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WAVEGUIDE PHASED ANTENNA ARRAY WITH APERTURE IRISES

ХВИЛЕВОДНА ФАЗОВАНА АНТЕННА РЕШІТКА З АПЕРТУРНИМИ ДІАФРАГМАМИ

In this paper the integral equation-based overlapping domain decomposition method is considered for the analysis of the electromagnetic wave diffraction process on inhomogeneities represented by thick irises in metal waveguides. The features of the developed computational algorithm for this type of inhomogeneities are shown on the sample problem of electromagnetic wave diffraction on an infinite parallel-plate phased antenna array. The dependence of the H_{10} wave type reflection coefficient magnitude and phase on the value of scan angle is shown as a result of the work. Also, a comparison of obtained results with the results obtained by other methods is shown in order to validate the developed algorithm.

***Keywords**: domain decomposition methods, integral equations, computational electromagnetics, diffraction.*

У роботі показано застосування методу часткових перетинних областей для аналізу процесу дифракції електромагнітної хвилі на неоднорідностях, що утворені діафрагмами кінцевої товщини у металевих хвилеводах. Особливості складання електродинамічного алгоритму для такого типу неоднорідностей показано на прикладі нескінченої фазованої антенної решітки з пласких хвилеводів, в апертурах яких розташована металева діафрагма заданої товщини.

В результаті роботи приведено залежність величини коефіцієнту відбиття хвилі типу H_{10} плаского хвилеводу від керуючого зсуву фаз антенної решітки та проведено порівняння з результатами, отриманими іншими методами.

***Ключові слова**: методи декомпозиції області, інтегральні рівняння, обчислювальна електродинаміка, дифракція.*

Problem’s formulation

The constant increasing of requirements for the electrical parameters of microwave waveguide devices, as well as the significant progress in the field of their production technology, raises more complex problems of their mathematical modeling. It is known that modern experimental research of microwave waveguide devices is an expensive and time-consuming task, which requires an automation of experiment with considerable time for its execution. Also, at microwave frequency range described approach may be inefficient. In some cases, experimental research cannot provide the required accuracy of the result.

At the same time, various devices based on rectangular waveguides are of great practical interest: waveguide transformers, filters, matching devices, etc. The production of such devices requires the use of expensive and highly precise technological processes.

Therefore, a necessary step in the development process of waveguide devices is an electromagnetic modeling, which can be performed using mathematical methods of various classes. Such classes include numerical analytical methods, which are less effective than numerical methods in terms of universality, however provide a significant gain in efficiency and a higher accuracy of calculations with lesser consumption of computing resources. Hence, the problem of developing of effective numerical and analytical methods for electromagnetic analysis of modern microwave devices is of interest.

Analysis of recent research and publications

Metal waveguides of various configurations are widely used in designing of microwave devices, particularly as radiators of antenna arrays [1, 2], filters and matching devices [3, 4]. Inhomogeneities in the form of thick irises are widely used in the design of such devices [5, 6].

The progress of modern microwave devices and a further increasing of their complexity require also a development of mathematical methods to perform accurate analysis of electromagnetic wave propagation and scattering in such devices.

The further development of the methods of mathematical modeling of the electromagnetic wave propagation process is called for the effective design and analysis of the of microwave device characteristics, considering further development and proceeding complication of such devices.

The effectiveness of each method depends on the class of problems to which it is applied to. Thus, the analysis of each specific electromagnetic problem requires accurate choice of the most effective mathematical method and its further development, which must take into account main features of considered problem [7, 8].The integral equation method based on Green's functions belongs to a group of such kind effective methods [9, 10].

The group of methods called domain decomposition methods [11, 12] are based on subdividing a whole field definition domain on overlapping or non-overlapping subdomains. These methods can be combined with integral equation and Green’s functions methods [13, 14]. Such combination allows performing a quite rigorous analysis of physical processes running in waveguide structures with taking into account key features of each specific problem at a stage of its formulation.

Thus, the problem of a developing of numerical methods based on domain decomposition methods for the analysis of an electromagnetic wave diffraction process in waveguide devices is of scientific and practical interest.

Formulation of the study purpose

The numerical algorithm, which applies the DDM to inhomogeneities interacting on higher wavetypes in rectangular waveguides, is represented in [14]. The initial problem is formulated as a system of integral equations for each partial subdomain which is solved using a system of linear algebraic equations. Described approach allows obtaining a problem solution not only in one partial subdomain, like in earlier formulations of this method presented in [13], but also in other subdomains.

Such formulation of a problem allows reducing significantly an analytical part of a problem solving. This main feature of an approach, developed in [14], may be useful for ingomogeneities that do not interact on higher order modes but still lead to large amount of analytical transformations to be performed. The irises of finite thickness in metal waveguides belong to specified type of inhomogeneities.

Simplified mathematical models are often used in order to analyze the main features of each specific electromagnetic problem. The mathematical model of infinite array antenna allows considering only a single unit cell of the array and the initial problem can be represented as junction of an array's waveguide and a spatial waveguide. In particular conditions the rectangular waveguide of an array can be represented as a parallel plate waveguide. For this, waveguide walls that are normal to a vector of an incident wave can be assumed infinitely thin and the beam scanning must be performed in an *H*-plane. Thus, the initial problem becomes scalar and one dimensional, which allows simplifying significantly the formulation of an electromagnetic problem.

Therefore, the main goal of this work is to develop the integral equation based domain decomposition method for the analysis of an electromagnetic wave diffraction process in metal waveguides with presence of finite thickness irises.

Presenting main material

Consider an infinite phased array antenna (PAA) consisting of rectangular waveguides placed in the nodes of a rectangular grid. There are irises of finite thickness located in the waveguide apertures (Fig. 1). The initial problem can be reduced to the problem of the junction of a unit cell planar waveguide with a spatial waveguide using the properties of an infinite antenna array. Then, the integral equations need to be composed only for the E_y component of the electric field vector, which must satisfy the boundary conditions on the perfectly conducting waveguide walls and the radiation conditions.

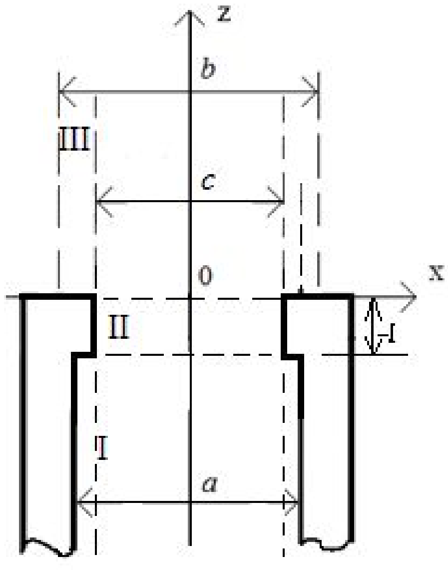


Fig. 1. A unit cell of PAA

The Schwartz's alternating method can be applied to solve related problems [13, 14]. This method allows obtaining the solution of boundary value problems, in which the entire complex field definition domain can be split into simple partial domains for which the Green's functions are known or can be easily obtained. Thus, the entire field definition volume of the unit cell is divided into the following three partial overlapping domains (Fig. 1).

1. Domain *sw* ($-a/2 \le x \le a/2$) is represented by the semi-infinite parallel plate waveguide ($-\infty \le z \le -l$). The boundaries of this domain are formed by the metal walls of the waveguide, on which the field function becomes equal to zero, the aperture of the iris with the boundary values defined at $z = -l$ by other domain, and the plane $z \to -\infty$, on which the H_{10} wavetype of parallel plate waveguide is excited with taking into account the metal wall at $z = -l$.

2. Domain *w* ($-c/2 \le x \le c/2$) is the infinite parallel plate waveguide ($-\infty \le z \le \infty$), which represents a volume inside the iris. For this domain the boundaries are represented by the metal walls of the waveguide at $l \le z \le 0$, on which the field function is equal to zero, and the imaginary waveguide walls extended to infinity, on which the boundary values of the field function are defined as a solution in other regions. On the boundaries at $z \to \pm\infty$ the radiation conditions and the limiting absorption principle are applied.

3. Domain *f* ($-b/2 \le x \le b/2$, $-\infty \le z \le \infty$) — radiation domain, or spatial waveguide. The boundaries of this region are formed by the imaginary walls of the spatial waveguide, the metal ends of the parallel plate waveguide, on which the field function becomes equal to zero, the surface at $z \to \infty$, for which the radiation conditions are satisfied, and the boundary values of the field function that are defined at the aperture — $c/2 \le x \le c/2$ and $z = 0$ as a solution in other regions.

Green's functions for similar regions are well known, for example, from work [13]. At this point, when the subdomain boundaries and their mutual location are determined, a system of integral representations for the unknown fields can be composed for each partial subdomain using Green's second identity:

$$E_y^{sw}(x,z) = E_{inc}(x,z) + \int_{-\frac{c}{2}}^{\frac{c}{2}} E_y^{w}(x',z') \cdot \frac{\partial G^{sw}(x,z;x',z')}{\partial \vec{n}'} dx' ; \quad (1)$$

$$E_y^{f}(x,z) = \int_{-\frac{c}{2}}^{\frac{c}{2}} E_y^{w}(x',z') \cdot \frac{\partial G^{f}(x,z;x',z')}{\partial \vec{n}'} dx' ; \quad (2)$$

$$E_y^{w}(x,z) = \int_{-\infty}^{l} E_y^{sw}(x',z') \cdot \frac{\partial G^{w}(x,z;x',z')}{\partial \vec{n}'} dz' + \int_{0}^{\infty} E_y^{f}(x',z') \cdot \frac{\partial G^{w}(x,z;x',z')}{\partial \vec{n}'} dz' . \quad (3)$$

Here $E_y^{sw}, E_y^{f}, E_y^{w}$ are unknown fields of the corresponding subdomain, E_{inc} is an incident field, G^{sw}, G^{f}, G^{w} are Green's functions of the corresponding subdomain, $\vec{n}$ is an inner normal vector of the corresponding subdomain, x, z, x' and z' — the coordinates of source points and observation points. The Green's function for subdomain *sw* is composed in "sourcewise" form:

$$G^{sw}(x,z) = \sum_{p} \Phi_p(x)\Phi_p(x')F_p(z,z') .$$

Here Φ_p is a transverse eigenfunction of parallel plate waveguide [13], F_p is a longitudinal Green's function of semi-infinite parallel plate waveguide placed in left semi space ($z < l < 0$) at a distance l from xOy plane:

$$F_p(z,z') = \frac{1}{j\theta_p} \cdot exp\left(j\theta_p\left[z-l\right]\right) \cdot sh\left(-j\theta_p\left[z'-l\right]\right); \quad z' > z .$$

Here $и_p$ is a longitudinal wave propagation coefficient. The incident wave function can be found using the previous formula and the second Green's identity:

$$E_{inc}(z,z') = \Phi_1(x) \cdot \left[e^{-j\theta_1[z-l]} - e^{j\theta_1[z-l]}\right].$$

Next, the directions of the normals and the source and observation points coordinates in the integral representations (1)—(3) must be determined according to the geometry of the entire field definition domain, which allows to obtain a system of integral equations for the unknown field functions in each subdomain. The obtained system of integral equations, according to the Schwartz's method, is solved by the iterative method. It is suitably to use the incident wave field as an initial approximation. The values of the unknown functions in the first approximation can be obtained by substituting the initial approximation into each equation of the system as a function $E^{\langle 0 \rangle sw}_{y}$. In the same way the second and the higher order approximations of the field functions for each partial domain can be found by substituting their previous approximations into the system of integral equations.

It is obvious that the presented procedure for problem solving is quite cumbersome and requires a large number of analytical transformations, which significantly complicates the application of the Schwartz method for problems with more than two partial subdomains. In works [13, 14] it is shown that previously described iterative sequence can be represented as a solution of a system of linear algebraic equations using iterative method. To do this, the unknown functions in the integral equations can be represented as a series expansion on the eigenfunctions of corresponding subdomain with unknown amplitude coefficients. Finally, the unknown functions in each integral equation of the system are presented in the next form:

$$E^{sw}(x,z)=E_{inc}(x,z)+\sum_{p=1}^{\infty}R_p^{sw}\Phi_p(x)\exp\left(j\theta_p[z-l]\right); \quad (4)$$

$$E^{w}(x,z)=\sum_{q=1}^{\infty}\left[T_q^{w}\varphi_q(x)\exp\left(-j\gamma_q[z-l]\right)+R_q^{w}\varphi_q(x)\exp\left(j\gamma_q z\right)\right]; \quad (5)$$

$$E^{f}(x,z)=\sum_{m=-\infty}^{\infty}T_m^{f}\psi_m(x)\exp\left(-j\Gamma_m z\right). \quad (6)$$

Here $R_p^{I}, T_q^{II}, R_q^{II}, T_m^{III}$ are unknown expansion coefficients, which have physical meaning of transmission and reflection coefficients for specific wave mode.

Substituting of (4)—(6) and expressions for Green's functions into the system (1)—(3) allows applying a property of eigenfunction orthogonality and, in that turn, represent initial problem as a system of linear algebraic equations (SLAE) for the unknown expansion coefficients. The obtained SLAE can be represented in a matrix form, which is solved by iterative method:

$$\mathbf{X}^{\langle i\rangle}=\mathbf{X}^{\langle 1\rangle}+\mathbf{Б}\cdot\mathbf{X}^{\langle i-1\rangle}. \quad (7)$$

Here **X** is a column matrix containing values of unknown expansion coefficients, **A** is a square matrix containing coefficients formed by multiplication of eigenfunctions and corresponding integral equation kernel, $\mathbf{X}^{<1>}$ is a column matrix of free terms, that contains information about incident wave and represents an initial approximation of the solution.

Therefore, equation (7) defines values of expansion coefficients in each partial subdomain, which are combined in a column matrix **X** at *i*-order approximation. The value of the reflection coefficient for H_{10} wave at *i*-order approximation is defined by the first element of the column matrix $\mathbf{X}^{<i>}$.

Therefore, the described numerical algorithm represents the integral equation based Schwartz alternating method in which the initial problem is reduced to a matrix equation solved by iterative method.

A similar approach is used in the method of overlapping partial domains, where the initial problem is also reduced to SLAE, but after it is solved by exact methods instead of approximate iterative methods. It is possible because integral equations of the system (1)—(3) are the equations with degenerate kernel. Such type of integral equations allows their rigorous solution by limiting the number of accounted wavetypes by finite value. Hence, the initial system is reduced to finite SLAE [14]. The finite SLAE for the unknown expansion coefficients combined in column matrix X_N has the next form:

$$X_N-\sum_{J=1}^{Q}A_{NJ}\cdot X_J=B_N, \quad N,J=1,2\ldots Q. \quad (8)$$

Here B_N are the elements of the column matrix of free terms, A_{NJ} are the elements of a square matrix defined in equation (7), Q is a number of considered wavetypes. Introduce a square matrix **Д** with dimensions $Q\times Q$, with its elements defined in the next form:

$$\Delta_{NJ}=\delta_{NJ}-A_{NJ}.$$

Here Δ_{NJ} is a Kronecker symbol. Here the SLAE (8) can be represented in a matrix form:

$$\mathbf{Д}\cdot\mathbf{X}=\mathbf{B}.$$

Elements of the matrix **X** contain unknown values of expansion coefficients for each wavetype of a corresponding partial domain. The reflection coefficient for specific wavetype represents a reflected-incident wave ratio. At the same time the incident wave is a superposition of a H_{10} unit wave (1) and the wave reflected from metal walls of an iris. Therefore, the full field in subdomain *sw* takes the form:

$$E^{sw}(z,z')=\Phi_1(x)\cdot exp^{-j\theta_1[z-l]}-\Phi_1\cdot(x)exp^{j\theta_1[z-l]}+\sum_{p=1}^{\infty}R_p^{sw}\cdot\Phi_p(x)exp\left(j\theta_p[z-l]\right).$$

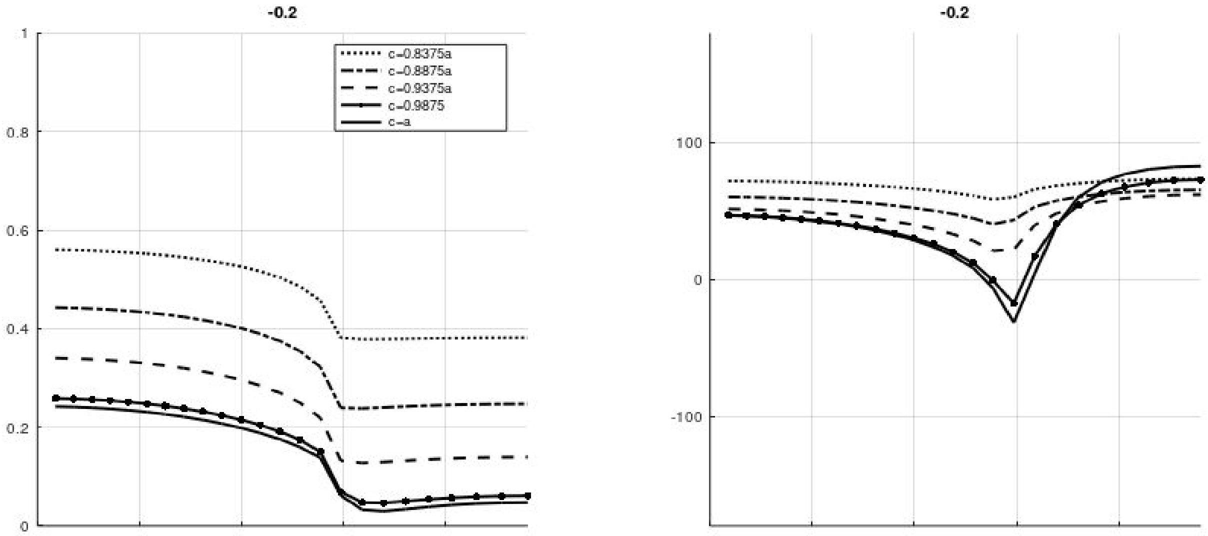
Finally, the value of a H_{10} — wave reflection factor in the subdomain *sw* through the expansion coefficient can be obtained as follows:

$$\Gamma_{10} = R_1^{sw} - 1 \,.$$

The described mathematical method was implemented in the form of a numerical algorithm for calculating the coefficients of transmission and reflection of waves in the field definition domain. The calculation was performed for PAA with parameters b/λ = 0,6724, b = 0,937a. The iris width for each case was chosen in the range from c = 0,8125a to $c = a$, and its relative thickness l was set in a range from 1,5λ to 0,2λ. Also, the phenomenon of a full reflection of an incident wave can be observed when the iris width is set lesser than c < 0,8125a which makes impossible to propagate the main type of waves in the considered waveguide. During the numerical experiment the values of reflection factor magnitude and phase were calculated for H_{10} wavetype in a parallel plate waveguide as a dependence on the value of scanning angle of PAA for different sets of dimensions of an aperture iris. The obtained results were compared with known ones, acquired by HFSS.

The results of the numerical experiment show that the presence of an iris at the waveguide apertures generally leads to an increase of the reflection coefficient magnitude. However, array waveguides can show better matching, compared to the absence of such inhomogeneity, at some values of the iris width and thickness. Also, the waveguide iris of specific size allows obtaining an almost constant value of the phase of the reflection coefficient in a wide range of scanning angles, which can be applied to design more complex PAA matching devices.

Fig. 2 shows the dependence of the reflection coefficient magnitude and phase for an iris with thickness of 0.2λ at different values of its width. As can be seen, the value of the reflection coefficient uniformly decreases with a decrease of the iris width.



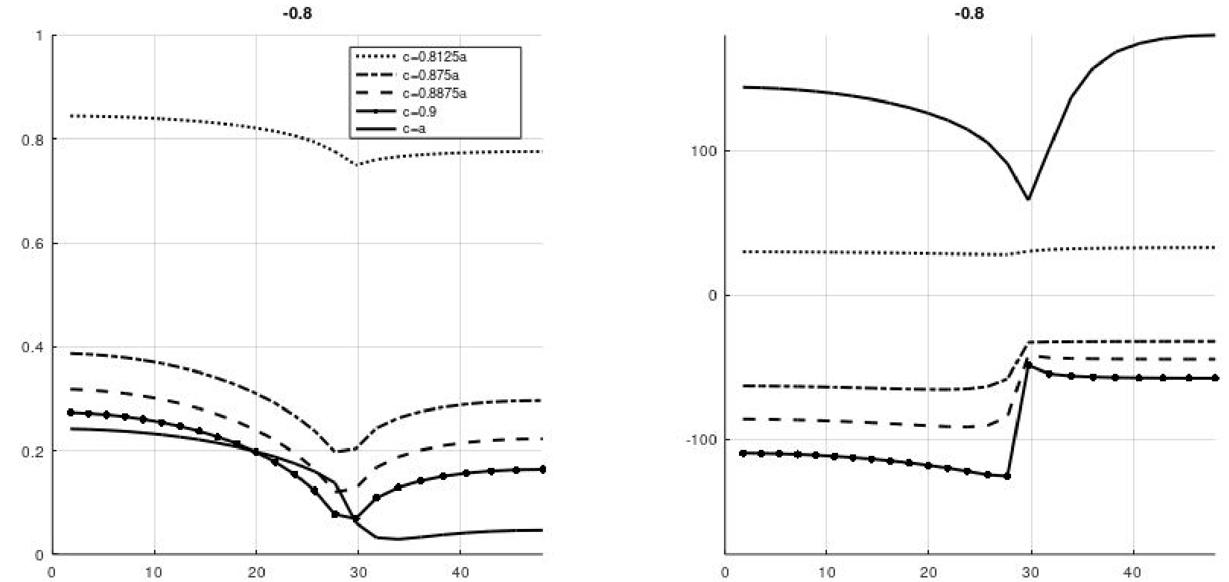
(a) (b)

Fig. 2. The dependence of the modulus (a) and phase (b) of the reflection coefficient for the case l = 0.2λ, obtained using DDM

A similar pattern remains constant for the aperture thickness value up to 0.7λ. While the general form of the figure curves is unchanged, the range of the reflection coefficient magnitude and the phase variations increases.

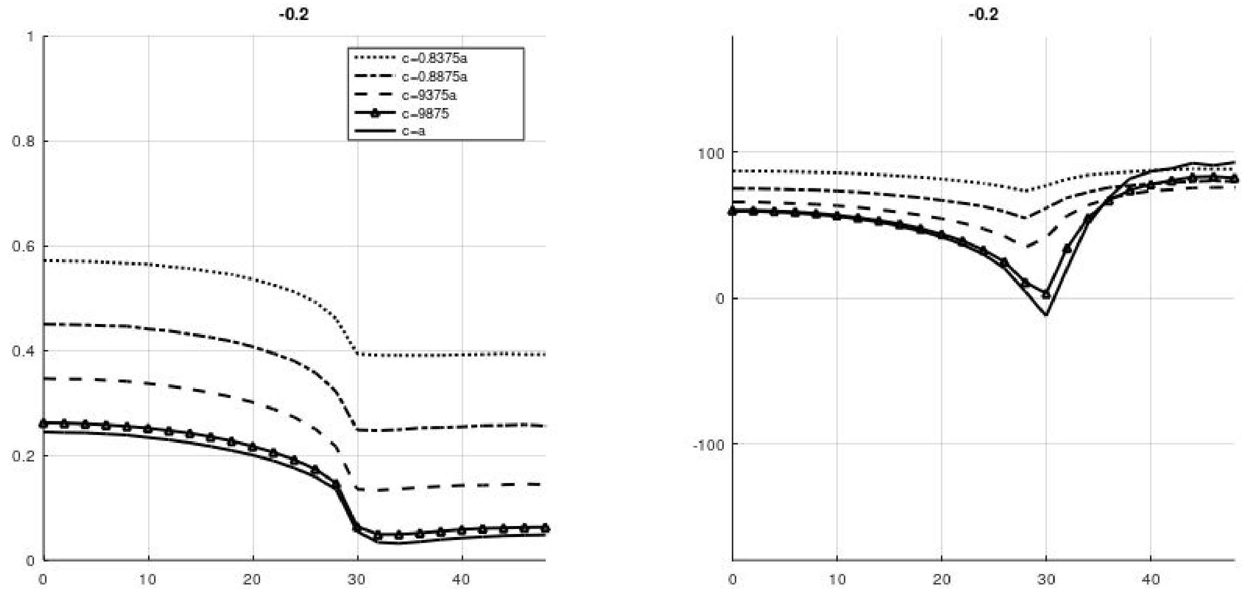
Fig. 3 shows the dependence of the calculated value of the reflection coefficient for the iris thickness 0,8λ. In this case, an almost constant value of the reflection coefficient phase can be observed at values of the iris relative width *c/a* from 0.8125 to 0.8875 when scanning angle value range provides the single-beam mode of PAA operation (approximately up to 30 degrees). The values of *c/a* ratio equal to 0.875, 0.8875 and 0.9 give a relatively small value of the reflection coefficient magnitude, which is comparable to complete absence of the iris. A further decrease of the iris width makes the dependence of the reflection coefficient magnitude and the phase similar to a case of iris absence.

Cases of relatively narrow irises (c/a = 0.8125 and 0.825) allows obtaining more constant value of the phase of the reflection coefficient, as well as to provide a smaller range of variations of the modulus relative value on a given range of scanning angles. However, a high (more than 0.8) value of the modulus of the reflection coefficient is maintained.



(a) (b)

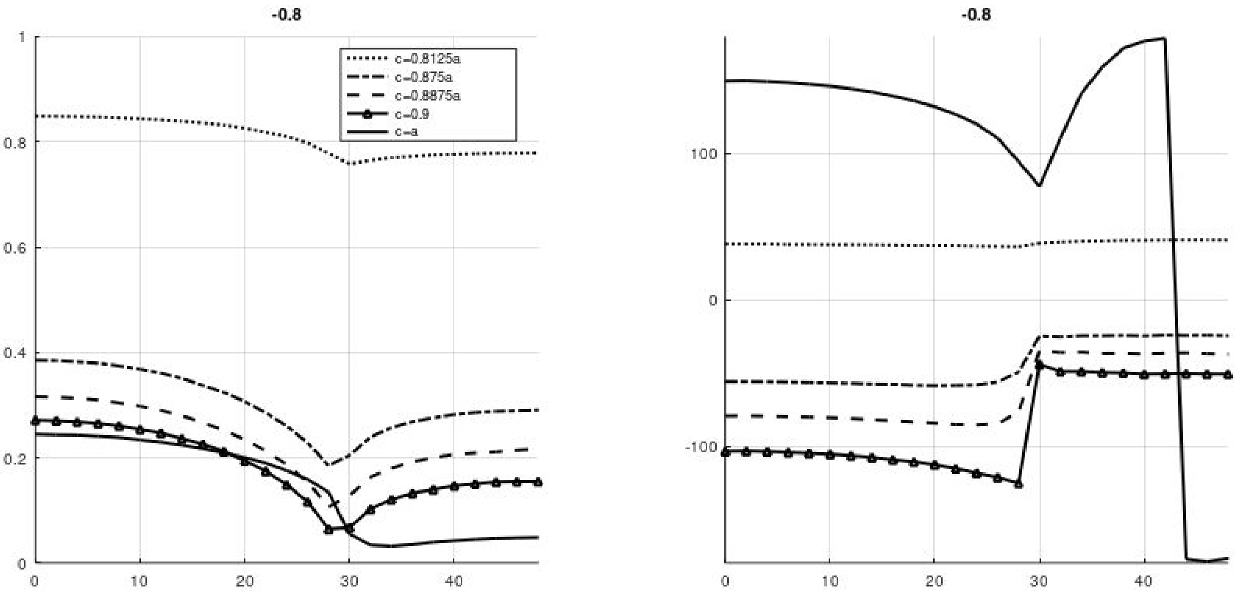
Fig. 3. The dependence of the modulus (a) and phase (b) of the reflection coefficient for the case $l = 0.8\lambda$, obtained using DDM



(a) (b)

Fig. 4. The dependence of the modulus (a) and phase (b) of the reflection coefficient for the case $l = 0.2\lambda$, obtained using HFSS

A further increasing of the iris thickness leads to a decreasing of the described features scale. A mathematical modeling of a similar PAA was performed using HFSS in order to validate the obtained results. The calculation was performed for the considered values of the width and thickness of the irises in the waveguide apertures. Thus, fig. 4 shows the dependence of the magnitude and phase of the H_{10} wave reflection coefficient from the value of the scanning angle for the case of the iris thickness $l = 0{,}2\lambda$. Fig. 5 shows similar characteristics for the case $l = 0{,}8\lambda$. As can be seen, the developed numerical algorithm based on the Schwartz method is correct and allows analyzing the features of electromagnetic wave propagation in the waveguide PAA with the presence of aperture inhomogeneities.



(a) (b)

Fig. 5. The dependence of the modulus (a) and phase (b) of the reflection coefficient for the case $l = 0.8\lambda$, obtained using HFSS

Conclusions

In the paper, the development of the Schwartz method and the IE based DDM was performed in order to solve the problem of electromagnetic wave diffraction on inhomogeneities represented by irises of finite thickness in the waveguide apertures of an infinite PAA. The obtained theoretical results can be used for further development of the presented methods for solving more complex vector problems.

The influence of the iris thickness and width on the dependence of the reflection modulus and phase in the array waveguides on the scanning angle was investigated. It is shown that at specific sizes of the irises the reflection coefficient phase becomes almost constant in a wide range of scanning angle. In addition, the range of variation of reflection coefficient magnitude is also reduced.

A comparison of the obtained results with the results obtained with commercial electromagnetic programs was performed. The comparison confirmed the correctness of the developed electromagnetic algorithm for solving such diffraction problems.

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