МОДЕЛЮВАННЯ ТА ОПТИМІЗАЦІЯ В ТЕХНОЛОГІЇ КОНСТРУКЦІЙНИХ МАТЕРІАЛІВ

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NON-STATIONARY SURFACE TEMPERATURE FIELD LAYER OF METAL WITH PERIODIC ENERGY ACTION ON SURFACE

Under the influence of temperature fluctuations, the properties of metals can change. Thermocyclic treatment of metals, in particular steels, requires accurate detection of patterns of non-stationary temperature field in the treated metal layer. An expression was obtained for the deviation of the temperature from the value of its instantaneous equilibrium distribution for a one-dimensional homogeneous model of finite length under conditions of periodic energy action on its active surface. This solution was applied to the low carbon steel sample.
Keywords*: periodic temperature action; surface layer of metal; thermal conductivity equation; method of distribution of variables.*

Під впливом температурних коливань властивості металів можуть змінюватися. Термоциклічна обробка металів, зокрема сталей, потребує точного виявлення закономірностей нестаціонарного температурного поля в оброблюваному шарі металу. Було отримано вираз для відхилення температури від величини миттєвого рівноважного її розподілу для одновимірної однорідної моделі конечної довжини в умовах періодичної енергетичної дії на її активну поверхню. Вказане рішення було застосовано для зразка із низько вуглецевої сталі.
Ключові слова*: періодична температурна дія; поверхневий шар металу; рівняння теплопровідності; метод розподілу змінних.*

Problem's Formulation

Metals, semiconductors, most other materials in the solid state mainly function in conditions of variable temperatures. In the process of manufacturing or restoration, they are also subject to variable thermal action. Under the influence of temperature fluctuations, due to thermal stresses caused by temperature gradients and phase transformations, the properties of materials may not change irreversibly. Local thermal non-uniform expansion is fixed by diffusion fluxes of vacancies and dislocated atoms, which leads, in addition to macroscopic changes in shape and size, to the formation of new physical properties in the surface layers of the material. Thermocycling of metals, in particular steels [1]—[5], attracts attention in the context of high-intensity pulse technologies that can speed up the processing phase or save resources [6]. It is possible to increase the set upper limits of heating and intensify diffusion processes. Pulsed energy action is easy to reproduce as part of local heat treatment using available local energy sources. A clear temperature conditionality of diffusion fluxes, phase transformations requires the detection of patterns of non-stationary temperature field in the surface layer of the metal under conditions of pulsed energy action on the surface to carry out a balanced point effect on this layer.

For transient nonequilibrium processes under conditions of local heating and cooling of the metal surface, it is difficult to experimentally determine the temperature, the rate of its change, and other parameters of heat fluxes. Mathematical modeling of the studied phenomena greatly simplifies the result, saves time and resources.

Analysis of recent research and publications

In the analytical study of the thermal conductivity process at time t, sufficiently distant from the initial moment t0, the effect of the initial temperature distribution is leveled. In this case, the problem is to find a solution of the equation of thermal conductivity that satisfies only the boundary conditions.

A typical boundary condition in the case of thermal conductivity problems without initial conditions reflects the change in temperature at the boundary surface according to the law of periodic, in particular sinusoidal, function (Fourier problem) [7, p. 242]. The simplest approach to finding the solution of the Fourier problem is carried out by introducing a complex variable with the subsequent selection of the real part of the solution [7, p. 242, 243], [8, p. 177].

The analytical solution of the equation of thermal conductivity in the one-dimensional approximation for a semi-infinite homogeneous model of the medium requires the setting of only one boundary condition — the dependence of the temperature on the surface of the medium on time. At periodic change of the specified temperature with frequency ω, after long enough time interval deducted from the initial moment of process, in depth of the environment fluctuations of temperature with the same frequency are also established [7, p. 247].

The Fourier problem without initial conditions for a bounded segment requires the setting of two boundary conditions and also leads to a solution in the form of a harmonic function or superposition of harmonics [7, p. 244].

This problem for a bounded homogeneous segment $0 \le x \le l$ in the standardized form of boundary conditions has the form [7, p. 243]:

$$\left.\begin{aligned} &\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2}, \\ &v(0,t) = A\cos\omega t, \\ &v(l,t) = 0. \end{aligned}\right\}, \tag{1}$$

where $v(x,t)$ is the temperature of a one-dimensional homogeneous bounded model of the medium, and a^2 is the coefficient of thermal conductivity of the medium.

Search for the solution of equation (1) in a complex form

$$v^*(x,t) = X(x)e^{-i\omega t} \tag{2}$$

with boundary conditions

$$v^*(0,t) = Ae^{-i\omega t}, \quad v^*(l,t) = 0,$$

leads to the equation for the function X (x)

$$X'' + \gamma^2 X = 0, \quad \gamma = \sqrt{\frac{i\omega}{a^2}} = \sqrt{\frac{\omega}{2a^2}}(1+i) \tag{3}$$

with additional conditions

$$X(0) = A, \quad X(l) = 0. \tag{4}$$

The general form of the solution of equation (3) has the form:

$$X(x) = C_1 \cos\gamma x + C_2 \sin\gamma x,$$

where C_1 and C_2 are integration constants. Additional conditions (4) are met by the following values of these constants

$$C_1 = A, \qquad C_2 = -Actg\gamma l .$$

Then, taking into account the sine formula of the difference of the two angles, we obtain

$$X(x) = \frac{A}{\sin\gamma l}\sin\gamma(l-x) . \tag{5}$$

Converting the function $X(x)$ to a standard complex form

$$X(x) = X_1(x) + iX_2(x),$$

and, taking into account the general form of solution (2), by selecting the real part of the function, we find the solution of the original problem in the form

$$v(x,t) = X_1(x)\cos\omega t + X_2(x)\sin\omega t . \tag{6}$$

Define the explicit expression of the functions $X_1(x)$ and $X_2(x)$.

We introduce in expression (5) the notation $\sqrt{\frac{\omega}{2a^2}} = k$, then $\gamma = k(1+i)$ and

$$X(x) = A\frac{\sin\gamma(l-x)}{\sin\gamma l} = A\frac{\sin k(1+i)(l-x)}{\sin k(1+i)l} = A\frac{\sin\left[k(l-x)+ik(l-x)\right]}{\sin(kl+ikl)} .$$

We use the expression of the sine function for the complex number z:

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} .$$

Then

$$X(x) = A\cdot\frac{e^{i[k(l-x)+ik(l-x)]} - e^{-i[k(l-x)+ik(l-x)]}}{e^{i(kl+ikl)} - e^{-i(kl+ikl)}} = A\cdot\frac{e^{-k(l-x)}\cdot e^{ik(l-x)} - e^{k(l-x)}\cdot e^{-ik(l-x)}}{e^{-kl}\cdot e^{ikl} - e^{kl}\cdot e^{-ikl}} .$$

For the obtained expression we apply Euler's formula:

$$e^{iz} = \cos z + i\sin z .$$

Then

$$X(x) = A\cdot\frac{e^{-k(l-x)}\cdot\left[\cos k(l-x) + i\sin k(l-x)\right] - e^{k(l-x)}\cdot\left[\cos k(l-x) - i\sin k(l-x)\right]}{e^{-kl}\cdot(\cos kl + i\sin kl) - e^{kl}\cdot(\cos kl - i\sin kl)} .$$

After conversion

$$X(x) = A\cdot\frac{\left[e^{-k(l-x)} - e^{k(l-x)}\right]\cdot\cos k(l-x) + i\left[e^{-k(l-x)} + e^{k(l-x)}\right]\cdot\sin k(l-x)}{\left(e^{-kl} - e^{kl}\right)\cdot\cos kl + i\left(e^{-kl} + e^{kl}\right)\sin kl} .$$

Entering notation

$$a = \left[e^{-k(l-x)} - e^{k(l-x)}\right]\cdot\cos k(l-x);\ b = \left[e^{-k(l-x)} + e^{k(l-x)}\right]\cdot\sin k(l-x);$$

$$c = \left(e^{-kl} - e^{kl}\right)\cdot\cos kl;\ d = \left(e^{-kl} + e^{kl}\right)\sin kl ,$$

obsessed

$$X(x) = A\cdot\frac{a+ib}{c+id} .$$

Multiplying the numerator and denominator of this fraction by (c-id)

$$X(x)=A\cdot\frac{(a+ib)(c-id)}{(c+id)(c-id)}=A\cdot\frac{(ac+bd)+i(bc-ad)}{c^2+d^2}.$$

So, so much $X(x)=X_1(x)+iX_2(x)$, so

$$X_1(x)=A\cdot\frac{ac+bd}{c^2+d^2};\ X_2(x)=A\cdot\frac{bc-ad}{c^2+d^2}. \quad (7)$$

And define auxiliary expressions:

$$c^2+d^2=\left(e^{-kl}-e^{kl}\right)^2\cdot\cos^2 kl+\left(e^{-kl}+e^{kl}\right)^2\sin^2 kl=e^{-2kl}+e^{2kl}-2\cos 2kl;$$

$$ac=\left[e^{-k(l-x)}-e^{k(l-x)}\right]\cos k(l-x)\cdot\left(e^{-kl}-e^{kl}\right)\cos kl;$$

$$bd=\left[e^{-k(l-x)}+e^{k(l-x)}\right]\sin k(l-x)\cdot\left(e^{-kl}+e^{kl}\right)\sin kl;$$

$$ad=\left[e^{-k(l-x)}-e^{k(l-x)}\right]\cos k(l-x)\cdot\left(e^{-kl}+e^{kl}\right)\sin kl;$$

$$bc=\left(e^{-kl}-e^{kl}\right)\cos kl\cdot\left[e^{-k(l-x)}+e^{k(l-x)}\right]\sin k(l-x).$$

The actual part of the complex temperature $v^*(x,t)$ is determined by formula (6).

However, these solutions correspond to moments of time that do not cover the temperature fields of the initial time interval of energy action, which are fundamental for the pulse processing mode.

Formulation of the study purpose

The problem of analytical study of the temperature field of a one-dimensional homogeneous metal model of finite length l under conditions of periodic (sinusoidal) energy action with a given frequency ω on its one active surface is maintained, while maintaining a constant temperature on another surface bordering the substrate. In the process of energy action on the active surface of the sample, its temperature periodically changes from the minimum temperature level T_1 to the maximum level T_1 + 2A, where T_1 is the initial surface temperature, A is the amplitude of temperature fluctuations on the surface. The solution should cover not only the remote time interval from the beginning of the heat treatment, but also the starting time.

Presenting main material

The goal is to solve a one-dimensional homogeneous equation of thermal conductivity

$$\frac{\partial T}{\partial t}=a^2\frac{\partial^2 T}{\partial x^2}, \quad (8)$$

where $T = T(x, t)$ is the temperature in the sample, t is the processing time, x is the distance from the processing surface towards the substrate, is the coefficient of thermal conductivity of the homogeneous metal, with the appropriate additional conditions:

$$\mu_1(t)=T(0,t)=T_1+A-A\cos\omega t, \quad (9)$$

$$\mu_2(t)=T(l,t)=T_2, \quad (10)$$

- boundary conditions of the sample,

$$\varphi(x)=T(x,0)=T_1-\frac{T_1-T_2}{l}x. \quad (11)$$

- initial temperature distribution in the sample (equilibrium distribution with temperatures T_1 at $x = 0$ and T_2 at $x = l$).

We look for a solution to this problem in a standard way

$$T(x,t) = u(x,t) + v(x,t), \tag{12}$$

where *u(x,t)* is the basic auxiliary function that specifies the instantaneous equilibrium temperature distribution in the sample:

$$u(x,t) = \mu_1(t) + \frac{x}{l}\left[\mu_2(t) - \mu_1(t)\right] = T_1 + A - A\cos\omega t + \frac{x}{l}(T_2 - T_1 - A + A\cos\omega t) \tag{13}$$

and *v(x,t)* — some unknown function that has the meaning of deviation from *u(x,t)*.

For the function *v(x,t)* equation (8) is converted to equation

$$\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2} + f(x,t), \tag{14}$$

with homogeneous additional conditions

$$v(x,0) = \varphi(x) - u(x,0) = \left(T_1 - \frac{T_1 - T_2}{l}x\right) - \left(T_1 + \frac{x}{l}(T_2 - T_1)\right) = 0,$$

$$v(0,t) = \mu_1(t) - u(0,t) = 0,$$

$$v(l,t) = \mu_2(t) - u(l,t) = 0,$$

where

$$f(x,t) = -\left(\frac{\partial u}{\partial t} - a^2\frac{\partial^2 u}{\partial x^2}\right) = -\left(A\omega\sin\omega t - \frac{x}{l}A\omega\sin\omega t\right) = A\omega\left(\frac{x}{l} - 1\right)\sin\omega t.$$

The inhomogeneous equation of thermal conductivity (14) with zero initial and boundary conditions has a solution in the form of a decomposition on the interval (0, l) in the Fourier series by functions $\left\{\sin\frac{\pi n}{l}x\right\}$, where n = 1, 2, 3,…:

$$v(x,t) = \sum_{n=1}^{\infty} v_n(t)\sin\frac{\pi n}{l}x,$$

where

$$v_n(t) = \int_0^t \exp\left[-\left(\frac{\pi n}{l}\right)^2 a^2(t-\tau)\right]\cdot f_n(\tau)d\tau,$$

and $f_n(t)$ are the coefficients of the decomposition of the function $f(x,t)$ into a Fourier series over the variable x:

$$f(x,t) = \sum_{n=1}^{\infty} f_n(t)\sin\frac{\pi n}{l}x; \qquad f_n(t) = \frac{2}{l}\int_0^l f(\xi,t)\sin\frac{\pi n}{l}\xi d\xi.$$

Let's define $f_n(t)$:

$$f_n(t) = \frac{2}{l}\int_0^l A\omega\left(\frac{\xi}{l} - 1\right)\sin\omega t\cdot\sin\frac{\pi n}{l}\xi d\xi = A\omega\sin\omega t\left(\frac{2}{l}\int_0^l\frac{\xi}{l}\sin\frac{\pi n}{l}\xi d\xi - \frac{2}{l}\int_0^l\sin\frac{\pi n}{l}\xi d\xi\right) =$$

$$= A\omega\sin\omega t\left(\frac{2}{l^2}\int_0^l\xi\sin\frac{\pi n}{l}\xi d\xi + \frac{2}{\pi n}\cos\frac{\pi n}{l}\xi\Big|_0^l\right) = A\omega\sin\omega t\left[\frac{2}{\pi n}\left(-\cos\pi n\right)\right] + A\omega\sin\omega t\left[\frac{2}{\pi n}(\cos\pi n - 1)\right] =$$

$$= -\frac{2}{\pi n}A\omega\sin\omega t.$$

(Integral $\int\limits_0^l \xi \sin \frac{\pi n}{l} \xi d\xi = -\frac{l^2}{\pi n} \cos \pi n$ taken in parts).

Next we define $v_n(t)$:

$$v_n(t) = \int\limits_0^t \exp\left[-\left(\frac{\pi n}{l}\right)^2 a^2 (t-\tau)\right] \cdot f_n(\tau) d\tau = \exp\left[-\left(\frac{\pi n}{l}\right)^2 a^2 t\right] \times$$

$$\times \int\limits_0^t \exp\left[\left(\frac{\pi n}{l}\right)^2 a^2 \tau\right] \cdot \left(-\frac{2}{\pi n} A\omega \sin \omega\tau\right) \cdot d\tau = -\frac{2}{\pi n} A\omega \cdot \exp\left[-\left(\frac{\pi n}{l}\right)^2 a^2 t\right] \int\limits_0^t \exp\left[\left(\frac{\pi n}{l}\right)^2 a^2 \tau\right] \cdot \sin \omega\tau \cdot d\tau =$$

$$= -\frac{2}{\pi n} A\omega \cdot \exp\left[-\left(\frac{\pi n}{l}\right)^2 a^2 t\right] \cdot I .$$

Integral $I = \int\limits_0^t \exp\left[\left(\frac{\pi n}{l}\right)^2 a^2 \tau\right] \cdot \sin \omega\tau \cdot d\tau$ is taken by double application of the formula of integration by parts and use of a recurrent formula:

$$I = \frac{1}{\left[\left(\frac{\pi n}{l}\right)^2 a^2\right]^2 + \omega^2} \cdot \left\{\exp\left[\left(\frac{\pi n}{l}\right)^2 a^2 t\right] \cdot \left[\left(\frac{\pi n}{l}\right)^2 a^2 \sin \omega t - \omega \cos \omega t\right] + \omega\right\}.$$

Then

$$v_n(t) = -\frac{2}{\pi n} A\omega \cdot \frac{1}{\left[\left(\frac{\pi n}{l}\right)^2 a^2\right]^2 + \omega^2} \cdot \left\{\omega \exp\left[-\left(\frac{\pi n}{l}\right)^2 a^2 t\right] + \left[\left(\frac{\pi n}{l}\right)^2 a^2 \sin \omega t - \omega \cos \omega t\right]\right\}.$$

So much

$$v(x,t) = \sum_{n=1}^{\infty} v_n(t) \sin \frac{\pi n}{l} x ,$$

then

$$v(x,t) = -\sum_{n=1}^{\infty} \frac{2}{\pi n} A\omega^2 \frac{1}{\left[\left(\frac{\pi n}{l}\right)^2 a^2\right]^2 + \omega^2} \left\{\exp\left[-\left(\frac{\pi n}{l}\right)^2 a^2 t\right] + \left[\frac{\left(\frac{\pi n}{l}\right)^2 a^2}{\omega} \sin \omega t - \cos \omega t\right]\right\} \sin \frac{\pi n}{l} x \cdot$$

You can interpret the expression for deviation differently *v(x,t)*. So much $\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$, then the expression $\left[\left(\frac{\pi n}{l}\right)^2 a^2 \sin \omega t - \omega \cos \omega t\right]$ can be submitted as $\left(\frac{\pi n}{l}\right)^2 a^2 \sin\omega t - \omega\cos\omega t = -\cos(\omega t + \varphi_n)$, where $\sin\varphi_n = \left(\frac{\pi n}{l}\right)^2 a^2$, $\cos\varphi_n = \omega$;

or, more precisely, $tg\varphi_n = \dfrac{\left(\frac{\pi n}{l}\right)^2 a^2}{\omega}$. Then the deviation

$$v(x,t) = -\sum_{n=1}^{\infty} \frac{2}{\pi n} A\omega^2 \frac{1}{\left[\left(\frac{\pi n}{l}\right)^2 a^2\right]^2 + \omega^2} \left\{ \exp\left[-\left(\frac{\pi n}{l}\right)^2 a^2 t\right] - \frac{1}{\omega}\cos(\omega t + \varphi_n) \right\} \sin\frac{\pi n}{l} x,$$

after entering the notation

$$B_n = -\frac{2}{\pi n} A\omega^2 \cdot \frac{1}{\left[\left(\frac{\pi n}{l}\right)^2 a^2\right]^2 + \omega^2}; \quad C_n = \frac{2}{\pi n} A\omega \cdot \frac{1}{\left[\left(\frac{\pi n}{l}\right)^2 a^2\right]^2 + \omega^2};$$

can be submitted as.

$$v(x,t) = \sum_{n=1}^{\infty} \left\{ B_n \cdot \exp\left[-\left(\frac{\pi n}{l}\right)^2 a^2 t\right] + C_n \cos(\omega t + \varphi_n) \right\} \cdot \sin\frac{\pi n}{l} x. \tag{15}$$

The temperature field *T (x, t)* of the model is determined by relations (12), (13) and (15). The problem is solved.

The behavior of a certain non-stationary temperature field was interpreted on the example of a specific model material — low-carbon steel (base metal for a wide class of technological problems), with the length of the model samples within 2—20 mm (l_1 = 2 mm, l_2 = 5 mm, l_3 = 10 mm, l_4 = 20 mm). The initial temperature level of the active surface T_l = 550 °C and the magnitude of temperature fluctuations 2*A* = 380 °C corresponded to the typical modes of periodic temperature action on the metal during thermal cycling [3]—[5]. The coefficient of thermal conductivity for the selected temperature range of the model was $6{,}9\cdot10^{-6}$ m^2/s [9].

A wide range of frequencies of energy action was studied. The frequency of energy action was determined by the heating and cooling rates of the active surface. For maximum heat treatment rates: from very slow (≈0.1 K/s) to high speed (≈1000 K/s and even up to ≈10^6 K/s), according to the selected surface temperature range, the cycle period may vary from 4000 s to 0.4 s (up to 0.4 ms). Cyclic processing frequency at the same time makes $1{,}57\cdot10^{-2}$—1,57 rad/s. Limit regimes with frequencies were also investigated 15,7 rad/s and $15{,}7\cdot10^3$ rad/s. Depending on the heating — cooling rate, four processing modes were distinguished: A-mode — for low heating rates ≈1 K/s; B-mode — for average speeds ≈100 K/s; C-mode — for high speeds ≈1000 K/s; D-mode — for high-speed heating ≈10^6 K/s.

To control the non-stationary temperature field in the surface layer, a characteristic point (D-point) was selected at a distance of 1 mm from the active surface, in which the temperature behavior was monitored in the context of promoting the saturation of the surface layer with the alloying component. In fig. 1 and fig. 2 shows the resulting oscillations in the control D-point of sample l_4 for B- and C-modes during the first cycles. The initial temperature fluctuations on the surface *T*(0, *t*), instantaneous equilibrium fluctuations *u*(*x,t*) and deviations *v*(*x,t*) at the indicated points are also shown.

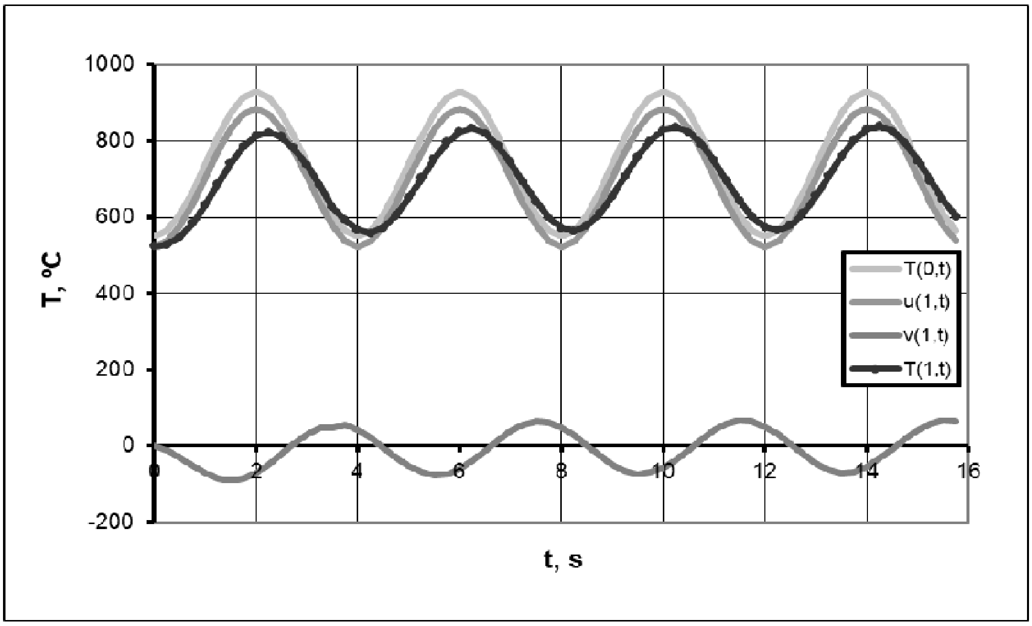

Fig. 1. Temperature deviation $v(1,t)$, nstantaneous equilibrium $u(1,t)$ and resulting $T(1,t)$ temperature fluctuations at the D-point of the sample l_4 = 20 mm in the conditions of the B-mode of processing; $T(0,t)$ — initial temperature fluctuations on the active surface of the sample

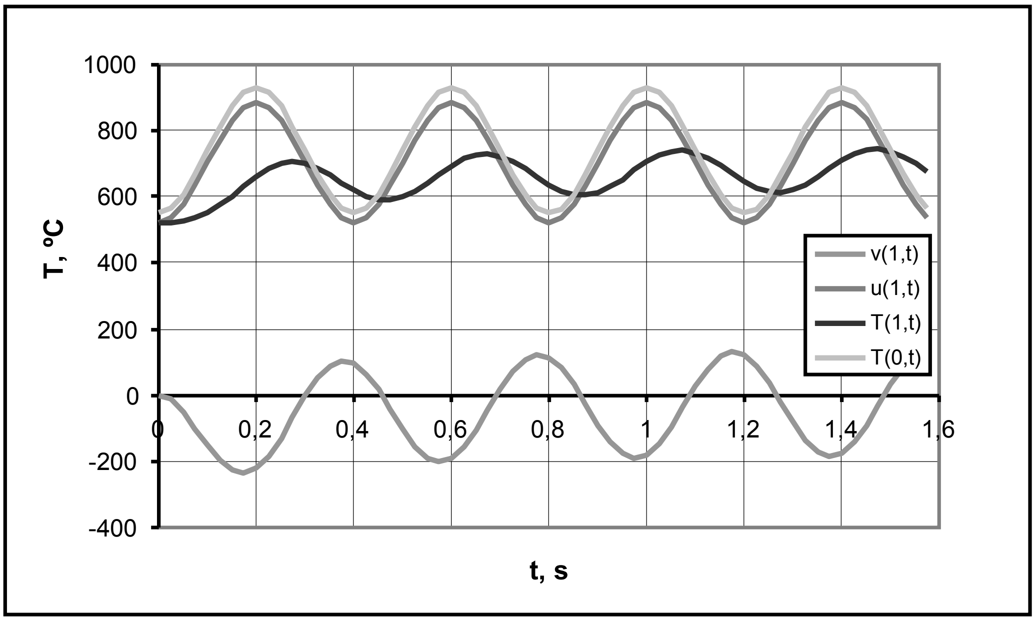

Fig. 2. Temperature deviation $v(1,t)$, instantaneous equilibrium $u(1,t)$ and the resulting $T(1,t)$ temperature fluctuations at the D-point of the sample l_4 = 20 mm in the conditions of the C-mode of processing; $T(0,t)$ — initial temperature fluctuations on the active surface of the sample

As can be seen from the graphs, the amplitude and phase deviation increase with increasing speed of heat treatment. The point of maximum deviation of temperature $v(x,t)$ from its equilibrium value is shifted to the active surface (for these models, it was localized near the coordinates $x_3 \approx 6$ mm and $x_4 \approx 3$ mm).

In Fig. 3 shows the resulting temperature fluctuations at the D-point of the sample l_4 = 20 mm in the case of high-speed thermal action (D-mode) for the first and remote (101–102) cycles. As can be seen from the figure, at the control point there is a significant increase in background temperature over time.

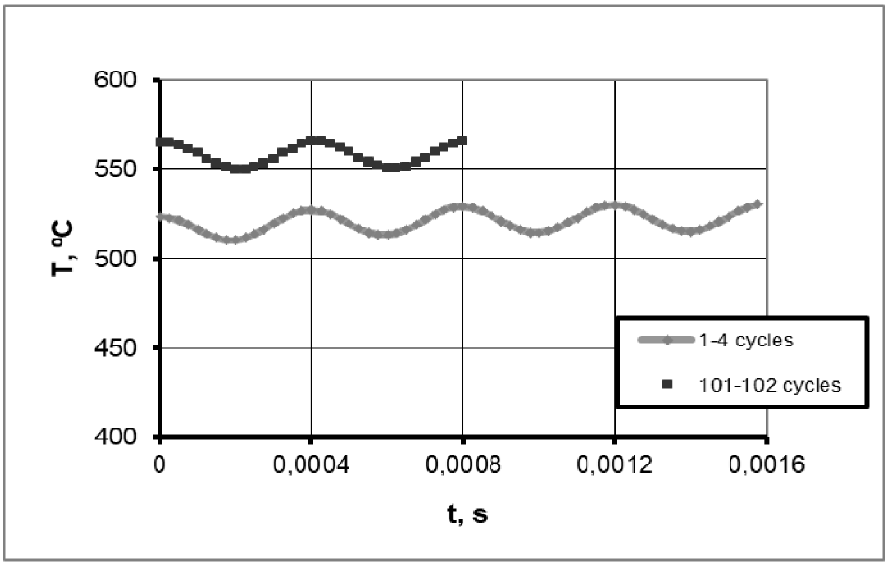

Fig. 3. The resulting temperature fluctuations at the control point 1 mm under conditions of D-mode processing for the first (1—4) and remote (101—102) cycles in the sample l_4 = 20 mm

The phase shift of the temperature oscillations at the points of the surface layer of the sample, in relation to the initial oscillations on the active surface, can be interpreted as the propagation of the temperature pulse in the medium — the temperature wave. The speed of temperature waves in the investigated metal c depends on the length of the model and the processing mode and was of the order of several mm/s [10], [11].

Conclusions

By analyzing the temperature field of a one-dimensional homogeneous metal model of finite length under conditions of periodic (sinusoidal) temperature action on its treated surface, a Fourier method was used to deviate the temperature from its instantaneous equilibrium distribution depending on the distance to the surface and processing time. The obtained solution has significant differences from similar analogues, due to the different formulation of the problem in relation to the time of the process. This solution was applied to a sample of low carbon steel of a given length.

References

[1] Ageenko, S.B. (2005) Influence of cyclic electrothermal treatment on the properties of the shaft-turbo wheel welded joint of steels 40X and 40G with alloy SZhL-800. *Equipment and technologies for heat treatment of metals and alloys:* Sat. report. 6th international conference. Section II. Thermal and thermomechanical processing technologies. (pp. 127–128). Kharkiv [in Ukrainian]

[2] Baranov, A.A., Slusarev, V.Y., & Leyrikh, I.V. (1990) Thermal cycling treatment of steel. *Chernaya metallurhiya*. № 3. (pp. 32–41). [in Russia]

[3] Kidin, I.N., Astafyev, E.V., Akopov, E.S. (1974) Structural state of steel after cyclic electric heat treatment. *Izv. vuzov. Chernaya metallurhiya*. № 11. (pp. 142–146). [in Russia]

[4] Rudenko, A.G. (1982) Behavior of 1Kh18N9T steel during thermal cycling. *Izv. AN USSR. Met*. № 6. (pp. 137–141). [in Russia]

[5] Biront, V.S., Nosovets, N.G., & Komendorovskaya, O.N. (1982) Thermal cycling treatment of hypoeutectoid steel. *Thermocycle. processing metal. products.* (pp. 68–71). Leningrad [in Russia]

[6] Riadno, A.A., Tsotsko, V.I., & Peleshenko, B.I. (2008) Numerical-analytical determination of the rate of crystallization of the melt in the conditions of axial symmetry by the example of the lugs of track castings. *Mathematical modeling. Dniprodzerzhynsk State Technical University.* 2 (19). (pp. 22–25). [in Ukrainian]
[7] Tikhonov, A. N., & Samarskiy, A. A. (1972) Equations of mathematical physics. *[Textbook for universities]*. Moscow: Nauka [in Russia].
[8] Sivukhin, D.V. (1979) General physics course. Thermodynamics and Molecular Physics. *[Study guide for students of physical specialties of higher educational institutions]*. Moscow: Nauka [in Russia].
[9] Tsotsko, V.I., & Denysenko, A. I. (2004) Temperature characteristics of the surface layer of low-carbon steels in conditions of linear heating of the surface. *Bulletin of Dnipropetrovsk University.* (Vols. № 2/2), (pp. 72–77). Dnipropetrovsk [in Ukrainian].
[10] Tsotsko, V.I., & Spiridonova, I.M. (2010.) Temperature fluctuations in the metal. *Bulletin of Dnipropetrovsk University*. (Vols. 18, № 2), (pp. 61–68). Dnipropetrovsk [in Ukrainian].
[11] Tsotsko, V.I., Spiridonova, I.M., & Peleshenko, B.G. (2010) Formation of temperature waves under conditions of periodic energy action on the metal surface. *Solid state physics and chemistry.* (Vols. 11, № 4), (pp. 1037–1043). [in Ukrainian].

НЕСТАЦІОНАРНЕ ТЕМПЕРАТУРНЕ ПОЛЕ ПОВЕРХНЕВОГО ШАРУ МЕТАЛУ ПРИ ПЕРІОДИЧНІЙ ЕНЕРГЕТИЧНІЙ ДІЇ НА ПОВЕРХНЮ

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Реферат

Метали, напівпровідники, більшість інших матеріалів у твердому стані переважно функціонують в умовах змінних температур. Під впливом температурних коливань властивості матеріалів можуть не зворотно змінюватися. Термоциклічна обробка металів, зокрема сталей, як і широкий клас інших високоінтенсивних імпульсних технологій, дозволяють прискорити фазу обробки або зекономити витрати ресурсів. Чітка температурна зумовленість дифузійних потоків та фазових перетворень в оброблюваному шарі металу потребує виявлення закономірностей нестаціонарного температурного поля в ньому в умовах імпульсної енергетичної дії на поверхню для проведення виваженого поточкового впливу на даний шар.

Відомі аналітичні дослідження не охоплюють з достатньою точністю весь інтервал циклічної термообробки поверхневого шару металу, особливо в початковій його стадії, що є принципово для імпульсного режиму обробки.

Була поставлена задача знаходження поля температур одновимірної однорідної металічної моделі конечної довжини в умовах періодичної (синусоїдальної) енергетичної дії з заданою частотою на її активну поверхню, при підтримці сталої температури на іншій, граничній з підкладкою, поверхні упродовж усього часу обробки.

Методом розподілу змінних було отримано вираз для відхилення температури від величини миттєвого рівноважного її розподілу в залежності від відстані до поверхні та часу обробки. Одержаний розв’язок має істотні відмінності від подібних аналогів, що зумовлено різною постановкою задачі по відношенню до часу процесу. Вказане рішення було застосовано для зразка із низько вуглецевої сталі заданої довжини. Виявлена залежність глибини проникнення температурного збурення від частоти термічної дії може бути використана для керування структурою та фізичними властивостями поверхневого шару металу.

Література

1. Агеенко С.Б. Влияние циклической электротермической обработки на свойства сварного соединения вал-турбоколесо из сталей 40Х и 40Г со сплавом СЖЛ-800. *Оборудование и технологии термической обработки металлов и сплавов*: сб. доклад. 6-й международной кон-

ференции. Раздел II. Технологии термической и термомеханической обработки. Харьков, 2005. С. 127–128.
2. Баранов А. А., Слюсарев В. Ю., Лейрих И. В. Термоциклирующая обработка стали. *Черная металлургия.* 1990. № 3. С. 32–41.
3. Кидин И. Н., Астафьев Е. В., Акопов Е. С. Структурное состояние стали после циклической элекротермообработки. *Изв. вузов. Черная металлургия.* 1974. № 11. С. 142–146.
4. Руденко А. Г. Поведение стали 1Х18Н9Т при термоциклировании. *Изв. АН СССР. Мет.* 1982. № 6. С. 137–141.
5. Биронт В. С., Носовец Н. Г., Комендоровская О. Н. Термоциклирующая обработка доэвтектоидной стали. *Термоцикл. обраб. метал. изделий.* Ленинград, 1982. С. 68–71.
6. Рядно А. А., Цоцко В. И., Пелешенко Б. И. Численно-аналитическое определение скорости кристаллизации расплава в условиях осевой симметрии на примере проушин отливок траков. *Математичне моделювання. Дніпродзержинський державний технічний університет.* 2008. 2 (19). С. 22–25.
7. Тихонов А. Н., Самарский А. А. Уравнения математической физики: учебное пособие для университетов. Москва: Наука, 1972. 736 с.
8. Сивухин Д. В. Общий курс физики. Термодинамика и молекулярная физика: учебное пособие для студентов физических специальностей высших учебных заведений. Москва: Наука, 1979. 552 с.
9. Цоцко В. И., Денисенко А. И. Температурные характеристики поверхностного слоя низкоуглеродистых сталей в условиях линейного нагрева поверхности. *Вісник Дніпропетровського університету.* Дніпропетровськ, 2004. № 2/2. С. 72–77.
10. Цоцко В. И., Спиридонова И. М. Температурные колебания в металле. *Вісник Дніпропетровського університету.* Дніпропетровськ, 2010. Т. 18, № 2. С. 61–68.
11. Цоцко В. І., Спиридонова І. М., Пелешенко Б. Г. Формування температурних хвиль в умовах періодичної енергетичної дії на поверхню металу. *Фізика і хімія твердого тіла.* 2010. Т. 11, № 4. С. 1037–1043.