МАТЕМАТИЧНІ МЕТОДИ В СУСПІЛЬНИХ І ГУМАНІТАРНИХ НАУКАХ

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CONCERNING THE IMPLEMENTATION OF THE MATHEMATICAL MODEL LOTKA-VOLTERRA AT THE RESEARCH OF ECONOMIC PROCESSES

ЩОДО РЕАЛІЗАЦІЇ МАТЕМАТИЧНОЇ МОДЕЛІ ЛОТКИ-ВОЛЬТЕРРИ ПРИ ДОСЛІДЖЕННІ ЕКОНОМІЧНИХ ПРОЦЕСІВ

The work considers the well-known Lotka-Volterra mathematical model. Examples are given that testify to the active use of this model in the study of various economic processes. It is emphasized that the nonlinearity of the mathematical model necessitates the use of numerical methods and means of their computer implementation. The proposed approach is based on the use of the well-known Runge-Kutta numerical method implemented in the MS Excel spreadsheet environment. The problem is solved in data manipulation mode without the need for programming. The described approach is implemented for the case of the study of the rational use of resources in the railway sphere of activity. The obtained results are in good agreement with the data of other authors.
Keywords*: model Lotka-Volterra, research of economic processes, numerical methods, table processor MS Excel, mode of manipulation data.*

У роботі розглядається математична модель Лотки-Вольтерри, первинним призначенням якої був аналіз відношення між двома біологічними видами, представники одного з яких виступали як хижаки, представники іншого – як жертви. Відзначено, що дана модель з успіхом може бути використана і в інших галузях, для яких характерні циклічні процеси зміни стану взаємодіючих агентів та сильна нелінійність. Наведені приклади, які свідчать про активне ви-

користання моделі Лотки-Вольтерри при дослідженні різноманітних економічних процесів. В той же час підкреслено, що до конкретних кількісних результатів слід відноситися з обережністю, оскільки вони сильно залежать від параметрів моделі, точні значення яких зазвичай невідомі.

Нелінійність математичної моделі зумовлює необхідність застосування чисельних методів розв'язання систем звичайних диференціальних рівнянь. В той же час безпосередня реалізація математичної моделі в конкретних випадках для фахівців в галузі економіки може викликати певні труднощі, оскільки потребує знань не тільки математики, а й засобів комп'ютерної реалізації математичних методів.

Для реалізації математичної моделі Лотки-Вольтерри запропонований підхід, в основі якого лежить використання відомого чисельного методу Рунге-Кутта, реалізованого в середовищі табличного процесора MS Excel. Розв'язання задачі проводиться в режимі маніпулювання даними без необхідності програмування. Основні розрахункові формули вводяться в один з рядків електронної таблиці. Для організації циклічних обчислень формули копіюються в інші рядки за допомогою методу "перетягнути і залишити". Виконання умови завершення циклу контролюється візуально за значеннями часу перебігу процесу або функцій, що характеризують поведінку взаємодіючих в системі агентів, залежно від цілей дослідження.

Описаний підхід реалізований для випадку дослідження раціонального використання ресурсів у залізничній сфері діяльності. Одержані результати добре узгоджуються з даними інших авторів.

***Keywords**: модель Лотки-Вольтерри, дослідження економічних процесів, чисельні методи, табличний процесор MS Excel, режим маніпулювання даними.*

Problem's Formulation

Various mathematical models are widely used in the study of economic processes. The special popularity of such models is not least due to their universality: the same mathematical model can be successfully used to solve problems related to completely different fields of science and technology. Thus, at one time, the efforts of A. Lotka and V. Volterra created a "predator-prey" model to describe the relationship between two biological species, representatives of one of which acted as predators, representatives of the other — as victims. As a result of modeling, the nature of changes in the number of representatives of each species over time was studied [1,2]. Later, it became clear that this model can be successfully used in other industries, which are characterized by cyclical processes of changing the state of interacting agents and strong nonlinearity. In particular, in economics, the mathematical model of Lotka-Volterra describes the processes of competition quite well [3,4].

Thus, the problem of implementing the mathematical model of Lotka-Volterra does not lose its relevance and needs further study to form an appropriate methodology for the study of economic processes.

Analysis of recent research and publications

The issue of using the Lotka-Volterra mathematical model was studied in the works of many domestic and foreign scientists. The general principles of construction and the main properties of the model were reflected in works [1—2]. Papers [3—8] are devoted to the peculiarities of the application of the Lotka-Volterra mathematical model in economics. At the same time, the authors give examples of specific economic processes in which the roles of predators and victims are distinguished; discuss the principles of determining model parameters. Considerable attention is paid to actual modeling using various methods and tools. In particular, paper [9] describes the procedure for implementing the Lotka-Volterra mathematical model using the classical Runge-Kutt method, paper [10] uses the MathCAD mathematical package, and papers [4, 6] use the MATLAB/Simulink package. In [3], a fairly simple Euler method was implemented using the MATLAB package to solve a system of ordinary differential equations corresponding to the Lotka-Volterra model.

Formulation of the study purpose

The purpose of the study is to show the possibilities of using the Lotka-Volterra mathematical model in the economy and to determine the features of its implementation using various methods and tools.

Presenting main material

As mentioned above, the classical mathematical model of Lotka-Volterra was created to describe the coexistence of two biological species, representatives of one of which acted as predators, representatives of the other — as victims (food for predators). When building a mathematical model, it was taken into account that an increase in the number of potential victims leads to an increase in the number of predators; satisfying the need for food of the latter leads to a decrease in the number of victims. This, in turn, causes a decrease in predators, as they lack food and weaker or less adapted organisms simply die. Those that remain eventually adapt to new conditions and the cycle of changes in the number of predators and prey repeats.

The process considered above is well described by a system of two nonlinear differential equations of the first order with appropriate initial conditions:

$$\begin{cases} \dfrac{dx}{dt} = x(t)\cdot(a - b\cdot y(t)) \\ \dfrac{dy}{dt} = y(t)\cdot(-c + d\cdot x(t)) \end{cases}; \qquad \begin{cases} x(t_0) = x_0 \\ y(t_0) = y_0 \end{cases}, \tag{1}$$

where $x(t)$, $y(t)$ — the number of victims and predators at a moment in time t; a, b, c, d — parameters of the model that characterize the interaction of victims and predators.

The cyclic course and competitive nature of the processes described by the mathematical model (1) also occurs in many other cases, in particular, in the economy. This gives reasons for various researchers to use the mathematical model of Lotka-Volterra for "... analysis of the course of many economic phenomena and objects, calculating, for example, the number of potential workers as "predators" and the number of workplaces — "victims", the state budget and GDP; the number of capitalists and workers; the number of employees employed in the private sector of the economy and in the state; the number of consumers and producers; supply and demand" [3, p. 171]. In fact, the transition from one meaningful task to another is determined by the interpretation of functions $x(t)$, $y(t)$ and parameters a, b, c, d. Tabl. 1 shows examples of such an interpretation for some problems.

Table 1. Examples of using the Lotka-Volterra model

Model (task)	**$x(t)$, $y(t)$**	**Model parameters**
Classical biological model [3]	Victims and predators	*a* – prey growth factor during the absence of the predator; *b* – loss ratio of prey during encounters with predators; *c* – predator loss ratio in the absence of prey; *d* – coefficient of frequency of encounters between predators and victims ending in a «meal».
Dynamic change of the capital of the economic system [4]	Specific profits, specific costs per unit of capital	*a* – coefficient of «monopoly»; *b* – coefficient of influence of expenses on obtaining profit (the rate of decrease of profit due to expenses); *c* – coefficient of accounting for costs not related to profits; *d* – growth rate of costs in subsystems secured by profits.
Rational use of resources in the railway sphere of activity [6]	Number of brake pads; the number of locomotives equipped with pads	*a* – coefficient of supply of brake pads to the depot; *b* – coefficient of use of pads on locomotives; *c* – rate of reduction in the number of locomotives during the absence of brake shoes; *d* – utilization ratio of locomotives equipped with brake pads.
Competition of potential workers for jobs [3]	Future workers, working places	*a* – rate of growth of vacancies in the absence of potential workers; *b* – coefficient of efficiency of "hunting" workers for places; *c* – coefficient of reduction in the number of potential workers; *d* – rate of appearance of new potential workers on the labor market.

Continue of the table 1.

Project knowledge management [8]		*a* – probability that the amount of useful and necessary information will increase; *b* – probability that available useful and necessary information will turn into knowledge; *c* – probability that the necessary information for the formation of knowledge will be missing; *d* – the probability that available useful information and established communication channels at the enterprise during project implementation will be sufficient to form and increase the amount of knowledge.

So, the mathematical model of Lotka-Volterra is quite actively used in the study of various economic processes. At the same time, the direct implementation of a mathematical model in specific cases for specialists in the field of economics can cause certain difficulties, since it requires knowledge not only of mathematics, but also of means of computer implementation of mathematical methods.

Considering the nonlinearity of the mathematical model (1), one of the numerical methods for solving systems of ordinary differential equations should be recognized as the most acceptable method for solving the problem. In particular, the Runge-Kutta method has proven itself well (see, for example, [9]). It has a fairly high fourth order of accuracy, is logically not very complex, although it requires a large amount of calculations.

With respect to problem (1), the Runge-Kutta method is given by the formulas:

$$\begin{gathered} t_{i+1} = t_i + h, \\ x_{i+1} = x_i + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right), \qquad i = 0,1,2,\ldots,n-1 \\ y_{i+1} = y_i + \frac{1}{6}\left(q_1 + 2q_2 + 2q_3 + q_4\right), \end{gathered} \tag{2}$$

Intermediate coefficients k_1, k_2, k_3, k_4, q_1, q_2, q_3, q_4 are calculated by formulas:

$$\begin{aligned} k_1 &= h \cdot x_i\left(a - b \cdot y_i\right) & q_1 &= h \cdot y_i \cdot \left(-c + d \cdot x_i\right) \\ k_2 &= h \cdot \left(x_i + \frac{k_1}{2}\right) \cdot \left(a - b \cdot \left(y_i + \frac{q_1}{2}\right)\right) & q_2 &= h \cdot \left(y_i + \frac{q_1}{2}\right) \cdot \left(a - b \cdot \left(x_i + \frac{k_1}{2}\right)\right) \\ k_3 &= h \cdot \left(x_i + \frac{k_2}{2}\right) \cdot \left(a - b \cdot \left(y_i + \frac{q_2}{2}\right)\right) & q_3 &= h \cdot \left(y_i + \frac{q_2}{2}\right) \cdot \left(a - b \cdot \left(x_i + \frac{k_2}{2}\right)\right) \\ k_4 &= h \cdot \left(x_i + k_3\right) \cdot \left(a - b \cdot \left(y_i + q_3\right)\right) & q_4 &= h \cdot \left(y_i + q_3\right) \cdot \left(a - b \cdot \left(x_i + k_3\right)\right) \end{aligned} \tag{3}$$

In fact, to implement the algorithm of the method, it is necessary to set the initial values (x_0, y_0, a, b, c, d, t_0=0, h); organize a cycle of sorting through the values of time t with a step h ($t_{i+1} = t_i + h$, i=0, 1, 2, …); within the cycle, calculate the intermediate coefficients according to formulas (3) and the values of the required functions $x_{i+1}=x(t_{i+1})$, $y_{i+1}=y(t_{i+1})$ according to formulas (2), output t_{i+1}, x_{i+1}, y_{i+1} for printing.

The implementation of this kind of algorithms is traditionally carried out on personal computers using independently created programs or special mathematical packages such as MathCAD or MATLAB [3, 4, 10]. Both approaches require skills that are not always available in economics. At the same time, the task can be solved in the environment of the MS Excel table processor, which is well known to all PC users. To do this, it is enough to create an MS Excel workbook sheet similar to the one shown in fig. 1.

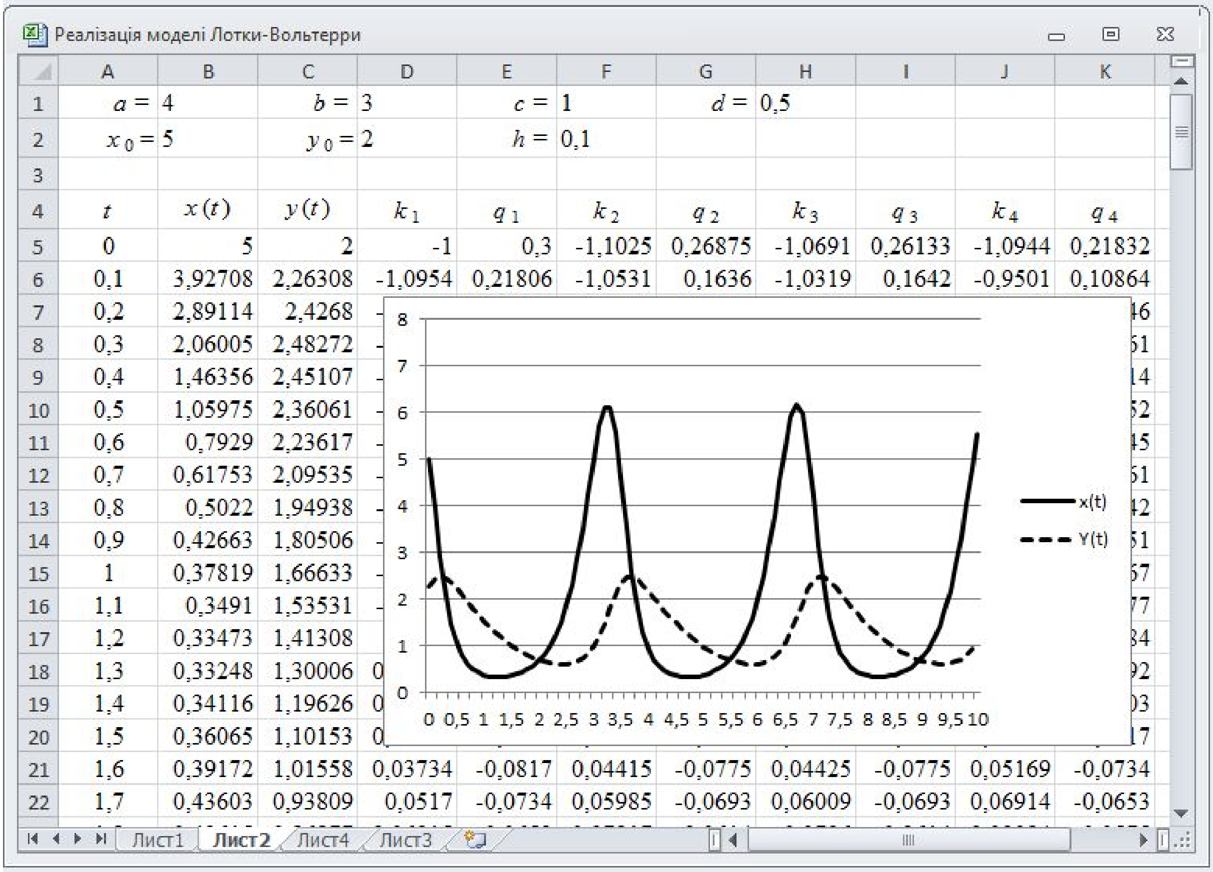

Fig. 1. Organization of the MS Excel worksheet

Cells A1, A2, C1, C2, E1, E2, G1, A4:K4 of the worksheet in Fig. 1 contain explanatory text; cells B1, B2, D1, D2, F1, F2, H1 — the values of the corresponding parameters of the model (the designations on the worksheet and in the text match).

The starting point of time (t = 0) and the initial values of the functions $x(t)$, $y(t)$ are entered in cells A5:K5. The rest of the cells of the worksheet contain formulas with the help of which the algorithm described above is implemented.

Formulas in cells D5:K5 and A6:C6 are entered manually using standard methods of working with the MS Excel spreadsheet (see, for example, [11, 12]). Other formulas are obtained by copying the specified formulas into the cells below using the drag-and-drop method. Content of formulas in cells D5:K5; A6:C6 is given in the tabl. 2.

Table 2. Calculation formulas of the worksheet "Implementation of the Lotka-Volterra model"

Cell	**Formula**
D5	=F$2*B5*(B$1-D$1*C5)
E5	=F$2*C5*(-F$1+H$1*B5)
F5	=F$2*(B5+D5/2)*(B$1-D$1*(C5+E5/2))
G5	=F$2*(C5+E5/2)*(-F$1+H$1*(B5+D5/2))
H5	=F$2*(B5+F5/2)*(B$1-D$1*(C5+G5/2))
I5	=F$2*(C5+G5/2)*(-F$1+H$1*(B5+F5/2))
J5	=F$2*(B5+H5)*(B$1-D$1*(C5+I5))

Continue of the table 2.

K5	=F$2*(C5+I5)*(-F$1+H$1*(B5+H5))
A6	=A5+F$2
B6	=B5+(D5+2*F5+2*H5+J5)/6
C6	=C5+(E5+2*G5+2*I5+K5)/6

Let's pay attention to the fact that copying formulas by the "drag and drop" method with this organization of the worksheet actually ensures the organization of cyclic calculations in accordance with the algorithm of the method. At the same time, the fulfillment of the cycle completion condition is visually controlled by the values of *t*, *x(t)* or *y(t)*, depending on the research objectives [13].

The worksheet was tested using an example from work [6]. At the same time, the functions *x(t)* and *y(t)* were interpreted as the number of hundreds of brake pads and the number of dozens of locomotives equipped with pads; the meaning of the model parameters is given in Tabl. 1, and the used values of the parameters are in the worksheet in Fig. 1. For clarity, the results of the test example are displayed in the form of a graph that fully corresponds to that given in [6]. Thus, the operability of the proposed quite simple approach to the implementation of the Lotka-Volterra mathematical model is confirmed.

Note that in most of the known works, a qualitative analysis of the studied process was carried out on the basis of the Lotka-Volterra mathematical model. Specific quantitative results should be treated with caution, as they are highly dependent on model parameters. This is clearly visible from fig. 2—3, which show the results of calculations for the above example with different values of parameters *a* and *b*.

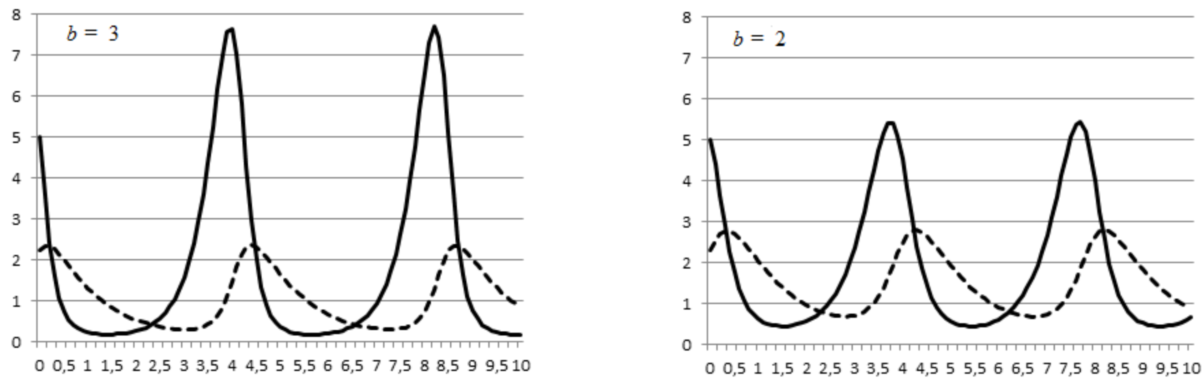

Fig. 2. Influence of model parameters on the course of the researched process (*a* = 3)

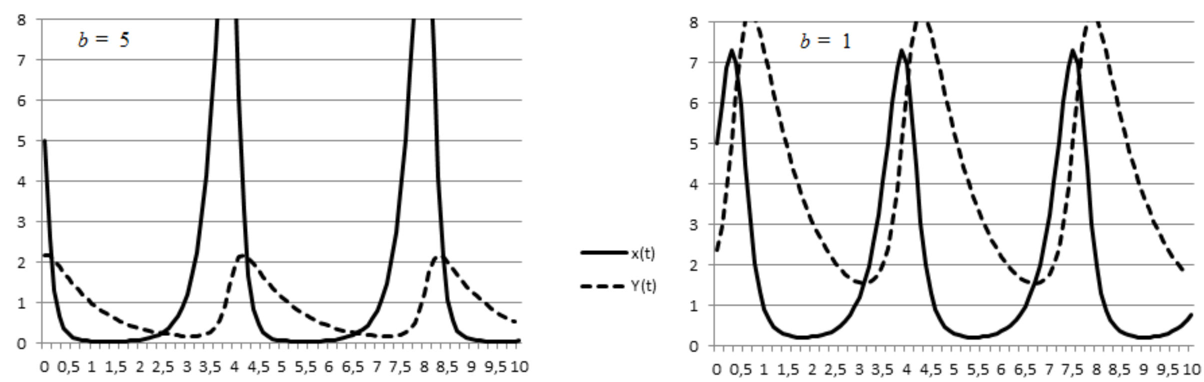

Fig. 3. Influence of model parameters on the course of the researched process (*a* = 4)

In practice, the exact values of model parameters are usually unknown, so approximate values obtained by processing statistical data are used. As a result, the calculated values of the functions *x(t)* and *y(t)* are also approximate, but in general they well reflect the laws of the studied process and provide an opportunity to propose measures to improve certain of its characteristics.

Conclusions

The mathematical model of Lotka-Volterra can be used in the study of regularities in the course of various economic processes. Specific quantitative results should be treated with caution, as they strongly depend on model parameters, the exact values of which are usually unknown.

To implement the Lotka-Volterra mathematical model, an approach based on the use of the well-known Runge-Kutt numerical method implemented in the MS Excel spreadsheet environment is proposed. The problem is solved in data manipulation mode without the need for programming.

References

[1] Maltsev, A. (2013). Matematychna i prohramna modeli spivisnuvannia populiatsii typu «khyzhak- zhertva» [Mathematical and software models of coexistence of "predator-prey" populations]. *Scientific news of NTUU "KPI"*, no. 3.

[2] Maevskyi, O., Pilkevich, I. (2010). Theoretical substantiation of the Lotka-Volterra population dynamics model [Teoretychne obhruntuvannia modeli dynamiky populiatsii Lotki-Volterra].*Bulletin of ZHTU (technical sciences)*, no. 3(54).

[3] Stavycjkyj, O., Djatlova, N. (2017). Metodologhija zastosuvannja matematychnoji modeli Lotky-Voljterry v ekonomici [Methodology of applying Lotka-Volterra mathematical model in economics]. *Pryazovsky Economic Bulletin*, vol. 2(02).

[4] Sokolov, Ju., Sokolov, A., Yljushko, V. (2010). Kompjjuterные tekhnologhyy v zadachakh pryrodu y obshhestva. Chastj 2. Modelj Lotky - Voljterra "khyshhnyk - zhertva" v zadachakh ekonomyky [Computer technologies in the problems of nature and society. Part 2. The model of Lotka - Volterra "predator - victim" in economic problems]. *Radioelectronic and computer systems*, no. 3.

[5] Kozyk, V., Sydorov, Ju. (2011). Problemy zastosuvannja modelej typu «khyzhak – zhertva» v ekonomichnij praktyci [Problems of using predator-prey models in economic practice]. *Science and innovations*, vol. 7, no. 1.

[6] Nerubacjkyj, V., Ghordijenko, D. (2018). Zastosuvannja klasychnoji modeli Lotky-Voljterry pry rozv'jazanni zadach racionaljnogho vykorystannja resursiv u zaliznychnij sferi dijaljnosti [Application of the classic Lotka-Volterra model in solving the problems of rational use of resources in the railway sphere of activity]. *Collection of scientific works of UkrDUZT*, vol. 178.

[7] Alilujko, A. (2013). Doslidzhennja konkurentnoji vzajemodiji na rynku poslugh mobiljnogho zv'jazku [Research of competitive interaction on the market of mobile communication services]. *Innovative economy*, no. 2.

[8] Chajkovsjka, I., Gharghasas, A. (2021). Osoblyvosti zastosuvannja modeli Lotky-Voljterry v upravlinni znannjamy proektu [Peculiarities of applying the Lotka-Volterra model in project knowledge management]. *Modeling the Development of the Economic Systems*, no. 2.

[9] Kostjushko, I., Ljubashenko, N., Tretynyk, V. (2021). *Metody obchyslenj [Calculation methods].* Kyiv : KPI im. Ighorja Sikorsjkogho, Vyd-vo «Politekhnika». [In Ukrainian].

[10] Boghach, I., Krakovecjkyj, O., Krylyk, L. (2020). *Chyseljni metody rozv'jazannja dyferencialjnykh rivnjanj zasobamy MathCAD [Numerical methods of solving differential equations by means of MathCAD].* Vinnytsia: VNTU [In Ukrainian].

[11] Kundrat, A., Kundrat, M. (2014). *Naukovo-tekhnichni obchyslennja zasobamy MathCAD ta MS Excel [Scientific and technical calculations using MathCAD and MS Excel].* Rivne: NUVGhP [In Ukrainian].

[12] Karimov, I. (2017). *Komp'juterni metody ta zasoby rozv'jazannja inzhenernykh zadach [Computer methods and means of solving engineering problems].* Kamianske: DDTU [In Ukrainian].

[13] Karimov, I., Karimov, H. (2020). *Komp'juterni tekhnologhiji v navchaljnomu procesi tekhnichnogho universytetu [Computer technologies in the educational process of a technical university]*. Kamianske: DDTU [In Ukrainian].

Список використаної літератури

1. Мальцев А.Ю. Математична і програмна моделі співіснування популяцій типу «хижак- жертва». *Наукові вісті НТУУ "КПІ"*. 2013. №3. С.70-74.
2. Маєвський О.В., Пількевич І.А. Теоретичне обґрунтування моделі динаміки популяцій Лоткі-Вольтерра. *Вісник ЖДТУ (технічні науки)*. 2010. №3(54). С.79-82.
3. Ставицький О.В., Дятлова Н.О. Методологія застосування математичної моделі Лотки-Вольтерри в економіці. *Приазовський економічний вісник*. 2017. Вип. 2(02). С. 168–171.
4. Соколов Ю.Н., Соколов А.Ю., Илюшко В.М. Компьютерные технологии в задачах природы и общества. Часть 2. Модель Лотки - Вольтерра "хищник - жертва" в задачах экономики. *Радіоелектронні і комп'ютерні системи*. 2010. № 3. С. 20–26.
5. Козик В.В., Сидоров Ю.І. Проблеми застосування моделей типу «хижак - жертва» в економічній практиці. *Наука та інновації*. 2011. Т. 7, № 1. С. 5–15.
6. Нерубацький В.П., Гордієнко Д.А. Застосування класичної моделі Лотки-Вольтерри при розв'язанні задач раціонального використання ресурсів у залізничній сфері діяльності. *Збірник наукових праць УкрДУЗТ*. 2018. Вип. 178. С. 41–49.
7. Алілуйко А.М. Дослідження конкурентної взаємодії на ринку послуг мобільного зв'язку. *Інноваційна економіка*. 2013. № 2. С. 221–226.
8. Чайковська І., Гаргасас А. Особливості застосування моделі Лотки-Вольтерри в управлінні знаннями проекту. *Modeling the Development of the Economic Systems*. 2021. №2. С. 54–61.
9. Костюшко І.А., Любашенко Н.Д., Третиник В.В. Методи обчислень : підручник. Київ : КПІ ім. Ігоря Сікорського, Вид-во «Політехніка», 2021. 243 с.
10. Богач І.В., Краковецький О.Ю., Крилик Л.В. Чисельні методи розв'язання диференціальних рівнянь засобами MathCAD: навч. посібник. Вінниця : ВНТУ, 2020. 106 с.
11. Кундрат А.М., Кундрат М.М. Науково-технічні обчислення засобами MathCAD та MS Excel. Рівне: НУВГП, 2014. 252 с.
12. Карімов І.К. Комп'ютерні методи та засоби розв'язання інженерних задач: навч. посібник. Кам'янське: ДДТУ, 2017. 283 с.
13. Карімов І.К., Карімов Г.І. Комп'ютерні технології в навчальному процесі технічного університету: монографія. Камянське : ДДТУ, 2020. 168 с.

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