DOI: 10.31319/2519-8106.2(49)2023.293117
UDC 511.11: 511.147

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PROPERTIES OF VOLUME NUMBERS WITH INTEGER COEFFICIENTS AND MATHEMATICAL MODELING

ВЛАСТИВОСТІ ОБ'ЄМНИХ ЧИСЕЛ З ЦІЛИМИ КОЕФІЦІЄНТАМИ І МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ

The nature of the distribution of volumetric numbers with integer coefficients on the corresponding spheres has been established, according to which the sets of numbers forming a family of rays of the first, second and third order, outlining a circle on the spheres beyond which there is a distribution of volumetric numbers with integer coefficients, have been determined. A theorem was also formulated about the possibility of the existence of at least a single volumetric number with integer real coefficients that would be located in the middle of the circles outlined by a family of rays of the first and second order on the corresponding spheres.
Keywords*: volume numbers, sphere, family of rays, integers, distribution.*

Об'ємні числа забезпечують при моделюванні навколишнього світу однозначну інтерпретацію не тільки суперечливих, а й двоїстих аспектів. Тому ширше дослідження властивостей об'ємних чисел з метою використання в математичному моделюванні представляє істотний інтерес. Крім того, враховуючи, що цілі числа, як і їх комбінації у вигляді суми, тією чи іншою мірою характеризують навколишній світ, то розгляд зони знаходження множини об'ємних чисел з цілими коефіцієнтами на відповідних сферах є одним з основоположних.

Встановлено характер розподілу об'ємних чисел з цілими дійсними коефіцієнтами $\{a,b,c\}$ *на відповідних сферах радіусу* ρ *, (* $\rho \in \{N\}$*), згідно з яким було визначено множини чисел* $\{a,b,c\}=\{n,2n,2n\}$, $\{a,b,c\}=\{4n,4n,7n\}$, $\{a,b,c\}=\{10n,10n,23n\}$, *що розташовуються на сферах радіусу* $\rho_k = 3^k n$ *, де:* $k=1,2,3$ *— номер сімейства;* n *— номер відповідної сфери, і утворюють сімейство променів першого, другого та третього порядку.*

Числа, які утворені множиною $\{a,b,c\} \in \{Z\}$ *на відповідних сферах, характеризуються рівністю суми коефіцієнтів* a,b,c *і розташовуються на одній відстані відносно точки (*V_0 *,* $a=b=c$ *), яка є геометричним центром сферичного трикутника. А для чисел першого та другого порядку відношення радіусу сфери до суми коефіцієнтів є постійним.*

Також встановлено, що зона розподілу об'ємних чисел з цілими коефіцієнтами розташовується за межами області, яку обмежує відповідне коло з центром V_0, яке утворюють числа першої та другої групи.

На підставі отриманих результатів була сформульована наступна теорема, для чіткого доказу отриманих результатів у загальному вигляді.

Теорема. Існує хоча б одне одиничне число V з цілими дійсними коефіцієнтами $\{a,b,c\}$, на довільній сфері радіуса $\rho_k = 3^k n$, сума яких задовольняла б умові:

$$\sum\{a,b,c\} > \sum\{n,2n,2n\}.$$

Цілі числа, в тій чи іншій мірі характеризують навколишній світ, тому отримані результати розподілу множини об'ємних чисел з цілими коефіцієнтами на відповідних сферах можуть бути використані для математичного моделювання не тільки подій які доступні нашому розумінню, але й подій інтуїтивно усвідомлених.

***Ключові слова**: об'ємні числа, сфера, сімейство променів, цілі числа, розподіл.*

Formulation of the problem

Expanding the number space through the introduction of the concept of a volume number significantly increases the range of use of algebra and mathematical analysis to describe and model comprehensive aspects of the surrounding world. The properties of the reciprocal mirror image of volumetric numbers provide an unambiguous interpretation of not only contradictory, but also dual aspects when studying the surrounding world. Therefore, a broader study of the properties of volume numbers for the purpose of using them in mathematical modeling is of significant interest.

Analysis of recent research and publications

The introduction by the authors of works [1, 2] of the concept of a spatially indefinite unit made it possible to move from a two-dimensional number field to a three-dimensional number field. It should be noted that a spatially indefinite unit is not an element of a two-dimensional numerical field and necessitates the expansion of the concepts of imaginary units in number theory.

According to [1, 2], the algebraic formula for the volume number is written as

$$V = a + bi + cj,$$

where a,b,c — real numbers; i — imaginary unit; j — a spatially indeterminate unit.

In trigonometric form

$$V = \rho(\sin\theta\cos\varphi + i\sin\theta\sin\varphi + j\cos\theta),$$

where ρ — the length of the radius vector of the corresponding point; φ — longitude; θ — polar distance.

The properties of volume numbers and their coefficients a,b,c were used in the development of a physical and mathematical model of the volumetric Universe [3—5]. These properties made it possible to mathematically model the location of the space-time continuum in the energy-information-time field of the volumetric Universe [6].

Despite the fact that the theory of volume numbers is only at the stage of its formation, its use makes it possible to model objects and processes of the surrounding world more qualitatively [3—6], which necessitates further research into the properties of these numbers.

Formulation of the research goal

An analysis of works [1—6] indicates that the set of volume numbers, which includes complex and real numbers, makes it possible to describe not only events that are accessible to our understanding, but also events that are intuitively realized. In addition, given that integers, as well as their combinations in the form of a sum [7], to some extent characterize the world around us, then consideration of the area where the set of volumetric numbers with integer coefficients a,b,c is located on the corresponding spheres is one of the fundamental ones.

Presentation of the main material

According to the geometric interpretation [1, 2], the volume number and its mirror images can be represented as follows: fig. 1.

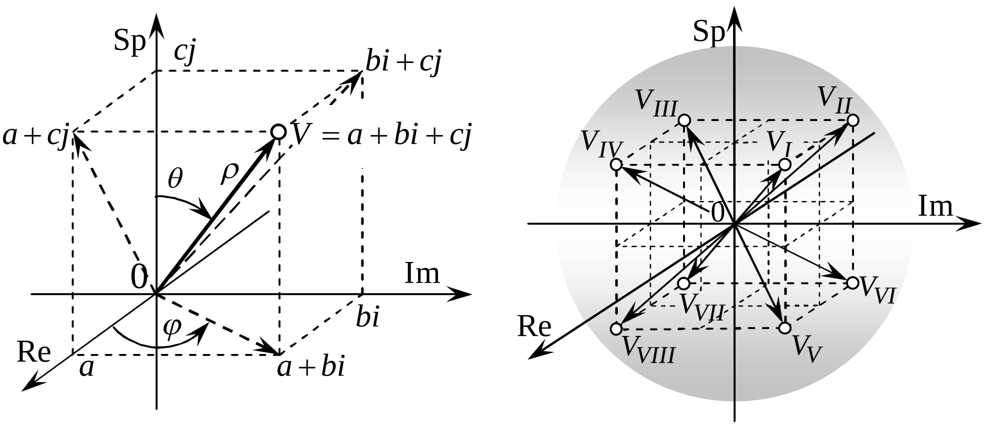

Fig. 1. Geometric representation of a volume number and its mirror images: Re — the real axis, Im — the imaginary axis, Sp —the spatially indefinite axis

In this paper, the task is to determine the nature of the distribution of volume numbers with integer coefficients on the corresponding spheres of radius ρ,

$$\rho = \sqrt{a^2 + b^2 + c^2}\ .$$

According to [7], the maximum sum of real coefficients $\Delta_{\Sigma\max} = 3a$ of the volume number $V_0 = a + bi + cj$ with coordinates

$$\begin{cases} \varphi = 45° \\ \theta = 54°44'8'' \end{cases}$$

takes place under the condition that $a = b = c$.

Hence

$$\rho = a\sqrt{3}\ .$$

Accordingly, on a sphere with radius $\rho = a\sqrt{3}$ there is at least one volume number with integer coefficients $\{a,b,c\} = Z$, for which condition $a = b = c$ is satisfied. But the corresponding radii do not satisfy condition $\rho \in \{N\}$. These spheres will be denoted by the radius ρ_o .

To determine the location zone of the set of volume numbers with integer coefficients $\{a,b,c\} = Z$ on the corresponding spheres, the radii of which satisfy the condition $\rho \in \{N\}$, we solve the system of equations

$$\begin{cases} \rho^2 - a^2 = p \\ p - b^2 = c^2 \end{cases},$$

successive selection of full squares of coefficients a,b,c that satisfy the condition:

$$\begin{cases} a^2 < \rho^2 \\ b^2 < p^2 \end{cases}.$$

For the sake of compactness in presenting the results, Tabl. 1 shows only positive numerical values of the coefficients a,b,c of the first octant.

The numbers that are formed by the set of coefficients of the form

$$\begin{cases} \{a,0,0\}; \{0,b,0\}; \{0,0,c\} \\ \{a,b,0\}; \{a,0,c\}; \{0,b,c\} \end{cases}$$

are not given in this table.

Table 1. The value of the coefficients a,b,c of the first octant

ρ	$\{a,b,c\}=Z$		
1	{0}		
2	{0}		
3	{1, 2, 2}	ρ_o	$\{a,b,c\}=Z$
4	{0}	1,732	{1, 1, 1}
5	{0}	3,464	{2, 2, 2}
6	{2, 4, 4}	5,196	{3, 3, 3}; {1, 1, 5}
7	{2, 3, 6}	6,928	{4, 4, 4}
8	{0}	8,660	{5, 5, 5}; {1, 5, 7}
9	{3, 6, 6}; {4, 4, 7}; {1, 4, 8}	10,392	{6, 6, 6}; {2, 2, 10}
10	{0}	12,124	{7, 7, 7}; {1, 5, 11}
11	{0}	13,856	{8, 8, 8}
12	{4, 8, 8}	15,588	{9, 9, 9}; {3, 3, 15}; {5, 7, 13}; {1, 11, 11}
13	{3, 4, 12}	17,321	{10, 10, 10}; {2, 10, 14}
14	{4, 6, 12}		
15	{5, 10, 10}; {2, 5, 14}; {2, 10, 11}		
16	{0}		
17	{8, 9, 12}		
18	{6, 12, 12}; {8, 8, 14}		
19	{1, 6, 18}; {6, 6, 17}; {6, 10, 15}		
20	{0}		
21	{7, 14, 14}; {4, 5, 20}; {4, 8, 19}; {6, 9, 18}; {4, 13, 16}; {8, 11, 16}		
22	{4, 12, 18}; {12, 12, 14}		
23	{3, 6, 22}; {3, 14, 18}; {6, 13, 18}		
24	{8, 16, 16}		
25	{9, 12, 20}; {12, 15, 16}		
26	{6, 8, 24}		
27	{9, 18, 18}; {12, 12, 21}; {10, 10, 23}; {2, 7, 26}; {2, 10, 25}; {2, 14, 23}; {3, 12, 24}		

An analysis of the set of volumetric numbers with integer coefficients indicates the existence of regularities in the distribution of a group of numbers on the corresponding spheres whose radii satisfy the condition $\rho \in \{3n \mid n \in N\}$, where n — is the number of the corresponding sphere. Namely:

$\rho = 3n$ — there is a distribution of the I group of numbers $\{n,2n,2n\}$;

$\rho = 9n$ — in addition to the I group, there is a distribution of the II group of numbers $\{4n,4n,7n\}$;

$\rho = 27n$ — except for I and II groups, the distribution of the III group of numbers $\{10n,10n,23n\}$.

The data of the groups of volume numbers of the first octant are presented in Tabl. 2.

Table 2. Groups of volumetric numbers of the first octant

ρ	N_c	$V_{N_c} = a + bi + cj$	ρ	N_c	$V_{N_c} = a + bi + cj$	ρ	N_c	$V_{N_c} = a + bi + cj$
3	I	$1+2i+2j$ $2+1i+2j$ $2+2i+1j$	30	I	$10+20i+20j$ $20+10i+20j$ $20+20i+10j$	57	I	$19+38i+38j$ $38+19i+38j$ $38+38i+19j$
6	I	$2+4i+4j$ $4+2i+4j$ $4+4i+2j$	33	I	$11+22i+22j$ $22+11i+22j$ $22+22i+11j$	60	I	$20+40i+40j$ $40+20i+40j$ $40+40i+20j$
9	I	$3+6i+6j$ $6+3i+6j$ $6+6i+3j$	36	I	$12+24i+24j$ $24+12i+24j$ $24+24i+12j$	63	I	$21+42i+42j$ $42+21i+42j$ $42+42i+21j$
	II	$4+4i+7j$ $4+7i+4j$ $7+4i+4j$		II	$16+16i+28j$ $16+28i+16j$ $28+16i+18j$		II	$28+28i+49j$ $28+49i+28j$ $49+28i+28j$
12	I	$4+8i+8j$ $8+4i+8j$ $8+8i+4j$	39	I	$13+26i+26j$ $26+13i+26j$ $26+26i+13j$	66	I	$22+44i+44j$ $44+22i+44j$ $44+44i+22j$
15	I	$5+10i+10j$ $10+5i+10j$ $10+10i+5j$	42	I	$14+28i+28j$ $28+14i+28j$ $28+28i+14j$	69	I	$23+46i+46j$ $46+23i+46j$ $46+46i+23j$
18	I	$6+12i+12j$ $12+6i+12j$ $12+12i+6j$	45	I	$15+30i+30j$ $30+15i+30j$ $30+30i+15j$	72	I	$24+48i+48j$ $48+24i+48j$ $48+48i+24j$
	II	$8+8i+14j$ $8+14i+8j$ $14+8i+8j$		II	$20+20i+35j$ $20+35i+20j$ $35+20i+20j$		II	$32+32i+56j$ $32+56i+32j$ $56+32i+32j$
21	I	$7+14i+14j$ $14+7i+14j$ $14+14i+7j$	48	I	$16+32i+32j$ $32+16i+32j$ $32+32i+16j$	75	I	$25+50i+50j$ $50+25i+50j$ $50+50i+25j$
24	I	$8+16i+16j$ $16+8i+16j$ $16+16i+8j$	51	I	$17+34i+34j$ $34+17i+34j$ $34+34i+17j$	78	I	$26+52i+52j$ $52+26i+52j$ $52+52i+26j$
27	I	$9+18i+18j$ $18+9i+18j$ $18+18i+9j$	54	I	$18+36i+36j$ $36+18i+36j$ $36+36i+18j$	81	I	$27+54i+54j$ $54+27i+54j$ $54+54i+27j$
	II	$12+12i+21j$ $12+21i+12j$ $21+12i+12j$		II	$24+24i+42j$ $24+42i+24j$ $42+24i+24j$		II	$36+36i+63j$ $36+63i+36j$ $36+36i+36j$
	III	$10+10i+23j$ $10+23i+10j$ $23+10i+10j$		III	$20+20i+46j$ $20+46i+20j$ $46+20i+20j$		III	$30+30i+69j$ $30+69i+30j$ $69+30i+30j$

According to the geometric interpretation of volume numbers [2]

$$\begin{cases} \varphi = arctg\,\dfrac{b}{a}, \\ \theta = arctg\,\dfrac{\sqrt{a^2+b^2}}{c}. \end{cases}$$

Therefore, for a certain group of numbers formed by the coefficients of the set $\{a,b,c\}=\{n,2n,2n\}$, $\{a,b,c\}=\{4n,4n,7n\}$ and $\{a,b,c\}=\{10n,10n,23n\}$, regardless of the number n of the sphere under consideration, the following conditions are satisfied:

$$\begin{cases} \dfrac{b}{a} = const, \\ \dfrac{\sqrt{a^2+b^2}}{c} = const. \end{cases}$$

Accordingly, the set of numbers of the form:

$$V_{I_1} = n+2ni+2nj\,,\ V_{I_2} = 2n+ni+2nj\,,\ V_{I_3} = 2n+2ni+nj$$

form a family of rays of the first order $N_c(I)$;

the numbers

$$V_{II_1} = 4n+4ni+7nj\,,\ V_{II_2} = 4n+7ni+4nj\,,\ V_{II_3} = 7n+4ni+4nj$$

form a family of rays of the second order $N_c(II)$;

and the numbers

$$V_{III_1} = 10n+10ni+23nj\,,\ V_{III_2} = 10n+23ni+10nj\,,\ V_{III_3} = 23n+10ni+10nj$$

are the family of rays of the third order $N_c(III)$.

To establish the general dependence of finding the necessary sphere, we use conditions $\rho = N$. Then, for I, II and III groups of numbers we have:

$$N = \sqrt{n^2+(2n)^2+(2n)^2} = 3n\,;$$

$$N = \sqrt{(4n)^2+(4n)^2+(7n)^2} = 9n = 3^2 n\,;$$

$$N = \sqrt{(10n)^2+(10n)^2+(23n)^2} = 27n = 3^3 n\,.$$

Hence

$$\rho_k = N = 3^k n\,,$$

where $k = 1,2,3$ — is the family number.

The volume numbers of the corresponding families on certain spheres are characterized by the equality of the sum of the coefficients a,b,c:

$$\sum\{a,b,c\} = \sum\{n,2n,2n\} = 5n\,;$$

$$\sum\{a,b,c\} = \sum\{4n,4n,7n\} = 15n\,;$$

$$\sum\{a,b,c\} = \sum\{10n,10n,23n\} = 43n\,.$$

According to [7], the numbers formed by the set $\{a,b,c\} = Z$ are located at the same distance relative to the point, which is the geometric center of the spherical triangle ($\Delta_{\Sigma\max} = 3a$, $a = b = c$).

Of particular interest are the volume numbers of the family of rays of the first and second order on spheres of radius $\rho_2 = 3^2 n$, since the condition of equality of the sum of their coefficients occurs a,b,c,

$$\sum\{a,b,c\} = \sum\{3n, 2\cdot 3n, 2\cdot 3n\} = 15n \ \equiv\ \sum\{a,b,c\} = \sum\{4n,4n,7n\} = 15n\,,$$

as well as the ratio of the sphere radius to the sum of coefficients a,b,c is constant:

$$\frac{\rho_1}{\sum\{n,2n,2n\}}=\frac{3n}{5n}=0{,}6\,;$$

$$\frac{\rho_2}{\sum\{4n,4n,7n\}}=\frac{9n}{15n}=0{,}6\,.$$

These numbers form a circle on a spherical triangle, the center of which is the volume number V_0, (Fig. 2).

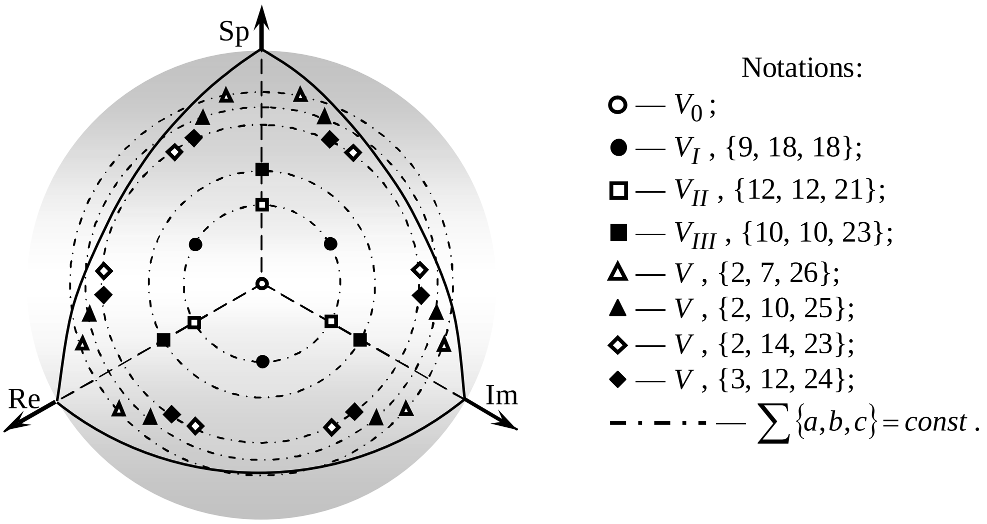

Fig. 2. Distribution of volume numbers with integer coefficients of the first octant on a sphere of radius $\rho=27$

If through the given numbers of an arbitrary sphere of radius $\rho_2=3^2 n$, draw a plane $m-m$ (Fig. 3), then the ray on which the numbers V_0 are located will be perpendicular to this plane, and the plane will be tangent to the sphere of radius ρ_o at the point $V_{0_0}=5n+5ni+5nj$.

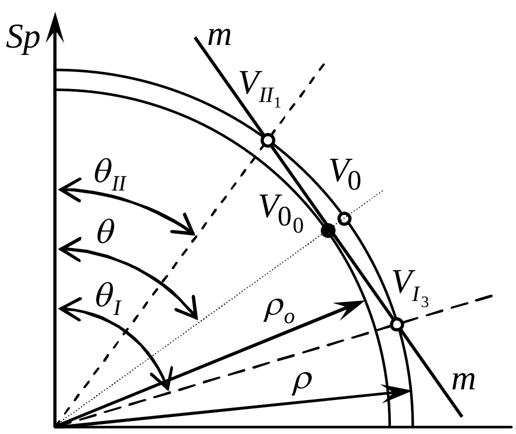

Fig. 3. Cross section $m-m$ at $\varphi=45^\circ$

From fig. 3 we have

$$\rho_o=\rho\cos(\theta_I-\theta)\,,$$

since: $\rho=\rho_2=9n$; $\cos(\theta_I-\theta)=0{,}96225 \Rightarrow$ $\rho_o=8{,}66025\cdot n\approx 8{,}66\cdot n$.

According to [7], under the condition that $a=b=c$

$$\rho_o=a\sqrt{3} \Rightarrow a=\frac{8{,}66\cdot n}{1{,}732}=5\cdot n \Rightarrow$$

$$V_{0_0}=5n+5ni+5nj$$

Studies show that on an arbitrary sphere of radius $\rho_k=3^k n$, $(1\le\rho_k\le 81)$ the zone of location of volume numbers with integer coefficients is located outside the area bounded by the corresponding circle with center V_0, (Fig. 2).

An exception is the volume numbers of a sphere of radius $\rho=21$, whose coefficients form the set $\{8,11,16\}$. Since,

$$\sum\{a,b,c\}=\sum\{n,2n,2n\}=\sum\{7,14,14\}=35 \equiv \sum\{a,b,c\}=\sum\{8,11,16\}=35,$$

then these numbers are located on the corresponding circle.

The number 21 is a Fibonacci number that is a multiple of three. It is possible that for Fibonacci numbers that are multiples of three 144, 987, etc., this exception will take place.

Conclusion

The nature of the distribution of volumetric numbers with integer real coefficients $\{a,b,c\}$ on the corresponding spheres of radius ρ, ($\rho \in \{N\}$), was established, according to which the sets of numbers located on spheres of radius $\rho_k = 3^k n$ were determined, forming a family of rays of the first, second and third order.

It has also been found that the zone of distribution of volumetric numbers with integer coefficients is located outside the area that is limited by the corresponding circle with the center V_0, which is formed by the numbers of the first and second groups.

Based on the results of numerical calculations, we can formulate the following theorem, for a clear proof of the results obtained in a general form.

Theorem. There is at least one single number V with integer real coefficients $\{a,b,c\}$, on an arbitrary sphere of radius $\rho_k = 3^k n$, whose sum satisfies the condition:

$$\sum\{a,b,c\}>\sum\{n,2n,2n\}.$$

An analysis of works [3—6] indicates that the set of volume numbers with their properties [1,2,7] makes it possible to describe not only events that are accessible to our understanding, but also events that are intuitively realized. In addition, given that integers to some extent characterize the world around us, the obtained results of the distribution of a set of volumetric numbers with integer coefficients a,b,c on the corresponding spheres can be used for mathematical modeling in various branches of science. This is especially confirmed by the exception that occurs on the sphere of radius $\rho = 21$. Especially if it turns out to be conforming to the laws of nature. Since the Fibonacci numbers are closely related to the concept of the “golden section”, which is considered one of the most harmonizing laws of the universe.

References

[1] Romaniuk A.D., & Romaniuk R.A. (2018). [On the question of expansion of numerical space] *Problemi matematichnogo modelyuvannya.: materiali vseuk. nauk.-metod. konf. 23–25 trav. 2018 – Problems of mathematical modeling: the materialsof the ukr. science-method. conf., 23–25 may. 2018.* (pp. 9–12). Kam'yanske: DSTU [in English].

[2] Romaniuk A.D., & Romaniuk R.A. (2018). [Theoretical foundations of expansion of the numerical space] *Matematychne modeliuvannia – Mathematical modeling,* 2(39), 20–28 [in English].

[3] Romaniuk A.D., & Romaniuk R.A. (2018). [Mathematical model of the world] *Problemi matematichnogo modelyuvannya.: materiali vseuk. nauk.-metod. konf. 23–25 trav. 2018 – Problems of mathematical modeling: the materialsof the ukr. science-method. conf., 23–25 may. 2018.* (pp. 6–9). Kam'yanske: DSTU [in English].

[4] Romaniuk A.D., & Romaniuk R.A. (2020). [General bases of the physical and mathematical model of a volume universe] *Matematychne modeliuvannia – Mathematical modeling,* 1(42), 7–15 [in English].

[5] Romaniuk A.D. (2021). *Enerhetychni aspekty fizyko-matematychnoi modeli obiemnoho Vsesvitu* [Energetic aspects of the physical and mathematical model of the volumetric Universe] *Matematychne modeliuvannia – Mathematical modeling,* 1(44), 7–16 [in Ukraine].

[6] Romaniuk A.D., & Romaniuk R.A. (2021). [Spatial-time continuum in the energy-information-time field of the volume universe] *Matematychne modeliuvannia – Mathematical modeling,* 2(45), 140–147 [in English].

[7] Romaniuk A.D., & Romaniuk R.A. (2022). [Properties of the sum of real coefficients of volume numbers and mathematical modeling] *Matematychne modeliuvannia – Mathematical modeling*, 2(47), 33–40 [in English].

Список використаної літератури

1. Romaniuk A.D., Romaniuk R.A. On the question of expansion of numerical space. *Проблеми математичного моделювання*: матеріали Всеукр. наук.-метод. конф., м. Кам’янське, 23-25 трав. 2018 р. Кам’янське, 2018. С. 9–12.
2. Romaniuk A.D., Romaniuk R.A. Theoretical foundations of expansion of the numerical space. *Математичне моделювання*. 2018. №2(39). С. 20–28.
3. Romaniuk A.D., Romaniuk R.A. Mathematical model of the world. *Проблеми математичного моделювання*: матеріали Всеукр. наук.-метод. конф., м. Кам’янське, 23-25 трав. 2018 р. Кам’янське, 2018. С. 6–9.
4. Romaniuk A.D., Romaniuk R.A. General bases of the physical and mathematical model of a volume universe. *Математичне моделювання*. 2020. №1(42). С. 7–15.
5. Романюк О.Д. Енергетичні аспекти фізико-математичної моделі об’ємного Всесвіту. *Математичне моделювання*. 2021. №1(44). С. 7–16.
6. Romaniuk A.D., Romaniuk R.A. Spatial-time continuum in the energy-information-time field of the volume universe. *Математичне моделювання*. 2021. №2(45). С. 140–147.
7. Romaniuk A.D., Romaniuk R.A. Properties of the sum of real coefficients of volume numbers and mathematical modeling. *Математичне моделювання*. 2022. №2(47). С. 33–40.

Надійшла до редколегії 19.09.2023