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# DEVELOPMENT OF A MATHEMATICAL MODEL OF THE METHOD OF ANALYZING HIERARCHIES FOR MAKING MANAGERIAL DECISIONS

# РОЗРОБКА МАТЕМАТИЧНОЇ МОДЕЛІ МЕТОДУ АНАЛІЗУ ІЄРАРХІЙ ДЛЯ ПРИЙНЯТТЯ УПРАВЛІНСЬКИХ РІШЕНЬ

*The study considers approaches to the implementation of the method of analyzing hierarchies and develops a mathematical model for solving management decision-making problems, namely choosing the optimal development project of a hotel business enterprise from several alternative projects.*

*The work also tested the technique of six filters, which combined the consistent use of all standard decision-making criteria under conditions of uncertainty, namely, the Wald, Savage, Hurwitz, average risk, Bayes-Laplace, Hodge-Lehmann criteria to confirm the consistency of the obtained results.*

***Keywords****: analytical hierarchy process (AHP), Saaty scale, priority vector, goal matrix, agreement index, random index.*

*У роботі досліджено підходи щодо реалізації методу аналізу ієрархій і розроблено математичну модель розв’язання задач прийняття управлінських рішень, а саме вибору оптимального проєкту розвитку з декількох альтернативних проєктів.*

*При виконанні дослідження було проаналізовано методику застосування методу аналізу ієрархій (МАІ), його основні характеристики та структуру, підходи до виконання попарних порівнянь в МАІ, процедуру визначення вектора пріоритетів та оцінювання міри погодженості отриманих оцінок. В роботі була апробована методика шести фільтрів, що поєднала послідовне використання всіх стандартних критеріїв прийняття рішень в умовах невизначеності, а саме, критерії Уальда, Севіджа, Гурвіца, середнього ризику, Бейеса-Лапласа, Ходжа-Лемана.*

*За даною методикою було розроблено модель для оптимального вибору прогнозного проєкту розвитку підприємства готельного бізнесу, перевірено його узгодженість з результатами методики використання шести фільтрів-критеріїв прийняття рішень в умовах невизначеності. Завдяки застосованій методиці МАІ можливо не лише виконати прогнозування і визначити оптимальний проєкт чи напрям розвитку, а й визначити технологію коригування — впливу на систему для реалізації бажаного проєкту чи асиміляції декількох проєктів з високою ймовірністю успішної реалізації. При виконанні попарних порівнянь на кожному рівні були визначені найважливіші чинники впливу на досліджуваний показник, що також є корисною інфо-*

*рмацією для адміністрації підприємства для наступних аналітичних досліджень і коригування проєкту розвитку у випадку змін на певних рівнях ієрархії. Чисельні розрахунки було виконано в системі комп'ютерної математики MathCAD і в ET Excel.*

*Запропонована модель може бути застосована для розв'язання задач прийняття управлінських рішень при прогнозуванні діяльності, вибору оптимального проєкту подальшого розвитку підприємств готельного, видавничого, ресторанного бізнесу та інших; для виявлення найважливіших чинників впливу на складові прогнозного проєкту, що є корисним при виникненні необхідності коригування обраного проєкту в процесі його реалізації.*

***Ключові слова**: метод аналізу ієрархій, шкала Сааті, вектор пріоритетів, матриця цілей, індекс погодженості, випадковий індекс.*

## Problem's Formulation

When modeling enterprise development strategies, when making management decisions, there is a need to take into account a large number of influencing factors and develop various scenarios of system development, since the studied processes have a high level of uncertainty. Uncertainty arises in the tasks of making management decisions, in which the expert, the evaluator does not know the entire set of influencing factors, which, in addition, can themselves change and depend on other factors. The situation of uncertainty is characterized by the fact that the choice of a certain project can lead to any result from a certain set of consequences. Therefore, approaches for the development of mathematical models of managerial decision-making under conditions of uncertainty are currently urgent and relevant.

## Analysis of recent research and publications

The analytical hierarchy process is one of the best methods for solving multi-parameter problems under conditions of uncertainty [1]. The AHP was proposed by the American mathematician Thomas Saati, who had an idea based on the principle of prioritization. This opinion was later formed in the AHP. The analytical hierarchy process is considered a general approach to measurement theory, it is used to create scales that make it possible to perform both discrete and continuous pairwise comparisons in hierarchical structures [2].

The decision-making process in the AHP combines psychological aspects and mathematical apparatus, since the compared indicators receive expert evaluations in accordance with the researchers' understanding of the problem. Then the received information is structured and analyzed using the mathematical apparatus of linear and vector algebra. This method makes it possible to check the consistency of the expert's assessments. With the help of the AHP, the investigated system or problem is presented in the form of a hierarchy, which allows avoiding complex comparisons by replacing them with pairwise comparisons. For this, matrices of pairwise comparisons are built, which is not a difficult task for experts.

## Formulation of the study purpose

The purpose of the work is to research the methodology of implementing the hierarchy analysis method and its application for modeling management decision-making problems.

The object of the research is the implementation of the method of the analytical hierarchy process (AHP) for modeling the tasks of making managerial decisions. The subject of the research was a set of theoretical and practical questions related to the method of analyzing hierarchies and its implementation by means of linear algebra and checking the consistency of results according to known decision-making criteria under conditions of uncertainty.

During the work, the following tasks were defined:

– to study and analyze the methodology of the implementation of the AHP, its main characteristics and structure, the method of performing pairwise comparisons in the AHP, the procedure for determining the vector of priorities and evaluating the degree of agreement of the received assessments;

– to develop a model for solving the problem of management decision-making, namely, choosing the optimal development project from several alternative development projects of the hotel business enterprise, verifying the obtained results using the six-filter method.

The work tested the technique of six filters, which combined the consistent use of all standard decision-making criteria under conditions of uncertainty, namely, Wald, Savage, Hurwitz, average risk, Bayes-Laplace, Hodge-Lehmann criteria.

**Presenting main material**

The method of the analytical hierarchy process (AHP) is a systematic procedure that is based on a hierarchical representation of the elements that make up the essence of a complex problem and an analysis of the interaction between them.

The problem is split into simpler elements with further evaluation of the relative level of interaction of the objects of the resulting hierarchical structure. The method is based on the principles of decomposition and synthesis, the implementation of which makes it possible to reduce the number of possible errors in the process of obtaining information from experts and evaluating the priority of the level factors and finding alternative solutions [2].

The construction of the hierarchy begins with the definition of the main research problem. Next, a hierarchy is built — a network containing the goal — the focus of the problem, intermediate levels (aspects of the goal, criteria), actors — groups of people involved in the system under study, alternatives that form the lowest hierarchical level. Subordination of elements of lower levels to elements of higher levels is established. The elements of each level are usually considered to be independent, although they may interact with each other. The main task in the constructed hierarchy is to evaluate the higher levels, analyzing the interactions of different levels, and not from direct dependence on elements at other levels [2—6]. For the clarity of the analysis, a structural model of the hierarchical representation of the components of the problem is being built. The structure of the AHP model of the decision-making problem in its general form is the diagram shown in fig. 1.

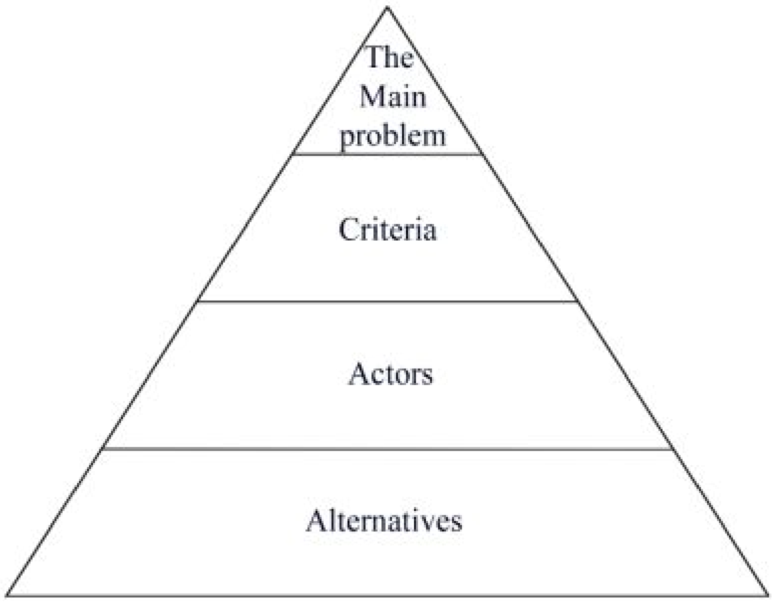


*Fig. 1.* Scheme of the AHP model of the decision-making problem

Consider the structure of the method.

Stage 1: determination of the main problem to be solved, for example, determination of the optimal development project.

Stage 2: delineation of the general system, definition of forces — criteria by which the quality of solving the problem will be evaluated.

Stage 3: determination of interested parties of the system — actors, their interests and goals, division of actors into groups. At the II and III stages, the problem is decomposed and its multi-level components are determined.

Stage 4: identification of alternatives — predictive development projects, construction of a hierarchical structure of the problem model.

Stage 5: expert evaluation of advantages: matrixes of pairwise comparisons are built; first, all factors are compared with each other to determine their importance for the focus of the problem. Then, similarly, for each of the factors, a pairwise comparison of the actors is performed to assess their influence on the factors and a pairwise comparison of alternative projects is performed to determine the degree of their influence on the actors.

Stage 6: construction of local priorities: in each of the matrices of pairwise comparisons, a normalized matrix is built, a vector of priorities is determined, the components of which determine the dominant factors at each level of the hierarchy.

Stage 7: assessment of the consistency of the matrices of pairwise comparisons. For this, for each matrix and its vector of priorities, a matrix-column of judgments is built and the values of the agreement index and the agreement ratio are found.

Stage 8: implementation of the synthesis of global priorities. To do this, it is necessary to multiply the matrix of local priorities of the influence of actors on factors by the matrix-column of the vector of priorities of the influence of factors on the main problem and obtain the matrix-column of weighting coefficients. Then multiply the priority matrix of the impact of alternative projects on the actors by the column matrix of the obtained weighting coefficients. We will get a vector — a column of priorities that correspond to the received decisions.

Stage 9: formulation of conclusions and proposals for decision-making, generalization of the main factors of influence at each level of the hierarchy.

Stage 10: formulation of the decision, conclusions of the study, determination of the priority development project.

After performing all the described stages of the method of analysis of hierarchies, the quantitative results of the importance of each component of the main problem are determined. In this way, the mathematical tools of linear algebra make it possible to obtain objective quantitative estimates of the importance of each of the factors and alternative projects. AHP transforms comparisons, which are often empirical, into numerical values that are mathematically processed using the rules of matrix operations. The ability to transform empirical data into mathematical models is the main advantage of the method of analyzing hierarchies compared to other methods of comparison [3—6]. The weight of each factor allows you to assess the importance of each element within the given hierarchy.

After carrying out all comparisons and determining the relative weight between the components of each level of the hierarchy, the numerical probability of each alternative — a forecast project or development strategy — is determined.

The method of analyzing hierarchies when building a scale for different components of a problem uses a measure of the degree of influence of each factor of one level on the components of a higher level or on the final goal. This measure is formed as a result of expressing judgments about the level of influence of these indicators.

Matrices of pairwise comparisons (MPC) of elements of a certain level are compiled according to the results of the evaluators' opinions, but the numerical advantages of the elements evaluated in pairs are determined according to the ranking according to Saati's psychometric scale [2] (Tabl. 1).

*Table 1.* Saati scale

| Intensity of relative importance (k) | Characteristic |
|---|---|
| 1 | Equally important |
| 3 | Moderate advantage |
| 5 | A significant advantage |
| 7 | A significant advantage |
| 9 | A very big advantage |
| 2, 4, 6, 8 | Intermediate values |
| 1/k | Inverted values |

To compare certain level n objects according to the intensity of their influence on a certain indicator of a higher level of the hierarchy, a square inverse-symmetric matrix of pairwise comparisons is made, the elements of the main diagonal of which are equal to units:

$$M = \begin{pmatrix} 1 & a_{12} & \dots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \dots & a_{2n} \\ . & . & . & . \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \dots & 1 \end{pmatrix}.$$

Each element of the matrix means a measure of the superiority of the factor $a_i$ over the factor $a_j$, which is expressed by the evaluator according to the Saati scale and takes values from 1 to 9. For each element of the matrix, the property is fulfilled: $a_{ij} = \frac{1}{a_{ji}}$ .

The next step of the research is determining the priority of the criteria, for this a set of local priorities is compiled from the matrix obtained above. For this:

1) The geometric mean value of the elements of each row of the matrix of pairwise comparisons is calculated: $V_i = \sqrt[n]{a_{i1} \cdot a_{i2} \cdot \dots \cdot a_{in}}, i = 1,2,\dots,n$ .

As a result, we get a vector $V = \begin{pmatrix} V_1 \\ V_2 \\ \dots \\ V_n \end{pmatrix}$

2) The vector of prioritities is determined. For this the vector *V* is normalized by dividing each element by the sum of all its components:

$$VP_i = \frac{V_i}{\sum_{i=1}^{n} V_i} .$$

With the help of this approach, it is possible to establish the ranking of the importance of elements in relation to a certain higher-level factor and their relative values.

An important stage of the research is the assessment of the degree of consistency of the obtained values. For this, the agreement index (IA) is calculated. This indicator helps to determine whether violations of numerical and transitive agreement have occurred.

To find the concordance index value:

1) determine the matrix-column of judgments л to calculate the component of which we add the elements of each column of the matrix M and multiply by the corresponding component of the priority vector *VP*:

$$\lambda_j = VP_j \cdot \sum_{i=1}^{n} a_{ij},\ j = 1,2,\dots,n ;$$

2) Calculated

$$\lambda \max = \sum_{j=1}^{n} \lambda_j ;$$

3) Calculate the index of agreement of local priorities (IA) according to the formula:

$$IA = \frac{\lambda \max - n}{n-1},$$

where *n* is the number of compared indicators, that is, the order of the matrix of pairwise comparisons.

To determine whether this agreement is acceptable, the agreement index (IA) is compared with the random index (IR) and the value of the agreement ratio is calculated: $RA = \frac{IA}{IR}$.

The random index (IR) is the index of agreement, which is calculated for a square inverse-symmetric matrix of the nth order, the elements of which are generated by a sensor of random numbers evenly distributed over the interval of values $\frac{1}{9}; \frac{1}{8}; \frac{1}{7}; \frac{1}{6}; \frac{1}{5}; \frac{1}{4}; \frac{1}{3}; \frac{1}{2}$; 1; 2; 3; 4; 5; 6; 7; 8; 9.

The ratio of agreement (RA) is considered good if its value is less than or equal to 0,1, and the degree of agreement is acceptable, and if , then the degree of agreement is considered unacceptable and experts need to revise their assessments.

In this study, the problem of forecasting the activity of the hotel business enterprise was solved using the method of analysis of hierarchies.

Stage 1 — is the definition of the main problem. The focus of the hierarchy, the main problem of the task is to determine the optimal hotel development project.In the following stages, the decomposition of the problem is performed.

Stage 2. The choice of indicators — forces, by which the quality of solving the problem will be evaluated — set *K:*

$k_1$ — price policy (optimal price for accommodation, meals, additional services, promotions);

$k_2$ — quality of service (staff, comfort, restaurant service, service in hotel rooms);

$k_3$ — loyalty program for regular customers (discounts, priority booking);

$k_4$ — safety (hygiene, sanitary safety of catering, personal safety, cyber safety, quality control);

$k_5$ — reliable and high-quality suppliers;

$k_6$ — organization of time (transfer, additional services, excursions, organizational issues).

Stage 3. Definition of interested parties of the system — actors of the hierarchy, which are described by the set *A:*

$a_1$ — hotel visitors (recipients of services);

$a_2$ — hotel employees;

$a_3$ — hotel administration;

$a_4$ — state bodies involved in the organization and control of hotel activities.

Stage 4. Determination of alternatives — predictive projects and construction of a hierarchical structure of the problem.

During the analysis and forecasting of the hotel's activities, 3 forecast projects were developed: Project 1 "Improvement of basic services, automation and digitization of services", Project 2 "Extension of additional services", Project 3 "Expansion of business services".

Based on the results of the problem decomposition and the identification of alternatives, a hierarchical structure of the problem was built (Fig. 2).

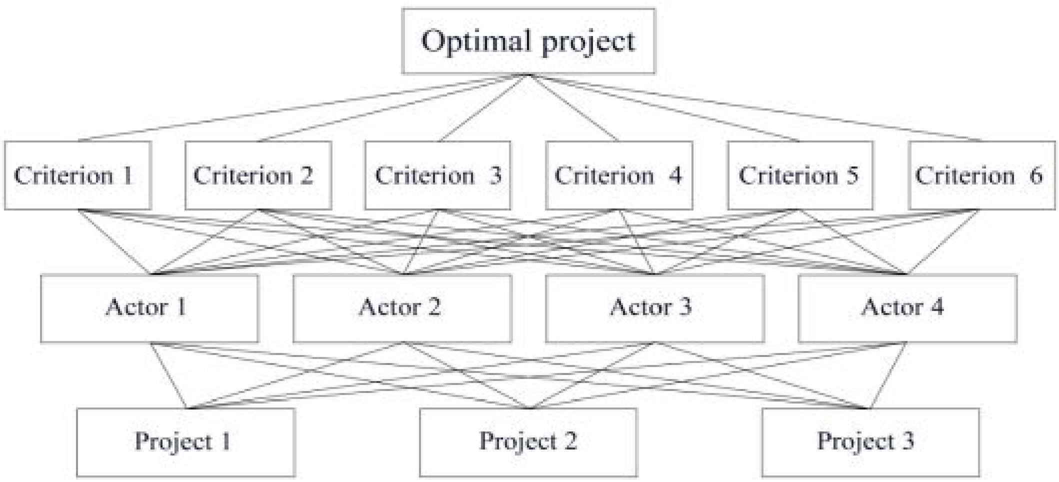


*Fig. 2.* Hierarchical structure of the problem

After defining the hierarchical model, they proceed to expert evaluation of advantages, determination of dominant factors at each level of the hierarchy, and verification of the degree of agreement of the obtained evaluations.

We perform 5—7 stages of the AP3 for elements of the corresponding levels.

The influence of criteria on the focus of the hierarchy. First, a comparison of indicators —

criteria, which constitute the first level of the hierarchy, is performed to determine the importance of their influence on the focus — the zero level of the hierarchy. To establish the relative importance of the elements of the hierarchy — criteria for the focus — the development project, we build a pairwise comparison matrix (PCM) of the corresponding indicators according to the Saati intensity scale:

$$M1 = \begin{pmatrix} 1 & 2 & 5 & 3 & 4 & 6 \\ 0,5 & 1 & 3 & 2 & 3 & 4 \\ 0,2 & 0,33 & 1 & 0,33 & 0,2 & 5 \\ 0,33 & 0,5 & 3 & 1 & 3 & 5 \\ 0,25 & 0,33 & 5 & 0,33 & 1 & 0,33 \\ 0,16 & 0,25 & 0,2 & 0,2 & 3 & 1 \end{pmatrix}.$$

We build a vector of priorities by dividing the components of the vector by the sum of its VS components:

$$VP = \begin{pmatrix} 0,386 \\ 0,234 \\ 0,068 \\ 0,182 \\ 0,077 \\ 0,053 \end{pmatrix}.$$

From the components of the priority vector, we can see that price policy is the dominant factor in influencing the development and forecasting of its activity (38.6 %). The quality of service (23.4 %) and the safety factor (18.2 %) also have a significant impact.

The index of agreement of local priorities (IA) is calculated according to the formula:

$$IA = \frac{\lambda\, max - n}{n-1} = \frac{6{,}631-6}{5} = \frac{0{,}631}{5} = 0{,}126,$$

where n is the number of indicators to be compared (at this level of the hierarchy $n$ = 6).

To determine whether this agreement is acceptable, we compare it with the random index (IR), the value of which at n=6 is equal to 1.24. Agreement relationship:

$$RA = \frac{0{,}126}{1{,}24} = 0{,}102 \approx 0{,}1.$$

So, degree of agreement is high.

Influence of actors on criteria. We perform a pairwise comparison for each indicator-criterion to determine the level of intensity of influence of the actors on the factors and determine the dominant elements in each comparison and build matrices of pairwise comparison of the level according to the algorithm given earlier. Based on the results of the research, we compile a matrix of actors' priorities, each column of which is a vector of actors' priorities relative to the corresponding factor:

$$PCM1 = \begin{pmatrix} 0{,}488 & 0{,}629 & 0{,}237 & 0{,}476 & 0{,}502 & 0{,}563 \\ 0{,}126 & 0{,}074 & 0{,}517 & 0{,}130 & 0{,}170 & 0{,}227 \\ 0{,}274 & 0{,}200 & 0{,}077 & 0{,}072 & 0{,}068 & 0{,}128 \\ 0{,}112 & 0{,}097 & 0{,}169 & 0{,}322 & 0{,}260 & 0{,}082 \end{pmatrix}.$$

The column values of the matrix can be seen visitors (48,8 %) and hotel administration (27,4 %) exert the greatest influence on price policy; visitors (62,9 %) and hotel administration (20 %) have the greatest influence on quality; employees (51,7 %) and visitors (23,7 %) are the most important factors affecting the loyalty program criterion $k_3$ for regular customers; visitors (47.6 %) and government bodies (32,2 %) have the greatest influence on the safety criterion $k_4$;

visitors (50,2 %) and government bodies (26 %) have the greatest influence on the supplier criterion $k_5$; according to the components of the last column of the matrix, we can see that visitors (56,3 %) and employees (22,7 %) have the greatest influence on the Time organization criterion $k_6$. The degree of agreement of local priorities for all obtained estimates is high.

The next step is to assess the impact of alternative projects on actors. As a result, we will get 4 matrices of the third order (since we have 3 alternative projects) and determine the components of their priority vectors, which will make up the matrix of actors' goals and show which of the projects is dominant for which actors. Let's evaluate the degree of agreement between the values of the provided expert assessments (when $n=3$, the random index $IR= 0,58$). The obtained research results are summarized in a tabl. 2.

*Table 2.* The results of a pairwise comparison of the impact of alternative development projects on actors

| Projects | Matrices of pairwise comparisons $(M_i)$ | Vector of priorities ($RA_i$) | Column matrix of judgments ($л_i$) |
|---|---|---|---|
| ***$a_1$* - hotel visitors (recipients of services)** | | | |
| Project 1 | $M_1 = \begin{pmatrix} 1 & 5 & 3 \\ 0,2 & 1 & 0,333 \\ 0,333 & 3 & 1 \end{pmatrix}$ | **0,637** | 0,977 |
| Project 2 | | **0,105** | 0,942 |
| Project 3 | | **0,258** | 1,119 |
| | | $\lambda$1max | **3,038** |
| **Index of agreement of local priorities (IA1)** | | | **0,019** |
| **Agreement relationship (RA1)** | | | **0,033** |
| ***$a_2$* - hotel employees** | | | |
| Project 1 | $M_2 = \begin{pmatrix} 1 & 0,333 & 3 \\ 3 & 1 & 5 \\ 0,33 & 0,2 & 1 \end{pmatrix}$ | **0,258** | 1,119 |
| Project 2 | | **0,637** | 0,977 |
| Project 3 | | **0,105** | 0,942 |
| | | $\lambda$2max | **3,038** |
| **Index of agreement of local priorities (IA2)** | | | **0,019** |
| **Agreement relationship (RA2)** | | | **0,033** |
| ***$a_3$* - hotel administration** | | | |
| Project 1 | $M_3 = \begin{pmatrix} 1 & 5 & 0,5 \\ 0,2 & 1 & 0,2 \\ 2 & 5 & 1 \end{pmatrix}$ | **0,352** | 1,127 |
| Project 2 | | **0,089** | 0,976 |
| Project 3 | | **0,559** | 0,951 |
| | | $\lambda$3max | **3,054** |
| **Index of agreement of local priorities (IA3)** | | | **0,027** |
| **Agreement relationship (RA3)** | | | **0,046** |
| ***$a_4$* - state bodies involved in the organization and control of the hotel's activities** | | | |
| Project 1 | $M_4 = \begin{pmatrix} 1 & 0,333 & 0,2 \\ 3 & 1 & 0,25 \\ 5 & 4 & 1 \end{pmatrix}$ | **0,101** | 0,905 |
| Project 2 | | **0,225** | 1,203 |
| Project 3 | | **0,674** | 0,977 |
| | | $\lambda$4max | **3,085** |
| **Index of agreement of local priorities (IA4)** | | | **0,043** |
| **Agreement relationship (RA4)** | | | **0,074** |

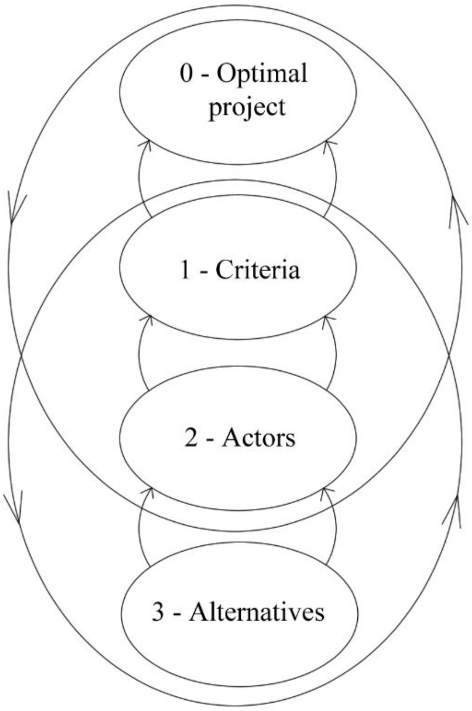


*Fig. 3.* Scheme of synthesis of priorities of different levels of the hierarchy

From the obtained pairwise comparisons according to the components of the priority vectors, it was determined that for visitors the dominant is the first project (63,7 %), for hotel employees — the second project (63,7 %), for the administration and state bodies — the third alternative project (55,9 % and 67,4 % respectively). The ratio of agreement of local priorities for all matrices of pairwise comparisons is less than 0,1, which indicates a high level of agreement of estimates.

We compile a matrix of priorities of alternative projects (PCM2) relative to different categories of actors:

$$PCM2 = \begin{pmatrix} 0{,}637 & 0{,}258 & 0{,}352 & 0{,}101 \\ 0{,}105 & 0{,}637 & 0{,}089 & 0{,}225 \\ 0{,}258 & 0{,}105 & 0{,}559 & 0{,}674 \end{pmatrix}.$$

Each column of which is a vector of project priorities relative to the corresponding category of actors.

After that, we perform 8—10 stages of the method. At the 8 stage, it is necessary to carry out a synthesis of priorities (Fig. 3). This procedure in linear algebra corresponds to the matrix multiplication operation.

The obtained system of matrices and their priority vectors allows to perform the analysis of forecast development projects and to determine the probabilities of their implementation. We make such an assessment starting from the top of the hierarchy. To assess the degree of importance of the actors in relation to the factors affecting the development and stable operation of the hotel, we multiply the matrix on the right, consisting of vectors of actors' priorities for each criterion, by the column matrix, which is a vector of priorities of six factors:

$$VWC = PCM1 \cdot VP = \begin{pmatrix} 0{,}488 & 0{,}629 & 0{,}237 & 0{,}476 & 0{,}502 & 0{,}563 \\ 0{,}126 & 0{,}074 & 0{,}517 & 0{,}130 & 0{,}170 & 0{,}227 \\ 0{,}274 & 0{,}200 & 0{,}077 & 0{,}072 & 0{,}068 & 0{,}128 \\ 0{,}112 & 0{,}097 & 0{,}169 & 0{,}322 & 0{,}260 & 0{,}082 \end{pmatrix} \cdot \begin{pmatrix} 0{,}386 \\ 0{,}234 \\ 0{,}068 \\ 0{,}182 \\ 0{,}077 \\ 0{,}053 \end{pmatrix} = \begin{pmatrix} 0{,}507 \\ 0{,}150 \\ 0{,}183 \\ 0{,}160 \end{pmatrix}.$$

We obtain a matrix-column of the weighting coefficients of the actors' influence on the success criteria of the forecast development project. The result of the product indicates that actors — visitors account for 50,7 % of the influence on success criteria, that is, they are the main actors for the analysis of forecast projects.

Next, we multiply the matrix of priorities of alternative projects (PCM2) with respect to the preferences of different groups of actors by the matrix-column of the influence of actors on the success criteria of the forecast development project (VWC):

$$WVP = PCM2 \cdot VWC = \begin{pmatrix} 0{,}637 & 0{,}258 & 0{,}352 & 0{,}101 \\ 0{,}105 & 0{,}637 & 0{,}089 & 0{,}225 \\ 0{,}258 & 0{,}105 & 0{,}559 & 0{,}674 \end{pmatrix} \cdot \begin{pmatrix} 0{,}507 \\ 0{,}150 \\ 0{,}183 \\ 0{,}160 \end{pmatrix} = \begin{pmatrix} 0{,}442 \\ 0{,}201 \\ 0{,}357 \end{pmatrix}.$$

In this way, the weight values of forecast development projects (WVP) were obtained, taking into account all the priorities specified in the statement of the task, which correspond to the probability of their successful implementation (Tabl. 3).

*Table 3.* Probability of successful implementation of forecast projects

| Projects | Weighting coefficients | Probabilities |
|---|---|---|
| Project 1 «Improvement of basic services, automation and digitization of» | 0,442 | 44,2 % |
| Project 2 «Extension of additional services» | 0,201 | 20,1 % |
| Project 3 «Expansion of business services» | 0,357 | 35,7 % |

According to the results of the implementation of the method of analysis of hierarchies, the first development project is optimal (44,2 %). Thanks to this methodology, it is possible not only to perform forecasting and determine the optimal project or direction of development, but also to determine the technology of adjustment — influence on the system for the implementation of the desired project or assimilation of several projects with a high probability of successful implementation. When performing pairwise comparisons at each level, the most important factors affecting the studied indicator were determined, which is also useful information for the enterprise administration for subsequent analytical studies and adjustment of the development project in case of changes at certain levels of the hierarchy.

To check the reliability of the obtained management decision according to the AHP method, we will apply the 6-filter method to determine the optimal hotel development project [7—9].

Development projects consist of a set of actions

$D = \{d_1, d_2, d_3\}$

where $d_1$ — improvement of basic services, automation and digitization of services;

$d_2$ — expansion of additional services;

$d_3$ — expansion of business services.

Each of the action projects can lead to certain consequences, representing a set of $C = \{c_1, c_2, c_3, c_4\}$

where $c_1$ — the implementation of the project will not require additional review within 2—3 years;

$c_2$ — the project needs to be reviewed in a year;

$c_3$ — the implemented project must be reviewed after six months;

$c_4$ — the project must be reviewed within 3 months of application.

Taking into account additional information about uncertainty, the probability distribution of Psi on the set of results for each action — profits and losses — was formed.

Let the following probability distributions for each of the development projects be obtained for the hotel and restaurant business enterprise using the expert evaluation method:

$P_{c1} = \{0,3; 0,35; 0,20; 0,15\};$

$P_{c2} = \{0,15; 0,26; 0,34; 0,25\};$

$P_{c3} = \{0,2; 0,3; 0,3; 0,2\}.$

Let the expected profits (thousands of hryvnias) be determined depending on the selected project and its results, which are given in the table of values of project selection parameters (Tabl. 4).

*Table 4.* Set of project selection parameter values (profits)

| A set of actions - projects | Application results (thousand UAH) | | | |
|---|---|---|---|---|
| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
| $d_1$ | 65000 | 50000 | 45000 | 27000 |
| $d_2$ | 38000 | 42000 | 47000 | 35000 |
| $d_3$ | 40000 | 33000 | 31000 | 44000 |

Thus, the state for decision-making is described by a matrix

$$A = \begin{pmatrix} 65 & 50 & 45 & 27 \\ 38 & 42 & 47 & 35 \\ 40 & 33 & 31 & 44 \end{pmatrix},$$

where elements aij are expected profits from the i-th profit according to the j-th consequence (million hryvnias).

We use each of the filters sequentially.

1) Wald's maximum criterion or a variant of extreme pessimism. According to this filter, the optimal project is the one for which the value of the expected profit is determined by the formula:

$$K_1 = \max_{i\in 1,2,\dots m}\left(\min_{j\in 1,2,\dots,n} a_{ij}\right).$$

Therefore, according to this filter, it is necessary:

– For each project, we choose the minimum utility value (million hryvnias):

$\min Kd_1 = \min\{65;\ 50;\ 45;\ 27\} = 27;$

$\min Kd_2 = \min\{38;\ 42;\ 47;\ 35\} = 35;$

$\min Kd_3 = \min\{40;\ 33;\ 31;\ 44\} = 31.$

– Choose the maximum value from the selected minimum values

$K_1 = \max\{\min Kdi\} = \max\{27; 35; 31\} = 35,\ \ i = 1,2,3.$

Hence, according to the Wald criterion, the second project is optimal.

2) Savage's criterion, according to which filtering is directed to the minimum risk. The optimal project is determined by the formula:

$$K_2 = \min_{i\in 1,2,\dots,m}\left(\max_{j\in 1,2,\dots n}\left(\max_{i\in 1,2,\dots,m} a_{ij} - a_{ij}\right)\right).$$

We compile the loss matrix $A_2$, each element of which is equal to the difference between the largest value of the element of this column and the given element

$$A_2 = \begin{pmatrix} 0 & 0 & 2 & 17 \\ 27 & 8 & 0 & 9 \\ 25 & 17 & 16 & 0 \end{pmatrix}.$$

So by this filter

$$\max Zd_1 = \max\{0;\ 0;\ 2;\ 17\} = 17;$$

$$\max Zd_2 = \max\{27;\ 8;\ 0;\ 9\} = 27;$$

$$\max Zd_3 = \max\{25; 17; 16; 0\} = 25;$$

$$K_2 = \min\{\max Zdi\} = \min\{17;\ 27;\ 25\} = 17,\ \ i = 1, 2, 3.$$

So, according to Savage's criterion, the first project is optimal.

3) Hurwitz's criterion of optimism-pessimism.

Let's consider the "pessimism coefficient", in the line of each project (Tabl. 5) we consider only the smallest and largest value and choose the optimal strategy based on the condition:

*Table 5.* Estimated values for determining the optimal project according to the Hurwitz criterion (thousand UAH)

| A set of actions - projects | The maximum value $c_i$ | The minimum value $c_i$ | Calculation of value $S_i = \alpha \cdot \min Kd_i + (1-\alpha) \cdot \max Kd_i$ |
|---|---|---|---|
| $d_1$ | 65000 | 27000 | $S_1 = 0{,}3 \cdot 27000 + 0{,}7 \cdot 65000 = 53600$ |
| $d_2$ | 47000 | 35000 | $S_2 = 0{,}3 \cdot 35000 + 0{,}7 \cdot 47000 = 43400$ |
| $d_3$ | 44000 | 31000 | $S_3 = 0{,}3 \cdot 31000 + 0{,}7 \cdot 44000 = 40100$ |

$$K_3 = \max\{53600; \ 43400; 40100\} = 53600.$$

So, according to the Hurwitz criterion, the first project is optimal.

4) Medium risk criterion, according to which the optimal project is chosen for which

$$K_4 = \min_{i \in 1,2,\ldots,m} \sum_{j=1}^{n} \left( \max_{i \in 1,2,\ldots,m} a_{ij} - a_{ij} \right) p_j .$$

Let's make an auxiliary table for calculating the estimated values (Tabl. 6).

*Table 6.* Calculated data for the average risk criterion

| A set of actions - projects | Values of strategy selection parameters and corresponding probabilities | | | | |
|---|---|---|---|---|---|
| $d_1$ | $\max a_{1j} - a_{1j}$ | 0 | 0 | 2 | 17 |
| | $P_{1c}$ | 0,3 | 0,35 | 0,2 | 0,15 |
| $d_2$ | $\max a_{2j} - a_{2j}$ | 27 | 8 | 0 | 9 |
| | $P_{2c}$ | 0,15 | 0,26 | 0,34 | 0,25 |
| $d_3$ | $\max a_{3j} - a_{3j}$ | 25 | 17 | 16 | 0 |
| | $P_{3c}$ | 0,2 | 0,3 | 0,3 | 0,2 |

$$R_1 = \sum_{j=1}^{n} \left(\max a_{1j} - a_{1j}\right) \cdot P_{1j} = 0 \cdot 0{,}3 + 0 \cdot 0{,}2 + 2 \cdot 0{,}2 + 17 \cdot 0{,}15 = 2{,}95;$$

$$R_2 = \sum_{j=1}^{n} \left(\max a_{2j} - a_{2j}\right) \cdot P_{2j} = 27 \cdot 0{,}15 + 8 \cdot 0{,}26 + 0 \cdot 0{,}34 + 9 \cdot 0{,}25 = 8{,}38;$$

$$R_3 = \sum_{j=1}^{n} \left(\max a_{3j} - a_{3j}\right) \cdot P_{3j} = 25 \cdot 0{,}2 + 17 \cdot 0{,}3 + 16 \cdot 0{,}3 + 0 \cdot 0{,}2 = 14{,}9;$$

$$K_4 = \min_{i \in 1,2,\ldots,m} \{R_1, R_2, R_3\} = \min_{i \in 1,2,\ldots,m} \{2{,}95; 8{,}38; 14{,}9\} = 2{,}95.$$

Therefore, according to the medium risk criterion, the first project is optimal.

5) Bayes-Laplace criteria. According to the Bayes criterion for each project, based on the values in Table 4 and the corresponding probability distribution, we calculate the mathematical expectation and choose the optimal project according to the formula

$$K_5 = \max M_i(c_{ij});$$

where $M_i$ is the mathematical expectation.

To perform calculations, we compile auxiliary tabl. 7.

*Table 7.* Estimated values for determining the optimal alternative according to the Bayes criterion

| A set of actions - projects | Values of projects selection parameters and corresponding probabilities | | | |
|---|---|---|---|---|
| $d_1$ | $C_{1j}$ | 65 | 50 | 45 | 27 |
| | $P_{1c}$ | 0,3 | 0,35 | 0,2 | 0,15 |
| $d_2$ | $C_{2j}$ | 38 | 42 | 47 | 35 |
| | $P_{2c}$ | 0,15 | 0,26 | 0,34 | 0,25 |
| $d_3$ | $C_{3j}$ | 40 | 33 | 31 | 44 |
| | $P_{3c}$ | 0,2 | 0,3 | 0,3 | 0,2 |

For each project, we calculate the value of the expected profit from its implementation:

$$M_1 = \sum_{j=1}^{3} c_{1j} \cdot P_{1j} = 65 \cdot 0{,}3 + 50 \cdot 0{,}35 + 45 \cdot 0{,}2 + 27 \cdot 0{,}15 = 50{,}05;$$

$$M_2 = \sum_{j=1}^{3} c_{2j} \cdot P_{2j} = 38 \cdot 0{,}15 + 42 \cdot 0{,}26 + 47 \cdot 0{,}34 + 35 \cdot 0{,}25 = 41{,}35;$$

$$M_3 = \sum_{j=1}^{3} c_{3j} \cdot P_{3j} = 40 \cdot 0{,}2 + 33 \cdot 0{,}3 + 31 \cdot 0{,}3 + 44 \cdot 0{,}2 = 30.$$

$$K_5 = \max M_i(c_{ij}) = \max\{50{,}05;\ 41{,}35;\ 30\} = 50{,}05.$$

Therefore, according to the Bayes criterion, the first project is optimal.

According to the Laplace criterion, all the consequences of project application are equally probable, so $P_{ij}$=0,25. We are recalculating. We obtain the following values of expected profits:

$$M_1 = \sum_{j=1}^{3} c_{1j} \cdot P_{1j} = 0{,}25(65 + 50 + 45 + 27) = 46{,}75;$$

$$M_2 = \sum_{j=1}^{3} c_{2j} \cdot P_{2j} = 0{,}25(38 + 42 + 47 + 35) = 40{,}5;$$

$$M_3 = \sum_{j=1}^{3} c_{3j} \cdot P_{3j} = 0{,}5 \cdot (40 + 33 + 31 + 44) = 37;$$

$$K_5 = \max M_i(c_{ij}) = \max\{46{,}75;\ 40{,}5;\ 37\} = 46{,}75.$$

Thus, the optimal project according to the Laplace criterion is also the first project.

6) Khoja-Lehman criterion. According to this criterion, a project that satisfies the formula is optimal

$$K_6 = \max L_i = \max(\nu \sum_{j=1}^{n} a_{ij} P_{ij} + (1-\nu) \min_{j=1,2,\dots n} a_{ij}), i = 1,2,..,m,$$

where the parameter н is the reliability of the applied probability distribution (in the calculations of the problem, н=0.95 is taken)

Using the data in Table 7, we calculate the value of $L_i$:

$$L_1 = \nu \sum_{j=1}^{n} a_{1j} P_{1j} + (1-\nu) \min_{j=1,2,\dots n} a_{1j} = 0{,}95 \cdot 50{,}05 + 0{,}05 \cdot 27 = 48{,}8975;$$

$$L_2 = \nu \sum_{j=1}^{n} a_{2j} P_{2j} + (1-\nu) \min_{j=1,2,\dots n} a_{2j} = 0{,}95 \cdot 41{,}35 + 0{,}05 \cdot 35 = 41{,}0325;$$

$$L_3 = \nu \sum_{j=1}^{n} a_{3j} P_{3j} + (1-\nu) \min_{j=1,2,\dots n} a_{3j}) = 0{,}95 \cdot 30 + 0{,}05 \cdot 31 = 30{,}05;$$

$$K_6 = \max L_i = \max\{48{,}8975;\ 41{,}0325;\ 30{,}05\} = 48{,}8975.$$

Therefore, according to the Hodge-Lehmann filter, the first project is also optimal.

According to the used model of six filters, the first project is optimal, which is consistent with the decision obtained by the method of analysis of hierarchies.

**Conclusions**

In the work, a model of the implementation of the method of analysis of hierarchies was developed to solve the problem of choosing the optimal project for the development of a hotel business enterprise. According to the developed methodology the task of choosing the optimal development project of the hotel business enterprise under the AHP was solved. The reliability of the results obtained by the MAI method was checked by the method of six filters — the sequential application of the Wald, Savage, Hurwitz, average risk, Bayes-Laplace, Khoja-Lehmann criteria.

The proposed approaches can be applied to solve other applied problems of making management decisions in conditions of uncertainty regarding activity forecasting, the optimal development project of hotel, publishing, restaurant business enterprises; identification of the most important factors influencing the components of the project, which is useful in the event of the need to adjust the selected project in the process of its implementation.

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