requirements: both high accuracy and speed. It was mentioned earlier that calculating QAS as an axisymmetric MS will give a completely incorrect result. In turn, the calculation of QAS as a three-dimensional MS is also unsatisfactory, as it is very inefficient and has difficulties with the convergence of the iterative process. Therefore, a two-level solution algorithm is required to calculate the high accuracy QAS. Namely, first, the MS is calculated in the axisymmetric approximation, then the working part is calculated in the three-dimensional approximation, with the main axisymmetric part of the magnetic system being the initial field for calculating the working three-dimensional part. The paper proposes an effective algorithm for calculating QAS magnetostatic fields by the IR method, consisting of two stages.

Presenting main material

The general equation of magnetostatics for the three-dimensional case can be written in the form:

$$\overline{M}(x) = 2\lambda(x)H_0(x) + \lambda(x)/(2\pi)\int_D [3(\overline{M}(y)\cdot\overline{R})/R^5 - \overline{M}(y)/R^3]dV + \lambda(x)M(x)/3\,. \quad (1)$$

Here $\overline{M}(x)$ — magnetization of the environment, $\overline{R}$ — radius vector from point x to y, $H_0(x)$ — primary sources field, $\lambda(x)$ — known function that depends on the relative magnetic permeability at point x, it can be calculated using the magnetization curve. The area of magnets is divided into elements that have no intersection over a area having a volume greater than zero:

$$\overline{H}_M(x) = 1/(4\pi)\int_S (\overline{n}\cdot\overline{M})/R^3 ds\,. \quad (2)$$

Here $\overline{M}$ — constant vector, and the vector $\overline{n}$ — outer normal of the surface S. In order to obtain the final field, we need to sum (2) over all elementary volumes, i.e.

$$\overline{H}_M(x) = \sum_{I=1}^{K} \overline{H}_M^I(x)\,. \quad (3)$$

QAS calculation algorithm

Main task of the QAS calculation is to calculate the magnetic field strength vector (3) at the specified points located in the three-dimensional working area. This calculation must be performed with maximum accuracy. From the previous, an effective algorithm for calculating such MSs follows. It is advisable to perform the calculation in two stages.

1) Let's divide the total MS into two parts: the working areas, where the field is three-dimensional, and the general axisymmetric part, where the field is practically axisymmetric. Let us denote the total MS by D, the axisymmetric part by D_V, and the working three-dimensional domain by D_T. It's obvious that $D = D_V \cup D_T$, and mes $(D_V \cap D_T) = 0$. Thus, the entire system consists of these areas only, and the D_V and D_T areas may have a common surface, but not a common volume. Let us form an array of centers of elementary areas $D_T = \cup D_i$, belonging to the working area, i.e., the area D_T. First stage of the calculation is performed in the axisymmetric approximation for the entire MS, and the D_T area is also considered in the axisymmetric approximation. To be more precise, the three-dimensional D_T area is replaced by the axisymmetric D_{TV} area, which is as close to it as possible. This means that the three-dimensional elements of areas ignored. For example, the area of holes in the D_T is treated as a solid metal. Next, the MS calculated in the axisymmetric approximation.

2) At the second stage, the part of the system $D_M = D \backslash D_{TV}$ source of the primary field, and the field of the D_{TV} area is not taken into account. To do this, at the points of the centers of the elementary D_T areas and the outer points where the magnetic field strength is to be found, the field from the D_M area is calculated by equation (3). It is possible because at the first stage, the field of the magnetization vector in the D_M area is known. Now, the second stage of the calculation can be performed — the three-dimensional one. To do this, a three-dimensional magnetostatics problem is solved for the D_T, area, with the constant axisymmetric field of the MS sources and the magnetization field of the D_M

area used as the primary field. The basis of this algorithm is the fact that the volume of the three-dimensional D_T area is insignificant, so its influence on the distribution of the magnetization vector in the D_V area is practically absent, or rather, it decreases rapidly. The most difficult point for calculating the QAS is to mentally separate the D_T and D_V areas.

Example of QAS calculation

As an example, we considered a magnetostatic multistellar klystron, an electronic optics device. In Fig. 1 shows a general sketch of the device. The system is quasi-axisymmetric, where the axis of rotation is the Y-axis, and the system also has a plane of symmetry, the Z-axis. Primary sources of the magnetostatic field are direct current coils, which are crossed out in the sketch. The D_T area is marked in black. It contains 8 through cylindrical holes, each with a diameter of 3 mm. The holes are used to control the flow of electrons passing through the device. These holes distort the axisymmetric field pattern in the D_T area. Fig. 2 shows an enlarged part of the magnetic system, the three-dimensional GT area is marked in black, as in Fig. 1.

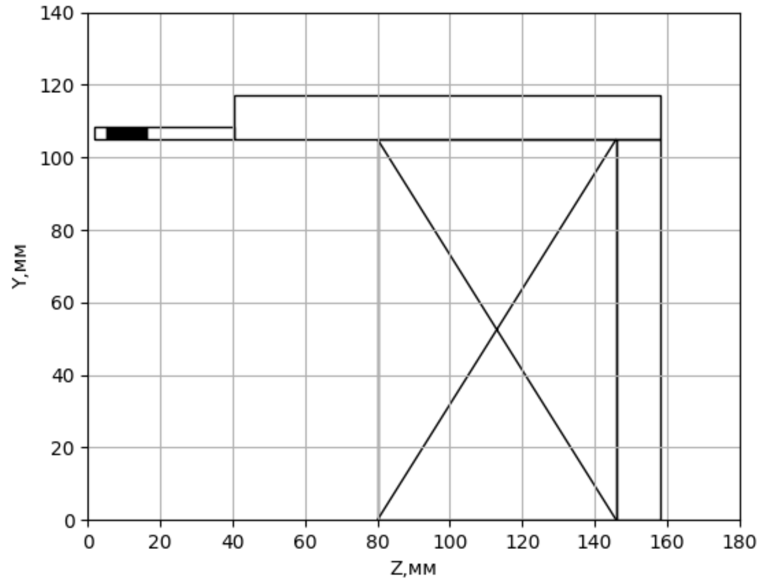

Fig. 1. Geometry of the axisymmetric part of the electronic optics device

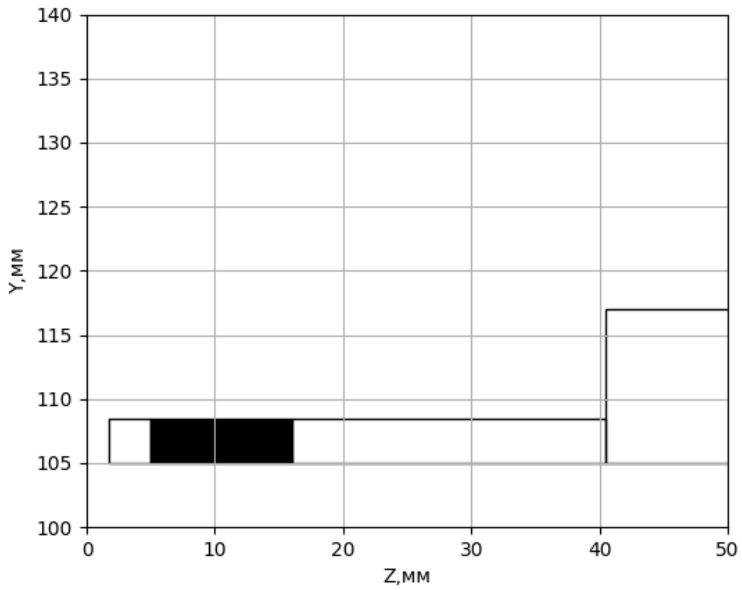

Fig. 2. Enlarged part of the workspace

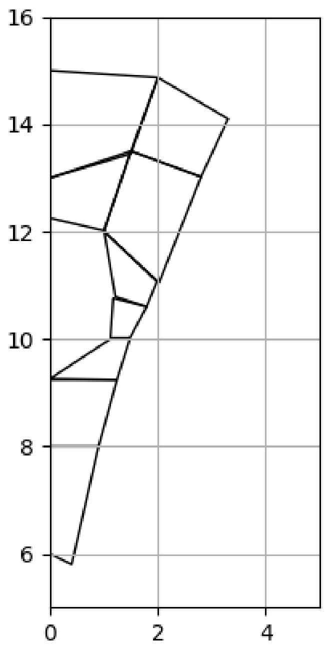

Fig. 3. The main fragment of the three-dimensional area and its initial discretization

Fig. 3 shows an enlarged main fragment of the three-dimensional areas, whose height does not exceed 11 mm, and its discretization into primary polyhedral. This is only half of the hole, the other half is symmetrical about the Y-axis. Each of the polyhedral was divided into elementary polyhedral, and there were 201 such polyhedral in this calculation. In addition, there were the same number in the symmetric part. QAS material is nonlinear, electrical steel. Fig. 4 and 5 show the results of QAS calculations. In Fig. 4 is the Bz(z) dependence, and Fig. 5 is the By(z) dependence. The observation points were selected in the center of the hole with coordinate Y=11, along the entire length of the hole along the Z axis. The number of observation points is N=21.

The developer was primarily interested in the By(z) curve, as the correct operation of this electronic optics device depended on it.

Experimental data confirmed the calculation results, with a discrepancy of 3—4%. It should be noted that with the increase in the number of elementary polyhedra, the calculation accuracy slowly increases. The process of calculating the SLAR coefficients for the three-dimensional domain took the most time. The time for solving the problem at the first stage is insignificant compared to the second, three-dimensional stage.

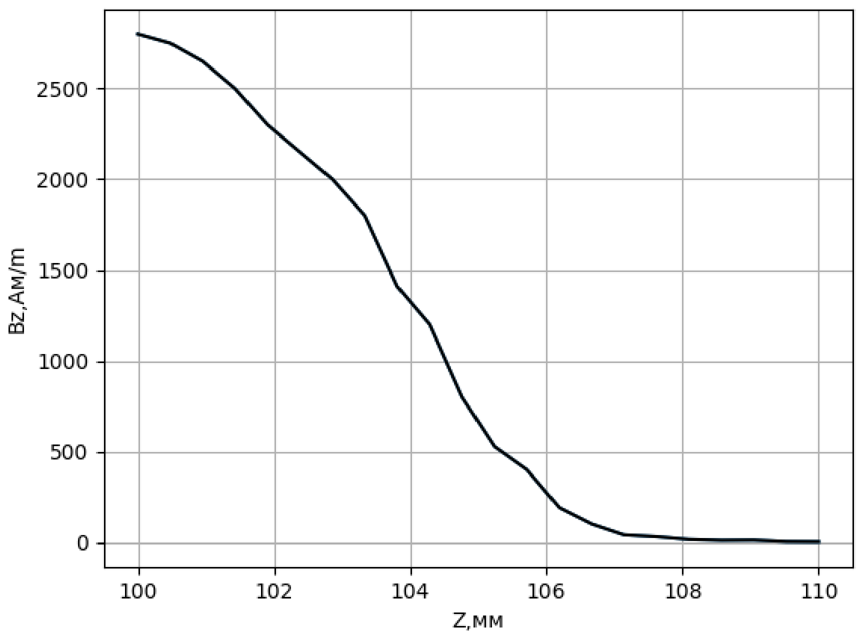

Fig. 4. Distribution of the Bz component of the field along the hole axis

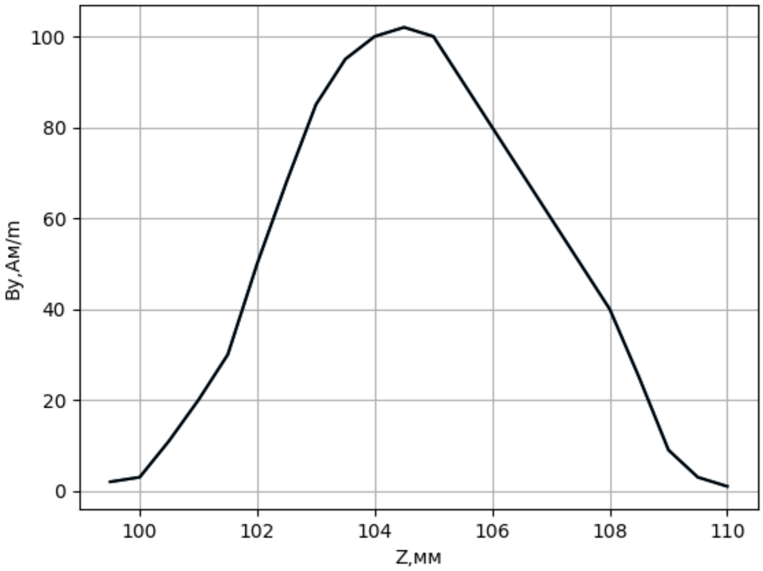

Fig. 5. Distribution of the By component of the field along the axis of the hole

Conclusions

1. We consider a subclass of MS that naturally combines the features of axisymmetric and three-dimensional systems. It is shown that the calculation of such MS has significant features.

2. An efficient algorithm for calculating QAS magnetostatic fields by the IR method, consisting of two stages, is proposed.

3. A practical calculation of the magnetic system is carried out.

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Надійшла до редколегії 15.01.2024