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MODELING OF EXPANDING SURFACES, WHICH INCIDENT BY TWO DIRECT DRIVE CURVESS

The geometric model of the expanding surface which is incident to two direction curves is considered. On the basis of the developed model, the conditions for the existence of the surface are derived, which makes it possible to apply the developed model to the design of machine-building products, in particular those working in a dense environment.
***Keywords**: geometry; surface; expansion surfaces; model; working bodies.*

Розглянуто геометричну модель розгортної поверхні яка інцидентна двом напрямним кривим. На основі розробленої моделі виведено умови існування поверхні, що дає змогу застосувати розроблену модель при проектуванні машинобудівних виробів, зокрема таких, що працюють у щільному середовищі.
***Ключові слова**: геометрія; поверхні; розгортні поверхні; модель; робочі органи.*

Formulation of the problem

For multi-nomenclature machine-building production the most important of the technological equipment is machining, the optimum service life does not exceed for 7 years. This equipment includes working bodies that work in dense environments: these are excavator buckets, bulldozer shelves, general-purpose and special-purpose plows, and many others that work in soil. At the heart of such working bodies is a surface that can be quite complex. An analysis of the working methods of working bodies shows that in the early stages of design it is very difficult to assess and take into account the variety of factors that affect the quality of its work. Therefore, there is a problem of choosing the design and technological parameters of the surfaces, in order to ensure the technological purpose of the working body. At the same time, there is no scientific and methodological base, including generalized surface models, which allows taking into account the design and technological features of the working bodies that are being designed. Thus, the development of surface models that allow the design of working bodies to work in a dense environment is a broad spectrum solution to an important problem.

Analysis of recent research and publications

One of the main directions of applied geometry development is the design of surfaces that meet different positional, metric, and differential conditions. At present, in these areas, certain results are obtained, which stand at the level of modern requirements, design engineering. In [1, 8], geometric models of ruler surfaces with multivariable sets of lines and special surface lines are presented, but a large number of parameters and difficult definition of special lines, for example, curvature lines complicate the design of working bodies. In [9, 10, 11] geometrical models of surfaces of soil tillage working bodies, in particular plows of agricultural purpose are considered. The disadvantage of these models is to specify them for a specific working body, and the application of the model for another working body is much more difficult. This leads to the fact that in the design of the working body in each case, you need to develop your algorithm for surface formation.

The main directions of development of analytical design of surfaces are formulated by II Kotov [2, 3] and consist in the development of methods of constructing lined surfaces, their continuous frames with observance of given differential-isometric conditions. Strengthening requirements for precision characteristics, increasing the speed and the need to improve the dynamic characteristics of the interaction of surfaces with a dense environment further complicate the required geometry of the surfaces. For example, for operated surfaces in dense environments or at high speeds.

The above causes the urgent need to develop new methods for obtaining mathematical models of surfaces that meet a large number of differential-geometric conditions with a given degree of accuracy. The energy costs thus depend on the geometric properties of the surfaces.

The use of working bodies with expandable surfaces is widespread in technology. This position is provided by the differential-parametric properties of the unfolding surfaces:

- the tangent plane touches the surface along the whole rectilinear formation and does not change its position in space when the point of contact changes;
- normal to the surface when moving the point of contact along the formation does not change its position.

Working bodies with unfolding surfaces have the advantage over non-expandable ones, first of all, because at the unfolding surface, the working body is made by a simple bend that eliminates plastic deformations, which leads to a violation of the shape of the surface. Secondly, if the working body is working in a dense environment, it has less traction resistance, because on the unfolding surface the dense medium will move more orderly because at each instant of time the unfolding surface is a plane.

Particularly noteworthy is that working bodies with expandable surfaces have linear wear due to their properties.

The greatest contribution to the theory of unfolding surfaces was made by V. M. Naidysh [5, 6]. Based on the differential properties of the unfolding surfaces, he developed a number of algorithms for their design. In [7] adaptation of tillage working bodies to certain conditions is given

These algorithms are generic in nature, so detailing them to save significant material and energy costs when designing and manufacturing working bodies is an important task.

Formulation of the purpose of the study

The purpose of the work is to provide rational choice of the parameters of the model of the unfolding surface when designing process equipment using automated systems.

The object of research is the design of surfaces of working bodies of machining and technological equipment, and working bodies of machines operating in conditions of dense environments, and made of structural materials of general mechanical engineering purpose.

The paper deals with the geometric model of a linear unfolding surface, the conditions of existence of such a surface, and presents a generalized algorithm for constructing a surface, regardless of the type of working body or engineering product.

Presenting main material

To develop a surface model, we define a Cartesian coordinate system $Oxyz$. In this system we define two curves:

$$m:\ x = x_1(u),\ y = y_1(u),\ z = z_1(u), \tag{1}$$

$$n:\ x = x_2(v),\ y = y_2(v),\ z = z_2(v), \tag{2}$$

where u and v — parameters.

These curves distinguish from the set of direct congruences

$$y = ax + b, \quad z = cx + d. \tag{3}$$

In this case, the position parameters a, b, c, d depend on u and v and are determined from the system of equations (1), (2), (3), so that

$$a = \frac{y_1 - y_2}{x_1 - x_2}; \quad b = y_1 - ax_1; \tag{4}$$

$$c = \frac{z_1 - z_2}{x_1 - x_2}; \quad d = z_1 - cx_1. \tag{5}$$

To remove the expanding torso surface, we introduce an additional condition in the form of compatibility, equations defining the equipment of both curves by the norms of the future surface,

$$\begin{cases} z_1' = p \cdot x_1' + q \cdot y_1' \\ z_2' = p \cdot x_2' + q \cdot y_2' \end{cases} \tag{6}$$

where the strokes denote the derivatives of the functions that set the curves by their parameter and the differential equation of the linear surface

$$p + aq - c = 0,$$

which is equivalent to the equation

$$\begin{vmatrix} x_1' & y_1' & z_1' \\ x_2' & y_2' & z_2' \\ x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \end{vmatrix} = 0. \quad (7)$$

Equation (6) expresses the fact that the surface normal at the corresponding points of the given curves (which belong to one straight line) are parallel, which is equivalent to having a common tangent plane to the surface at the indicated points. Indeed, given (4) and (5), equation (8) takes the form

$$\begin{vmatrix} x_1' & y_1' & z_1' \\ x_2' & y_2' & z_2' \\ x - x_2 & y - y_2 & z - z_2 \end{vmatrix} = 0, \quad (8)$$

that is, the vectors tangent to the curves and the generic coplanar.

Equation (8) allows us to determine the dependence between the parameters u and v on the surface unfolding condition

$$\varphi(u, v) = 0, \quad (9)$$

which together with (3), (4), (5) gives the equation of the search surface.

If one of the parameters (such as u) can be explicitly expressed in (9) through the second parameter,

$$u = \varphi_1(v), \quad (10)$$

then, given (4), (5) and (10), the surface equation will look like

$$y = a(v) \cdot x + b(v),\ z = c(v) \cdot x + d(v). \quad (11)$$

To find the edge of *l*, we differentiate (11) by *v*

$$x = \frac{b'(v)}{a'(v)} = -\frac{d'(v)}{c'(v)}. \quad (12)$$

One of equations (12) in conjunction with (11) defines *l*.

Obviously, not every two lines in their shape and position in space will allow for a torso surface. In solving the determinant (9), the following cases can occur (except for the one discussed above, which leads to (9)):

– the determinant is equal to zero (the curves are in the same plane, which is the desired surface regardless of the function φ);

– as a result of decoupling (8) it becomes an equation from one parameter (there is no torso surface, there are separate generators whose number is equal to the number of roots of equations where there is a common tangent plane to the given curves);

– the determinant (8) is not zero (the torso surface does not exist).

Thus, the desired result is only the case leading to (9).

Consider equations (6) and (8). The minors 2×2 of the first two lines of the determinant (8) are the coordinates of the surface normal $\vec{n}\{\Delta x, \Delta y, \Delta z\}$, so

$$p = \frac{-\Delta x}{\Delta z},\ q = \frac{-\Delta y}{\Delta z}, \quad (13)$$

where

$$\Delta x = -\begin{vmatrix} y_1' & z_1' \\ y_2' & z_2' \end{vmatrix};\ \Delta y = \begin{vmatrix} x_1' & z_1' \\ x_2' & z_2' \end{vmatrix};\ \Delta z = \begin{vmatrix} x_1' & y_1' \\ x_2' & y_2' \end{vmatrix}.$$

Surface design is greatly simplified if the guide curves are level lines. For example, if *m* and *n* are horizontal, then the condition of surface unfolding is $\Delta z = 0$, and for the fronts $\Delta y = 0$, etc.

Equation (7) allows one of the functions, for example, (2), to be defined to satisfy the existence of a spreading surface. In this case, the functions x_1, y_1, z_1 and x_2, y_2, and, must be known.

Write equation (8) as follows

$$A\begin{vmatrix} y_1' & z_1' \\ y_2' & z_2' \end{vmatrix} - B\begin{vmatrix} x_1' & z_1' \\ x_2' & z_2' \end{vmatrix} + \left(z_1 - z_2\right)\cdot C = 0,$$

where $A = x_1 - x_2$, $B = y_1 - y_2$, $C = \begin{vmatrix} x_1' & y_1' \\ x_2' & y_2' \end{vmatrix}$.

From the obtained expression we arrive at a differential equation of a relatively unknown function

$$z_2' - \frac{C}{F} z_2 = 0, \tag{14}$$

where $F = \left(A\cdot y_1' - B\cdot x_1'\right) - A\cdot y_2'\cdot z_1' - B\cdot z_1'\cdot x_2' + C\cdot z_1'$.

Surface construction algorithm

1. We select the curves *m* and *n* (equations (1) and (2)) according to the technological process. For example, for the design of a general-purpose plow shelf, these can be trajectories of the chuck motion.
2. Find the generators $x'_1, y'_1, z'_1, x'_2, y'_2, z'_2$.
3. We construct the determinant (8) as a result of the solution which we find (9). If dependence (9) on the determinant (8) does not follow, then there is no solution.
4. We determine the coefficients *a*, *b*, *c*, *d* according to (4) and (5).
5. Find *a, b, c, d* with (9).
6. Write the equation of the surface (3) which will be given by the set of rectilinear generators.

According to the specified model, a semi-screw general-purpose plow shelf is constructed in which the formation in each position will be determined by two angles of inclination to the projection planes.

The coordinate system $Oxyz$ is chosen such that the axis Ox is directed opposite to the movement of the plow body, the axis Oy is located in the horizontal plane and the axis is vertical. With this arrangement of the surface, the angle of projection of the genera on the horizontal plane of projections Oxy is denoted by γ the angle of projection of the genera to the axis Ox on the vertically longitudinal plane through β. Surface diagram and function graphs $a\left(x\right) = tg\gamma\left(x\right)$ та $b\left(x\right) = tg\beta\left(x\right)$ are presented in Fig. 1. Logarithmic spirals were adopted as the guide curves

$$r_1 = r_{01}e^{w_1\varphi_1} \text{ end } r_2 = r_{02}e^{w_2\varphi_2},$$

where r_1, r_2 — the current radius vectors of the guide curves; r_{01}, r_{02} — the initial radius vectors of the guide curves (1), (2); w_1, w_2 — tangents of angles between the current radius vector and the tangent; φ parameter, namely the polar angle of rotation of the radius of the vector. The coordinate is programmed in the radians of the polar angle φ_1.

Agro-technical assessment of the quality of work of plows with standard cultural and semi-screw shelves of buildings was carried out on the percentage of plowing of crop residues. Taking into account the working speed of the plows of 2.7 m/s, the percentage of plowing of crop residues for the plow with the half-screw shelves was 97.9%, and with the crop — 78.1%, which is higher on 19.8%. Experimental studies have shown the prospect of using semi-screw shelves on general-purpose shelf plows.

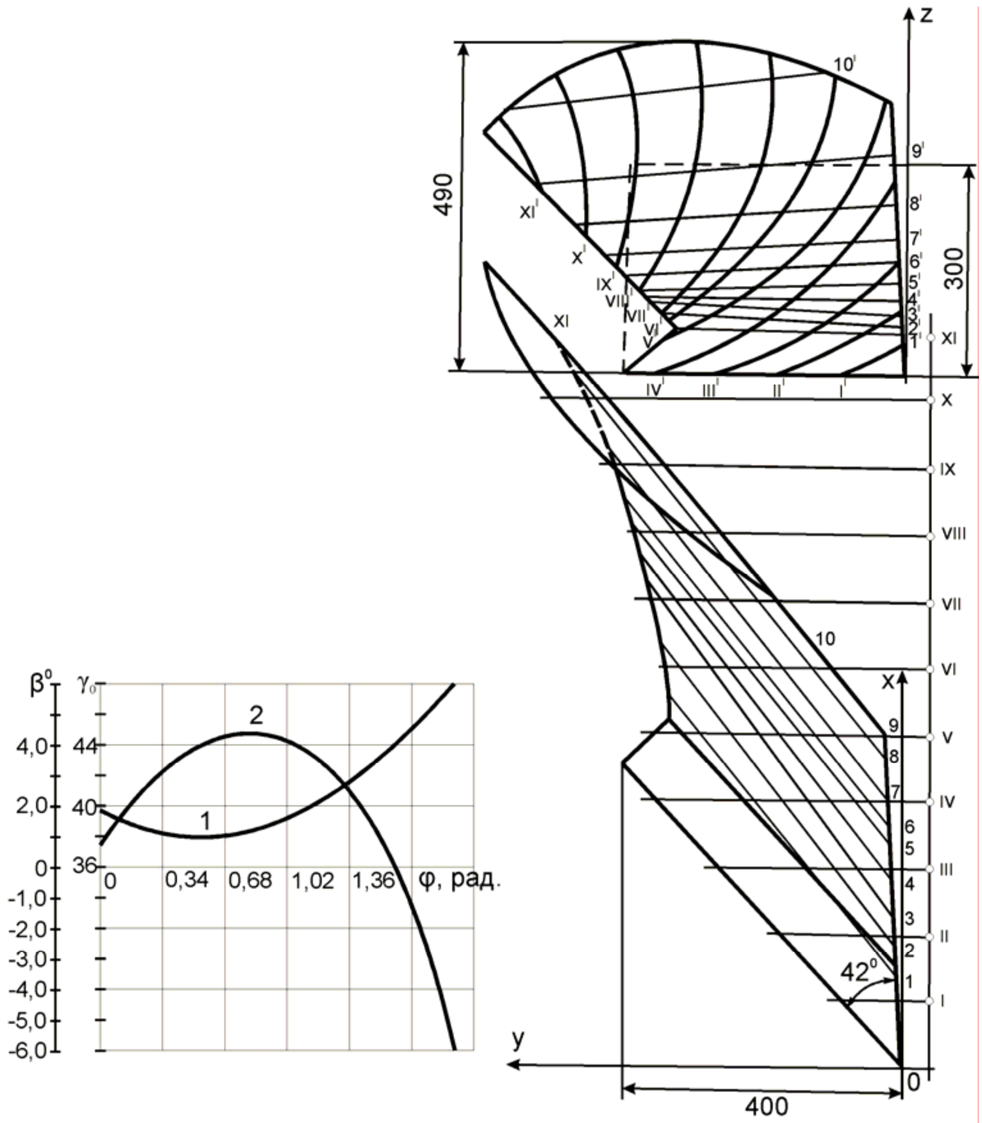

Fig. 1. The scheme of the surface of the plow body and the slope function of the projection of the genera: 1 — graph of the function $a(\varphi)$; 2 — function graph $b(\varphi)$

Conclusions and prospects for further research

The model of the unfolding surface, which is incident to the two guide curves, has been successfully applied in the design of the working surface of a general-purpose plow. Experimental studies have shown the advantage of expandable surfaces as working shelves for general-purpose plows. Thus, the geometric surface model is promising and can be used in the design of other shelves working bodies. Further studies should aim to determine the surface curvature to be taken into account in computer design.

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МОДЕЛЮВАННЯ РОЗГОРТНИХ ПОВЕРХОНЬ, ІНЦИДЕНТНИХ ДВОМ НАПРЯМНИМ КРИВИМ

Тищенко С.С.

Реферат

Для забезпечення раціонального вибору параметрів поверхонь деталей при проектуванні технологічного обладнання потрібно базуватися на методах штучного інтелекту, зокрема на узагальнених алгоритмах. У статті розглянуто геометричну модель лінійчатої розгортної поверхні, та умови її поверхні. Надається узагальнений алгоритм побудови розгортної поверхні незалежно від типу робочого органу чи виробів машинобудування. Однією з найпоширеніших у галузі поверхонь є розгортні поверхні, які займають особливе положення, завдяки своїм дифе-

ренціально-параметричним властивостям: дотична площина торкається поверхні вздовж прямолінійної твірної і не змінює свого положення в просторі при зміні точки контакту. Такі поверхні можуть бути виготовлені згинанням листового металу. Ці положення дозволяють виготовити продукцію, заощаджуючи значні матеріальні та енергетичні засоби; тому розробка геометричних моделей таких поверхонь є важливим завданням. Наведена геометрична модель розгортної поверхні, напівгвинтового корпусу плугу загального призначення.

Експериментальні дослідження показали перспективність застосування напівгвинтових полиць плугах, особливо в двоярусній оранці. Враховуючи робочу швидкість руху плуга 2,7 *м / с*, відсоток залишків сільськогосподарських культур на оранці з напівгвинтовими полицями становить 97,9%, а з культурними корпусами плугів — 78,1%.

Розроблені підходи до вирішення проблеми розпізнавання умов існування розгортних поверхонь при застосуванні програми по її концептуальній моделі. Побудовано відповідні процедури обчислювальних алгоритмів. Математична модель, що визначає вплив параметрів поверхонь на їх експлуатаційні властивості – силу опору. Теоретичні основи цієї моделі можуть бути використані у середовищі програмування для будь-якого інтерфейсу. В даний час ця модель може дати значні результати, які відповідають сучасним вимогам будівельної інженерії. У процесі параметричної оптимізації модель генерує безліч бажаних значень параметрів поверхонь. Модель дозволяє зображення розгортних поверхонь за формальними ознаками. Результатом є спряження розгортних поверхонь до певної групи, кожна з яких відповідає безлічі окремих параметрів обчислювальної моделі.

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