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MATHEMATICAL MODEL OF A PIPE WELDED CONNECTION UNDER VIBRATION

The article discusses the theoretical foundations for improving pipe welding processes through the use of vibration. A mathematical model of a pipe welded joint has been constructed, which makes it possible to determine the residual stresses in welded seams exposed to vibration during the welding process and the optimal time of vibration exposure. The constructed mathematical model makes it possible to obtain analytical values of residual stresses for each specific processed object having data on the natural vibration frequency, vibration damping coefficient, amplitude and frequency of the driving force of vibration.

Keywords*: residual stresses; vibration; pipe; weld; mathematical model.*

У статті розглядаються теоретичні основи вдосконалення процесів зварювання труб за рахунок застосування вібрації. Побудовано математичну модель трубного зварного з'єднання, що дозволяє визначати залишкові напруги в зварних швах, до яких застосовується вібрація в процесі зварювання і оптимальний час вібраційного впливу. Побудована математична модель дозволяє отримати аналітичні значення залишкових напружень для кожного конкретного оброблюємого об'єкта маючи дані про власну частоту вібрації, коефіцієнт затухання коливань, амплітуду і частоту вимушуючої сили вібрації.

Ключові слова*: залишкові напруги; вібрація; труба; зварений шов; математична модель.*

Problem's Formulation

Currently, welding is one of the most commonly used technologies for joining metal structures in many branches of modern industry, including metallurgy. According to experts [1,2], up to 2/3 of the world consumption of rolled steel goes to the production of welded structures and structures, almost all metals are welded in any conditions, and the thickness of welded units ranges from micrometers to meters, the mass of welded structures — from fractions grams to hundreds and thousands of tons. Welding is often the only possible or most effective way to create permanent joints of structural materials and obtain standard products. Its widespread use is due to a number of advantages, such as a reduction in the weight of structures, a reduction in the production time for products of complex shape, and great opportunities for mechanization and automation of the process. At the same time, welding also has some disadvantages, which include the occurrence of residual stresses in the heat-affected zone, which complicates the assembly of large-sized structures.

A similar situation arises in the production of welded pipes and is characterized by cooling of the metal and the appearance of stresses in it caused by unequal heating of the base and deposited metals, its shrinkage, structural changes due to heating and rapid cooling, a change in the solubility of gases in the welded seam upon cooling and, as consequence, its destruction.

Analysis of recent research and publications

The processes of occurrence of residual stresses in the welded elements were described in [3,4], methods for determining residual stresses are presented in [5,6]. Measures to prevent or reduce stresses and strains during welding are described in [7]. The works of Antonov A.A., Birger I.A., Vinokurov V.A. are fundamental in the development and improvement of experimental methods for determining residual stresses.

Each of the known methods of reducing residual stresses has a more or less limited area of rational application, therefore, it becomes necessary to both improve existing and search for fundamentally new methods of processing pipes in order to remove residual stresses, as well as hardening, stabilizing geometry, changing the structural state [8, nine].

To eliminate or reduce the level of residual stresses, constructive and technological measures are taken. The constructive method is based on the rational design of welded assemblies, and the technological method is based on the rational choice of the thermal regime, assembly method and welding technology.

Welding technology is currently of great importance for reducing residual stresses, especially with the use of vibration. This topic is discussed in the works of various leading research centers in Ukraine and the CIS (E.O.Paton Electric Welding Institute, National Academy of Sciences of Ukraine, Kiev, Ukraine; IPPM SB RAS, Tomsk, Russia; Gubkin Russian State University of Oil and Gas, Moscow, Russia). At the same time, research in the field of vibration processing of welded joints in the process of pipe production has been insufficiently carried out [10, 11].

Formulation of the study purpose

The purpose of this work is to construct a mathematical model of a pipe welded joint, taking into account the influence of the interaction of individual particles in the bulk of the material, in order to determine the residual stresses in the sample exposed to vibration during the welding process, as well as the optimal vibration exposure time during welding of the pipe billet.

Presenting main material

The essence of the proposed method of vibration processing in relation to welding consists in communicating vibration waves to the processed pipe billet during the welding process, thus preventing the formation of residual stresses in the welded seam and the heat-affected zone.

When constructing a mathematical model of vibration treatment of a weld, the following assumptions are introduced:

- The temperature field at each point of the welding surfaces does not depend on the coordinates of the point;
- Instant laying of the seam along the entire length;
- The deformation of the material is elastic-plastic;
- The bodies to be welded are elastic-plastic;
- The plastic flow of the material has a wave character.

The specific potential energy of a single layer is the sum of the potential energies of interaction of individual particles:

$$W = \sum_{i=1}^{n} W_i, \tag{1}$$

where W_i — specific potential energy i-th particle; n — number of particles in a layer.

For the specific potential energy of a particle, the following formula holds [12]:

$$W_i = \text{б} \cdot \frac{A}{r^m} + \text{в} \cdot \frac{B}{r^n}, n > m. \tag{2}$$

Here: $\text{б}, \text{в}$ — constants that take into account the interaction of particles with neighboring; r — distance between nearest neighboring particles; A, B — constants for a given substance ($A \geq 0$; $B \geq 0$); m — number of layers.

Expression (2) is called the Mie potential. The extreme value of functional (2) is found using differentiation, equating to zero and transforming the resulting expression:

$$W_i = \text{б} \cdot \frac{A}{d^m} (\frac{d^m}{r} - (\frac{d}{r})^n \cdot \frac{m}{n}). \tag{3}$$

According to [13], the change in potential energy during relaxation is described by the following equation:

$$\frac{dW_i}{dt} = C \cdot e^{(a(\text{ў}+\text{у}_a)/\text{у}_{-1})}, \tag{4}$$

where C — experimentally determined constant; ў — residual stresses; у_a — amplitude value of the dynamic stresses created by the vibration source; у_{-1} — material fatigue limit; a — vibration amplitude; t — vibration exposure time.

Integrating (4) over the variable t and equating the resulting expression to (3), we obtain the formula for finding the vibration processing time

$$t = \frac{\sum \text{б} \cdot \frac{A}{d^m}\left(\frac{d^m}{r} - \left(\frac{d}{r}\right)^n \cdot \frac{m}{n}\right)}{C \cdot e^{(a(\text{ў}+\text{у}_a)/\text{у}_{-1})}}. \tag{5}$$

In formula (5), the residual stresses $\breve{y}$ are considered as the operator of the stress tensor $y_{ij} = 1/\breve{y}$ and are found from the solution of the differential equation

$$\ddot{y} + 2д\dot{y} + щ_0^2 \cdot y = a \cdot \sin щt, \tag{6}$$

where y — residual stress level, $д$ — damping coefficient; $щ$ — forced vibration frequency; $щ_0$ — natural cyclic frequency of oscillations; t — time; a — forced vibration amplitude.

Equation (6) is a second-order linear inhomogeneous differential equation with constant coefficients and a special right-hand side.

Solution (6) has the form:

$$y = \bar{y} + y^*, \tag{7}$$

where $\bar{y}$ — general solution of a linear homogeneous second order differential equation with constant coefficients corresponding to an inhomogeneous differential equation; y^* — particular solution of a linear inhomogeneous differential equation (6).

We consider solution (6) for 3 cases: $D > 0; D = 0; D < 0$. Here $D = 4(д^2 - щ_0^2)$ is the discriminant of the characteristic equation

$$k^2 + 2д \cdot k + щ_0^2 = 0. \tag{8}$$

I. Case $D > 0$: roots k_1, k_2 of equation (8) are different, real. Then $д > щ_0$; $k_{1,2} = -д \pm \sqrt{D}$. General solution (6) has the form:

$$y_I = e^{(-д-\sqrt{D})\cdot t} c_1 + e^{(-д+\sqrt{D})\cdot t} c_2 + \frac{2\cdot д\cdot щ\cdot a}{4д^2щ^2+\left(щ^2-щ_0^2\right)^2}\cos(щt) +$$
$$+\frac{a\cdot\left(щ^2-щ_0^2\right)}{4д^2щ^2+\left(щ^2-щ_0^2\right)^2}\sin(щt). \tag{9}$$

II. Case $D = 0$: roots k_1, k_2 of equation (8) are multiple, real. Then $д = щ_0$; $k_{1,2} = -д$. General solution (6) has the form:

$$y_{II} = e^{-д\cdot t}(c_1 + c_2 t) + \frac{2 \cdot д \cdot щ \cdot a}{4д^2щ^2+(щ^2 - щ_0^2)^2} \cdot \cos(щt) +$$
$$+\frac{a\cdot\left(щ^2-щ_0^2\right)}{4д^2щ^2+\left(щ^2-щ_0^2\right)^2} \cdot \sin(щt). \tag{10}$$

III. Case $D < 0$: roots k_1, k_2 of equation (8) are complex conjugate. Then $д < щ_0$; $k_{1,2} = -д \pm i \cdot \sqrt{\breve{D}}$, where $\breve{D} = 4(щ_0^2 - д^2)$. General solution (6) has the form:

$$y_{III} = e^{-д\cdot t}\left(c_1 \cos\sqrt{\breve{D}}t + c_2 sin\sqrt{\breve{D}}t\right) + \frac{2\cdot д\cdot щ\cdot a}{4д^2щ^2+\left(щ^2-щ_0^2\right)^2}\cos(щt) +$$
$$+\frac{a\cdot\left(щ^2-щ_0^2\right)}{4д^2щ^2+\left(щ^2-щ_0^2\right)^2}\sin(щt). \tag{11}$$

Here c_1 and c_2 are arbitrary constants.

To find arbitrary constants c_1 and c_2, we use the initial conditions:

$$\begin{cases} y(0) = 0; \\ y'(0) = 0. \end{cases} \tag{12}$$

The solution to the Cauchy problem for case I is obtained in the form:

$$y_I = e^{(-д-\sqrt{D})\cdot t}\frac{a \cdot щ \cdot \left[2д \cdot \left(-д + \sqrt{D}\right) + щ \cdot (щ^2 - щ_0^2)\right]}{2\sqrt{D}(4д^2щ^2+(щ^2 - щ_0^2)^2)} +$$
$$+e^{(-д+\sqrt{D})\cdot t}\frac{-a\cdot щ\cdot\left[2д\cdot\left(д+\sqrt{D}\right)+щ\cdot\left(щ^2-щ_0^2\right)\right]}{2\sqrt{D}\left(4д^2щ^2+\left(щ^2-щ_0^2\right)^2\right)} + \frac{2\cdot д\cdot щ\cdot a}{4д^2щ^2+\left(щ^2-щ_0^2\right)^2}\cos(щt) +$$
$$+\frac{a\cdot\left(щ^2-щ_0^2\right)}{4д^2щ^2+\left(щ^2-щ_0^2\right)^2}\sin(щt). \tag{13}$$

The solution to the Cauchy problem for case II is obtained in the form:

$$y_{II} = e^{-д\cdot t}\left(-\frac{2\cdot д\cdot щ\cdot a}{4д^2щ^2+\left(щ^2-щ_0^2\right)^2} - \frac{a\cdot щ^2\cdot\left(щ^2-щ_0^2\right)+2д^2\cdot щ\cdot a}{4д^2щ^2+\left(щ^2-щ_0^2\right)^2}t\right) +$$
$$+\frac{2\cdot д\cdot щ\cdot a}{4д^2щ^2+\left(щ^2-щ_0^2\right)^2}\cos(щt) + \frac{a\cdot\left(щ^2-щ_0^2\right)}{4д^2щ^2+\left(щ^2-щ_0^2\right)^2}\sin(щt). \tag{14}$$

The solution to the Cauchy problem for case III is obtained in the form:

$$y_{III} = e^{-\text{д}\cdot t}\left(-\frac{2\cdot\text{д}\cdot\text{щ}\cdot a}{4\text{д}^2\text{щ}^2+\left(\text{щ}^2-\text{щ}_0^2\right)^2}\cdot\cos\sqrt{\breve{D}}\cdot t-\frac{a\cdot\text{щ}^2\cdot\left(\text{щ}^2-\text{щ}_0^2\right)+2\text{д}^2\cdot\text{щ}\cdot a}{4\text{д}^2\text{щ}^2+\left(\text{щ}^2-\text{щ}_0^2\right)^2}\cdot sin\sqrt{\breve{D}}\cdot t\right)+$$
$$+\frac{2\cdot\text{д}\cdot\text{щ}\cdot a}{4\text{д}^2\text{щ}^2+\left(\text{щ}^2-\text{щ}_0^2\right)^2}\cdot\cos(\text{щ}t)+\frac{a\cdot\left(\text{щ}^2-\text{щ}_0^2\right)}{4\text{д}^2\text{щ}^2+\left(\text{щ}^2-\text{щ}_0^2\right)^2}\cdot\sin(\text{щ}t). \qquad (15)$$

Thus, expressions (12)—(14) represent solutions to the differential equation (6) and make it possible to obtain analytical values of residual stresses having data on the natural vibration frequency, vibration damping coefficient, amplitude and frequency of the driving force of vibration.

Conclusions

1. The stresses and strains arising during welding have been sufficiently studied; On this topic, there are many works of researchers, in which the causes of occurrence, classification of welding stresses and strains, methods of their determination and methods of prevention have been disclosed.
2. In order to prevent the formation of residual stresses in the welded seam and the heat-affected zone, it is proposed to use vibration processing of a pipe billet during the welding process.
3. A mathematical model of vibration treatment of a weld and a formula for finding the time of vibration treatment, taking into account the influence of the interaction of individual particles in the volume of the material, has been built.
4. Determination of residual stresses during vibration processing of welded joints of pipes was reduced to solving the problem of mathematical programming. The presented formulas make it possible to obtain analytical values of residual stresses having data on the natural vibration frequency, vibration damping coefficient, amplitude and frequency of the driving force of vibration.

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МАТЕМАТИЧНА МОДЕЛЬ ТРУБНОГО ЗВАРНОГО З'ЄДНАННЯ, ЩО ЗНАХОДИТЬСЯ ПІД ВПЛИВОМ ВІБРАЦІЇ

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Реферат

Метою цієї роботи є побудова математичної моделі трубного зварного з'єднання, що враховує вплив взаємодії окремих частинок в обсязі матеріалу, з метою визначення залишкових напружень в зразку, до якого застосовано вібраційний вплив в процесі зварювання, а також оптимального часу вібраційного впливу при зварюванні трубної заготовки. Сутність запропонованого способу вібраційної обробки стосовно до зварювання полягає в повідомленні трубній заготовці, що оброблюється вібраційних хвиль в процесі зварювання, запобігаючи таким чином формування залишкових напружень у зварному шві і біляшовній зоні. Виникаючі при зварюванні напруги і деформації в достатній мірі вивчені; на дану тему існує безліч робіт дослідників, в яких були розкриті причини виникнення, класифікації зварювальних напруг і деформацій, методи їх визначення та способи запобігання. З метою запобігання формуванню залишкових напружень в зварному шві і біляшовній зоні запропоновано використання вібраційної обробки трубної заготовки в процесі зварювання. Побудовано математичну модель вібраційної обробки зварного шва і формула для знаходження часу вібраційної обробки, що враховує вплив взаємодії окремих частинок в обсязі матеріалу. Визначення залишкових напружень при вібраційній обробці зварних з'єднань труб було зведено до вирішення задачі математичного програмування. Представлені формули дозволяють отримати аналітичні значення залишкових напружень маючи дані про власну частоту вібрації, коефіцієнт затухання коливань, амплітуду і частоту вимушуючої сили вібрації

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