DOI: 10.31319/2519-8106.2(47)2022.268340
UDK 621.317
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ANALYSIS OF THE EFFICIENCY OF MATHEMATICAL MODELS OF THE METROLOGICAL RELIABILITY OF MEASUREMENTS IN THE PRODUCTION SYSTEM

The article presents a comparative analysis of mathematical models of metrological reliability of measuring instruments when solving the task of optimizing the functioning and organization of metrological control of measuring instruments (MC MI) in production.

Keywords: metrological reliability, metrological control of measuring instruments, analytical models of assessment of metrological reliability of measuring instruments.

У статті приведений аналіз математичних моделей метрологічної надійності засобів вимірювань при розв'язку задачі оптимізації функціонування і організації метрологічного контролю засобів вимірювань(МК ЗВ) на виробництві.

Ключові слова: метрологічна надійність, метрологічний контроль засобів вимірювань, аналітичні моделі оцінки метрологічної надійності засобів вимірювань.

Problem's formulation

The organization of metrological control of measuring instruments is related to the organization of the production system. The existing methods of solving the problems of assessing the metrological reliability of measuring instruments based on analytical models of operation and metrological control are limited by the hypothesis of exponential models of sudden and metrological failures. Along with this, metrological evaluation methods based on probabilistic and physical models of metrological failures have recently become widely used. Therefore, the task of comparative analysis and evaluation of the effectiveness of the application of mathematical models of metrological reliability of measuring instruments is relevant [1].

Analysis of recent research and publications

The model of the operation process and the metrological control (MC) MI, which are considered in the work [2], operate with the probability of finding the MI in each of the possible states, while only the stationary case is considered. When analyzing metrological reliability with the help of a state model, a Markov model is used for the analytical solution, which describes a random process with continuous time (differential equation of the process of the transition of the MI from one state to another). All flows of events that transfer the process from one state to another are poisson — the probability of the time spent by the MI in each of the states obeys the exponential law. In [3—4], a discrete-continuous model of operation of the control system is presented, which, unlike the markov model, takes into account the difference between random events of failures and deterministic events of periodic control of the control system. The conducted studies showed that the calculations made using the Marcov model, in comparison with the discrete-continuous model, underestimate the indicators of metrological reliability, but the general nature of their dependence on the periodicity of verifications is shown correctly.

Formulation of the study purpose

The development of methods for evaluating the reliability of the vehicle under the influence of various factors is a task, the solution of which allows the consumer to more accurately determine the reliability indicators at any moment of the operation of the vehicle in real conditions, to correctly determine the terms of inspections and preventive works.

Presenting main material

Metrological reliability as a property of a measuring instrument affects almost all its characteristics. The quality of metrological control is characterized by the following parameters:

 α_v — probability of false diagnosis of metrological failure during inspection; β_v — the probability of not finding (not revealing) a metrological failure during inspection; β_r — the probability of returning a repair order with a metrological failure (it is assumed that a repair order is not returned with a sudden failure).

As a rule, during the operation of the control system, it is necessary to determine the dependence of the control time for repair T_{mi} , on the control time for an undisguised (obvious) failure T_u , the control time for a metrological failure T_m , and the inter-check interval at the given values T_v of the metrological control indicators.

In general, the formula for T_{mi} has the form [3]

$$T_{mi} = \int_{0}^{\infty} t \dot{P}_{r}(t) dt,$$

where $\dot{P}_r(t) = \frac{d}{dt}(P_r(t))$, $P_r(t)$ — distribution of probabilities of sending MI for repair.

Let's introduce dimensionless relations

$$\Theta_{\rm r} = \frac{T_{mi}}{T_u}; \Theta_{\rm v} = \frac{T_{\rm v}}{T_u}; \Theta_m = \frac{\overline{T_m}}{T_u}$$

Then all time parameters will be expressed in relation to T_{μ} , which is a significant conveni-

ence for analysis, because
$$\Theta_u = 1, \Theta = \frac{t}{T_u}, dt = d\Theta \cdot T_u$$
.
 $\Theta_r = \int_0^\infty \Theta \dot{P}_r(\Theta) d\Theta$

Taking into account the exponential laws of probability distribution of the time of occurrence of manifest and metrological failures, we have:

$$\Theta_{r} = \sum_{k=1}^{\infty} n_{k} \left\{ (k\Theta_{v} + 1 - \Theta_{v}) - (k\Theta_{v} + 1)e^{-\Theta_{v}} \right\} + \sum_{k=1}^{\infty} k\Theta_{v} (n_{k}e^{-\Theta_{v}} - n_{k+1})$$

 n_k — the probability that after the completion of the k — inspection, the work permit will remain in operation, where $k = \left[\frac{\Theta}{\Theta_v}\right] + 1$.

An alternative to the considered model of metrological reliability analysis is the discretecontinuous model studied in works [2, 4]. The formalization of the process of metrological maintenance of the MI based on this approach is as follows: $p_{1,k}$ and $p_{2,k}$ — the probability of detecting the MI immediately after the k-th verification, respectively, in states 1 (MI work without failures) or 2 (MI work with a metrological failure) [4—6]. Probability $p_{\neg m,k}(t)$ that in the time interval $\tau_v + kT_v \le t < \tau_v + (k+1)T_v$ the MI will work without failures, and the probability $p_{m,k}(t)$ that in the same time interval the MI will have a metrological failure, where τ_v, T_v — is the time and period of verification of the MI, $p_m(...)$ and $p_u(...)$ are the probabilities of occurrence of metrological and obvious failures in the MI according to the moment of time given in parentheses.

The mathematical expectation of the time of the MI t_j being in a certain state *j* is expressed [3]

$$t_{j} = \int_{0}^{\infty} \xi \frac{dp_{j}(\xi)}{d\xi} d\xi = -\int_{0}^{\infty} \xi \frac{dp_{i}(\xi)}{d\xi} d\xi,$$

where is $p_i(\xi)$ — the probability of the MI remaining in the *j*-th state until the moment of time ξ .

In fig. 1 shows the solution for obtained using the developed model of analysis of complex reliability indicators. The obtained dependence sufficiently accurately approximates the dependence of the inter-repair interval in the case of a discrete-continuous model and Markov models of states.



Fig. 1. Dependencies T_{mi} on the T_v period for the case of ideal inspection and repair: 1 — discrete-continuous model; 2 — Markov model; 3 — the model of the analysis of the reliability of the MI

Recently, a number of works [1, 4—6] have been published, which are devoted to the study of properties and applications for solving the problems of assessing and predicting the reliability of technical systems of probabilistic-physical failure models. This approach is based on the analysis of physical processes of degradation that lead to failures. Let's consider the main schemes of formalization of mathematical and physical failure models and the possibility of their use for reliability analysis.

For the analysis of the metrological reliability of the MI, the input distributions of failures were used: for clear failure — an exponential model, for metrological — diffusion — monotonic (DM) and diffusion — nonmonotonic (DN) distribution (depending on the idealization of the random process of metrological failure occurrence). The failure model and maintenance interval T_{mi} in this case is expressed in [1, 4]

$$T_{mi} = T_u \left[\left(1 - e^{-\frac{\tau_v}{T_u}} \right) + \left(1 - e^{-\frac{T_v}{T_u}} \right) \sum_{k=0}^{\infty} p_{DM,k} \right],$$

where $p_{DM,k}$ — is the mathematical expectation of the total time of stay of MI in state 1 and 2 during the life cycle T_c .

The inter-repair interval $T_v \to \infty$ at for diffusion and exponential models goes towards the average working time for metrological failure. Studies show that diffusion models at values $T_m < 100,000$ hours have a non-stationary section (function failure) (Fig. 2 curves 1, 2), but at values $T_v >> T_m$ they go to the case of an exponential model. It should be noted that DM — a model unlike DN and exponential overestimates the mathematical expectation of the time between repairs. When the value $T_m (T_m >> T_u)$ increases, the discrepancy between these dependencies decreases.



Fig. 2. The family of curves of dependence of the mathematical expectation of the maintenance interval T_{mi} on the period T_v (logarithmic scale) for a number of values T_m : 1 — 464 hours, 2 — 10000 hours, 3 — 46416 hours.

Conclusions

The development of methods for assessing the metrological reliability of measuring equipment under the influence of various factors is a task, the solution of which will allow the consumer to more accurately determine the MH at any moment of the operation of the VT in real conditions, to correctly determine the terms of inspections and preventive works.

The use of DM and DN — metrological models and the exponential model of apparent failure to solve the problems of diagnosing and monitoring the metrological reliability of the equipment allows to increase the accuracy of the assessment of the parametric component of reliability due to the approximation of failure statistics by diffusion functions and a priori analysis of the physics of failure occurrence.

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АНАЛІЗ ЕФЕКТИВНОСТІ МАТЕМАТИЧНИХ МОДЕЛЕЙ МЕТРОЛОГІЧНОЇ НАДІЙНОСТІ ЗАСОБІВ ВИМІРЮВАНЬ В ВИРОБНИЧІЙ СИСТЕМІ Литвиненко В.А., С'янов О.М., Марченко С.В., Нескоромна Ю.О.

Реферат

У статті розглянуті математичні методи теорії метрологічної надійності(МН) засобів вимірювань. Проведено порівняльний аналіз і оцінка ефективності застосування математичних моделей метрологічної надійності засобів вимірювань (ЗВ) на основі статистичних і ймовірностио-фізичних моделей метрологічних відмов в задачах обчислення і прогнозування показників надійності ЗВ в будь-який момент часу експлуатації ЗВ.

Розробка методів оцінки метрологічної надійності засобів вимірювальної техніки під впливом різних факторів являється задачею, розв'язок якої дозволить споживачу більш точно визначати МН в будь-який момент часу експлуатації ЗВТ в реальних умовах, вірно визначити терміни перевірок і профілактичних робіт.

Використання DM і DN — моделей метрологічних та експоненціальної моделі явної відмови для розв'язку задач діагностики і моніторингу метрологічної надійності ЗВ дозволяє підвищити точність оцінки параметричної складової надійності за рахунок апроксимації статистики відмов дифузійними функціями і апріорного аналізу фізики виникнення відмов. Даний підхід базується на аналізі фізичних процесів деградації, які призводять до виникнення відмов. Розглянуті показники MH ЗВ дозволяють оцінити ступінь впливу параметрів метрологічного обслуговування ЗВ на якість контрольно-вимірювальних операцій в виробничій системі.

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