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## MATHEMATICAL MODELING OF LOCAL HEAT TREATMENT OF STRUCTURES MADE BY ELECTROSLAG WELDING

### МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ МІСЦЕВОЇ ТЕРМООБРОБКИ КОНСТРУКЦІЙ, ВИГОТОВЛЕНИХ МЕТОДОМ ЕЛЕКТРОШЛАКОВОГО ЗВАРЮВАННЯ

*The manufacture of large-sized products by electroslag welding leads to the occurrence of residual stresses, for the elimination of which further heat treatment of the product is required. A resource-saving option for such treatment is local heat treatment of a certain area around the weld. However, the modes of local heat treatment are still remain irrational, which determines the relevance of this work.*

*The purpose of the research is to develop and test a computer-oriented algorithm for modeling the full cycle of local heat treatment of board structures in order to design rational process modes.*

*The heat treatment schedule usually involves three stages: heating to a certain temperature in the seam area; exposure for a given time; cooling to the set temperature. Heating and cooling rates should not exceed the parameters specified in the schedule. The time of the full cycle of heat treatment will be minimal if the conditions for the implementation of the maximum permissible speeds are provided. To comply with such a schedule, it is necessary to be able to physically adjust the parameters of external heat transfer on the basis of appropriate mathematical dependencies. The paper proposes a computer-oriented algorithm for determining the change in time of the heat flux density  $q(\tau)$ , the supply of which to the surface of the plate provides optimization of the heat treatment process in terms of speed. The algorithm is based on the use of a nonlinear mathematical model, a local-one-dimensional method and the method of elementary heat balances to determine  $q(\tau)$ .*

*The algorithm is implemented in the form of a program with the help of which a computational experiment was carried out. The results showed a significant influence of the local nature of heat*

supply on the characteristics of the process. It has been established that such parameters as the width of the heat supply zone and the density of the heat flux (in fact: the power of the heating device) are interrelated. If the power of the heating device is insufficient, the desired result can be achieved by expanding the heat supply zone. If it is structurally impossible to implement a zone of sufficient width, an increase in the power of heating devices may be a way out of the situation.

In general, the results of the computational experiment confirmed the possibility of using the developed algorithm to control the rational modes of local heat treatment of slab structures.

**Keywords:** mathematical model, algorithm, computational experiment, local heat treatment, optimization.

У роботі представлено алгоритм математичного моделювання повного циклу місцевої термообробки плитних конструкцій, виготовлених методом електрошлакового зварювання. Алгоритм орієнтований на визначення зміни в часі щільності теплового потоку  $q(\tau)$ , підведення якого до поверхні плити забезпечує оптимізацію процесу термообробки за швидкістю. Використані нелінійна математична модель, локально-одновимірний метод та модифікований метод елементарних теплових балансів для визначення  $q(\tau)$ . Наводяться результати обчислювального експерименту, які продемонстрували суттєвий вплив локального характеру підведення теплоти на характеристики процесу і підтвердили можливість застосування розробленого алгоритму для управління раціональними режимами місцевої термообробки плитних конструкцій.

**Ключові слова:** математична модель, алгоритм, обчислювальний експеримент, місцева термообробка, оптимізація.

### Problem statement

The manufacture of large-sized products using the electroslag welding method leads to residual stresses, which require heat treatment of a certain area around the weld [1,2]. Traditionally, heat treatment has been performed by heating the entire product, followed by holding and cooling in accordance with a specified temperature schedule. This process is associated with high energy costs, and these costs can be considered unjustified, since the technology requires heat treatment of only a certain, rather small, area around the weld. Taking this fact into account, local heat treatment processes were introduced in practice, in which the thermal effect was carried out in a limited area around the weld [2]. Local heat treatment processes are energy-saving, but their widespread implementation is constrained by insufficient study of the features caused by local heat supply. The modes of local heat treatment are still irrational, since they are based on the laws of general heating established on the basis of analytical calculation methods. At the current level of scientific development, the theoretical substantiation of the modes of local heat treatment of large-sized products should be carried out on the basis of mathematical and computer modeling.

### Analysis of recent research and publications

The issues related to local heat treatment of large-sized products have been studied in the works of many domestic scientists, among whom I.N. Manusov, M.M. Bilyaev, V.A. Soroka, G.F. Alekseev and others should be mentioned. Their works investigated the thermal state of products under various conditions of external heat transfer, and mainly used analytical solutions to linear heat conduction problems [2].

There are also some works devoted to the use of numerical and analytical schemes for calculating welded plate systems [3] and the application of the finite difference method to study the heating kinetics of low-carbon steel during local heat treatment [4]. The two-dimensional nature and nonlinearity of heat transfer processes are taken into account in works [5, 6], in which a modified method of elementary heat balances was used to research the features of local heating of large-sized products.

It should be noted that the vast majority of known works are devoted to one of the stages of heat treatment, namely, heating the weld to a given temperature. As for the full cycle of heat treatment, this issue requires further research, in particular, the development of computer-oriented mathematical models and algorithms for their implementation.

**Formulation of the research objective**

The purpose of the research is to develop and test a computer-oriented algorithm for modeling the full cycle of local heat treatment of plate structures made by electroslag welding.

**Presentation of the main material**

Consider the process of local heat treatment of slab structures within the framework of the scheme proposed in [6] (Fig. 1).

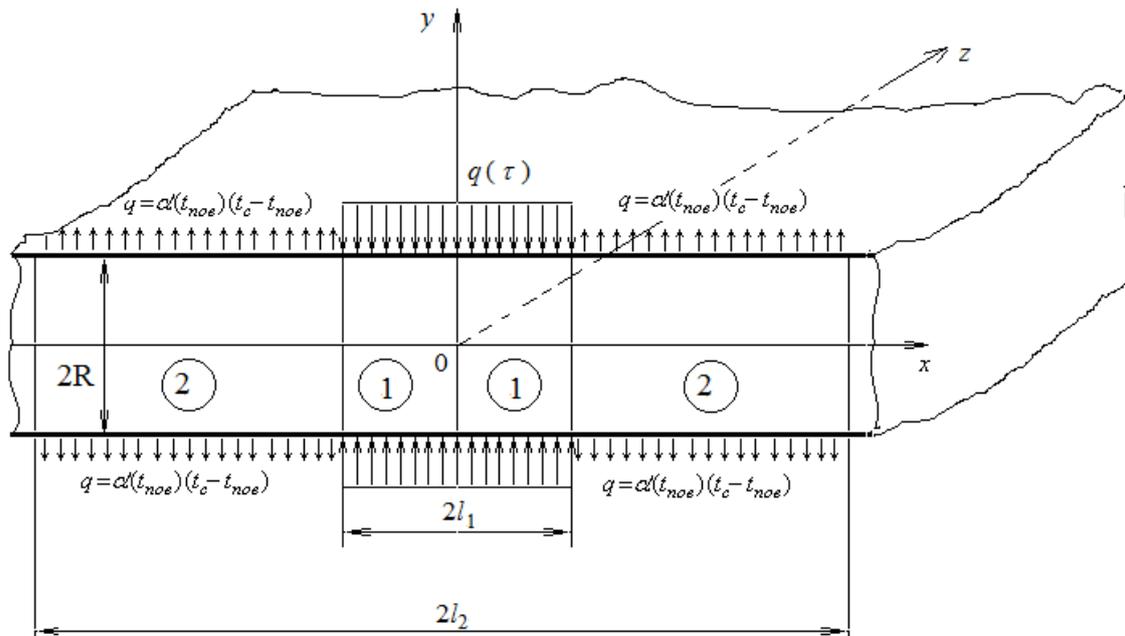


Fig. 1. Thermal scheme of local heat treatment of the plate [6]

It should be noted that heat treatment of large-sized products manufactured by the electroslag welding method is usually carried out in accordance with the schedule of changes in surface temperature in the weld zone (Fig. 2, a).

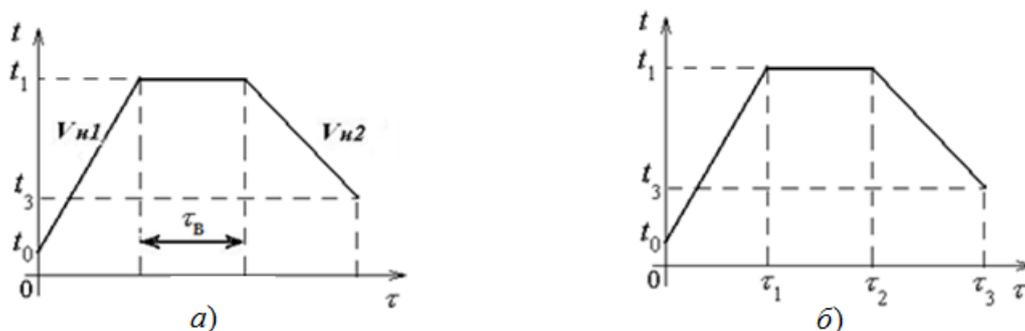


Fig. 2. Heat treatment schedules

Also note that the heat treatment schedule does not specify the heating and cooling time of the product. It is only required that the heating rate does not exceed the specified rate  $V_{n1}$ , and the cooling rate did not exceed the set speed  $V_{n2}$ . It is clear that the time for the heat treatment process as a whole will be minimal if the conditions for realizing the maximum permissible speeds are provided  $V_{n1}$  and  $V_{n2}$ . In this regard, a modified heat treatment schedule is proposed (Fig. 2, b). It provides for the regulation of the time of each stage of heat treatment of products so that the following ratios are observed

$V_{H1} = (t_1 - t_0)/\tau_1$ ,  $\tau_e = \tau_2 - \tau_1$ ,  $V_{H2} = (t_3 - t_1)/(\tau_3 - \tau_2)$ . The modified schedule does not contradict the traditional schedule, but provides.

In order to comply with the modified heat treatment schedule, it is necessary to be able to physically adjust the parameters of external heat exchange based on the corresponding mathematical dependencies  $q = q(t_{\text{ноб}})$  or  $q = q(\tau)$ . The main purpose of mathematical modeling of the local heat treatment process is to obtain the following dependencies.

To solve this problem, we will use a computer-oriented approach that has proven itself well in solving similar problems before (see, for example, [7, 8]). The approach is based on the use of a modified method of elementary heat balances and actually consists in solving two problems at each time step. First, it is necessary to find the parameters of the law using an explicit difference equation  $q = q(\tau)$ , that provide the temperature at point (0, R) in accordance with the modified heat treatment schedule, and then use economical implicit difference schemes to determine the temperatures at all other design points.

For the second of these tasks, a computer algorithm described in [6] can be used to solve the problem. It is based on the use of a nonlinear mathematical model in the form of a two-dimensional heat conduction equation with appropriate boundary conditions and a locally one-dimensional method for implementing the model. In this case, the boundary condition on the surface of the plate was set as follows;

$$\lambda(t) \frac{\partial t}{\partial y} \Big|_{y=R} = q_H(\tau), \quad (1)$$

where  $q_H(\tau)$  — density of the heat flux supplied to the surface of the plate (in accordance with the adopted scheme  $q_H(\tau)$  may vary in time, but is the same on the surface).

The first of the above tasks is to determine at each time step the value of  $q_H(\tau)$ , which would ensure compliance with the schedule shown in Fig. 2,b. Using the method of elementary heat balances, we obtain the formula for calculating the required heat flux density

$$q_H^n = R_0 V_H + R_1 (t_{1,M}^n - t_{2,M}^n) + R_2 (t_{1,M}^n - t_{1,M-1}^n), \quad (2)$$

where

$$V_H = \begin{cases} V_{H1} & , \quad 0 \leq \tau < \tau_1 \\ 0 & , \quad \tau_1 \leq \tau \leq \tau_2 ; \\ V_{H2} & , \quad \tau_2 < \tau \leq \tau_3 \end{cases} \quad (3)$$

$$R_0 = c_{v1,M} \cdot \frac{\Delta y}{2}; \quad R_1 = \frac{(\lambda_{1,M} + \lambda_{2,M}) \Delta y}{2 \Delta x^2}; \quad R_2 = \frac{(\lambda_{1,M} + \lambda_{1,M-1})}{2 \Delta y}. \quad (4)$$

In the following formulas  $t_{i,j}^n$  ( $i = 1, 2, \dots, N_2; j = 1, 2, \dots, M$ ) — temperature at a point  $x_i = (i-1)\Delta x$ ,  $y_j = (j-1)\Delta y$  at a point in time  $\tau_n = n\Delta\tau$ ;  $\lambda_{i,j}$  i  $c_{vi,j}$  — thermal conductivity and volumetric heat capacity at the same point, calculated from the temperature values at the previous time step;  $\Delta x$ ,  $\Delta y$ ,  $\Delta\tau$  — parameters of discretization of the computational domain (steps for variables  $x$ ,  $y$  and  $\tau$ , respectively);  $N_2 = 1 + l_2 / \Delta x$ ;  $M = 1 + R / \Delta y$ .

A generalized block diagram of the algorithm for modeling the full cycle of heat treatment is shown in Fig. 3. Arrays T1, T2, and T3 in the block diagram of the algorithm correspond to temperatures  $t_{i,j}^n$ ,  $t_{i,j}^{n+1/2}$  i  $t_{i,j}^{n+1}$ , respectively.

At the beginning of the algorithm (block 2 in Fig. 3, a), the functions are described  $\lambda(t)$ ,  $c_v(t)$  and  $\alpha(t)$ , which define the dependence of the thermal properties of the metal and the heat transfer coefficient in zone 2 on temperature. Next (block 3), all the necessary data are entered, in particular, the parameters of the difference grid  $\Delta x$ ,  $\Delta y$ ,  $\Delta\tau$ ,  $N_1$ ,  $N_2$ ,  $M$ ; parameters of heat treatment schedule ( $\tau_1$ ,  $\tau_2$ ,  $\tau_3$ ,  $t_0$ ,  $t_1$ ,  $t_3$ ), ambient temperature in zone 2  $t_c$ . Block 4 sets the start of the time

countdown. In block 5, the temperatures of the design points are entered into the T1 array in accordance with the initial condition. These temperatures are also entered in the T3 array. This is due to the fact that when refining the values of  $\alpha(t)$ ,  $\lambda(t)$ ,  $c_v(t)$  and  $q(t)$  array T3 will be used. Blocks 6-9 implement calculations at the first stage of the heat treatment process, blocks 10-13 correspond to the holding stage, blocks 14-17 correspond to the controlled cooling stage.

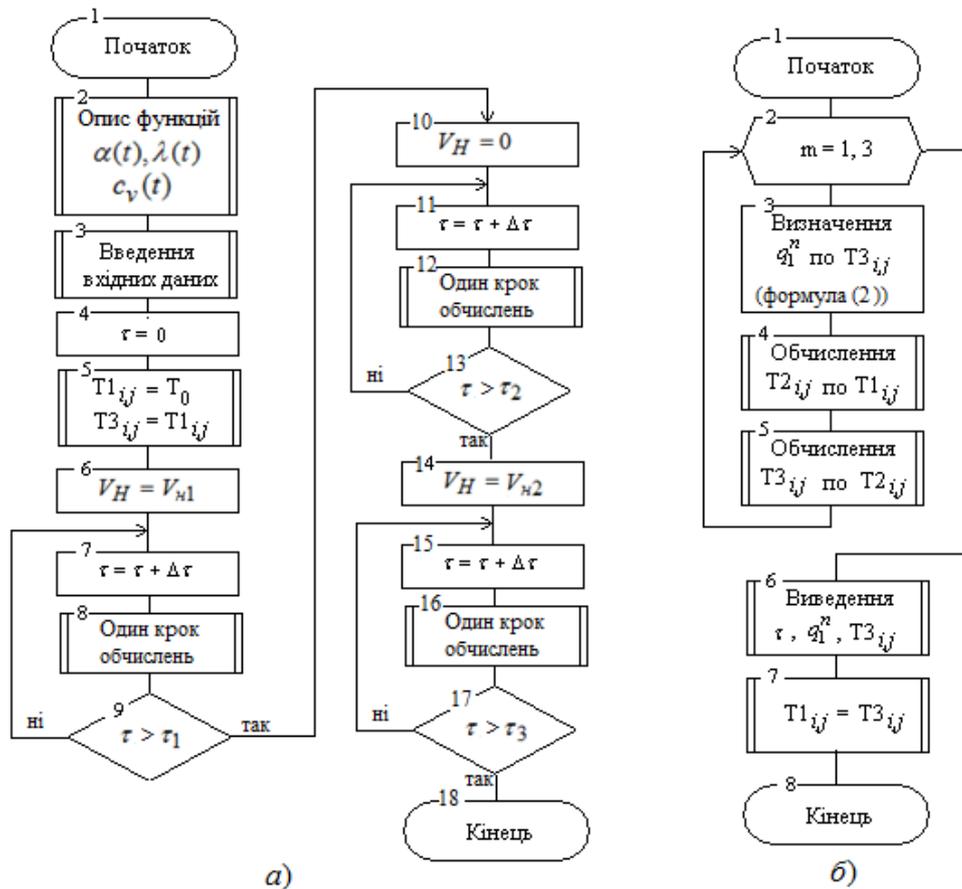


Fig. 3. Generalized block diagram of the heat treatment process modeling algorithm

Fig. 3 b shows the detail of the “One step of calculations” block, which occurs three times in the main block diagram (see blocks 8, 12, 16 to fig. 3, a). At the same time, block 2 provides refinement of the values  $\lambda(t)$ ,  $c_v(t)$ ,  $\alpha(t)$  and  $q(t)$ , which are initially calculated from the temperature values at the previous time step  $t_{i,j}^n$ . Using these coefficients, a full cycle of calculations is performed to determine the temperatures at a new time step  $t_{i,j}^{n+1}$ , and then all calculations are repeated three times, but with the coefficients  $\lambda(t)$ ,  $c_v(t)$ ,  $\alpha(t)$  and  $q(t)$ , which are listed by value  $t_{i,j}^{n+1}$ . Block 3 calculates the heat flux density that needs to be delivered to the surface of the product to meet the specified heat treatment schedule. Blocks 4-5 perform all the calculations necessary to determine the temperatures  $t_{i,j}^{n+1}$  on a new time step. These blocks are described in detail in [6]. Block 6 displays the calculation results on the screen, and block 7 prepares for calculations at a new time step.

We emphasize that the algorithm presented in Fig. 3, makes it possible to calculate temperature fields both at the heating stage ( $V_H > 0$ ), and during the aging stages ( $V_H = 0$ ) and cooling

( $V_H < 0$ ), this makes it possible to simulate the full cycle of heat treatment of a product. At the same time, control parameters are determined in the form of heat flux density, the supply of which to the surface of the product ensures the process in accordance with a given schedule.

The algorithm is implemented in the form of a program for a personal computer in the environment of the PascalABC.NET programming system. A computational experiment was conducted to confirm the effectiveness of the algorithm and establish the basic laws of the process of local heat treatment of plate structures according to a given schedule.

Some results of the calculations are shown in Fig. 4. The following parameters were used:  $R=0,2$ ;  $l_1 = 0,3 \text{ м}$ ;  $l_2 = 1,8 \text{ м}$ ;  $\Delta x = 0,05 \text{ м}$ ,  $\Delta y = 0,05 \text{ м}$ ,  $\Delta \tau = 60 \text{ с}$ ; plate material - steel 20. Parameters of the heat treatment schedule:  $\tau_1 = 12 \text{ year.}$ ,  $\tau_2 = 18 \text{ year.}$ ,  $\tau_3 = 24 \text{ year.}$ ,  $t_0 = 20^0 \text{ C}$ ,  $t_1 = 620^0 \text{ C}$ ,  $t_3 = 500^0 \text{ C}$ . The ambient temperature was assumed to be equal to  $t_c = 20^0 \text{ C}$ .

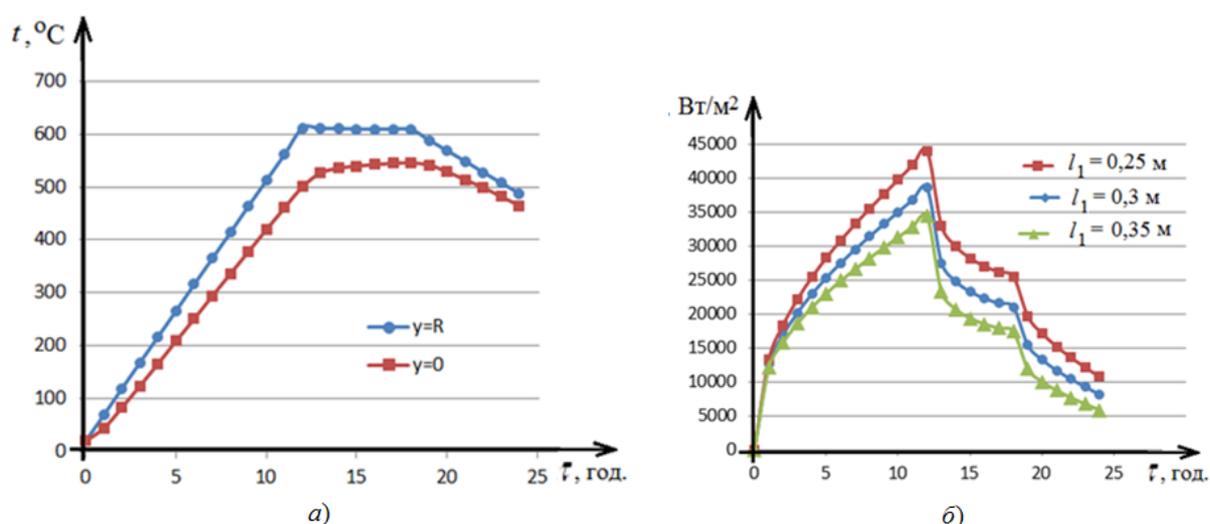


Fig. 4. Results of testing the algorithm for modeling the full heat treatment cycle

Fig. 4, a shows the dynamics of temperature changes on the surface ( $y = R$ ) and in the central plane of the plate ( $y = 0$ ) in the weld zone ( $x = 0$ ). Since the temperature on the surface of the plate corresponds to the specified schedule, it can be concluded that the developed algorithm makes it possible to calculate the value of the control parameter for the local heat treatment process, which follows the modified schedule, and thus implements the optimal technological process in terms of speed.

The pattern of change in the heat flux density over time (Fig. 4, b) is in good agreement with the known laws of heat transfer theory. The rate of change is the highest at the initial stage of the process, then it decreases, gradually approaching the regular regime. When switching to the holding mode, the heat flux density decreases sharply, as there is no need to increase the product temperature. Another dramatic change occurs when you move to the controlled cooling stage. This is also understandable, because the need for new heat portions becomes even smaller, as the surface of the product must be cooled.

From Fig. 4, b, it can also be seen that such parameters as the width of the heat supply zone and the heat flux density (in fact: the power of the heating device) are interrelated. If the power of the heating device is insufficient, the required result can be achieved by expanding the heat supply zone. If it is constructively impossible to realize a heat supply zone of sufficient width, increasing the power of the heating devices may be a solution.

### Conclusions

For the purpose of ensuring rational modes of the technological process of local heat treatment of plate structures, a computer-oriented algorithm for determining control parameters in the form of

heat flows, which are supplied to the surface of the product, has been developed to ensure that the process is carried out in accordance with a given heat treatment schedule. The process optimization in terms of speed is achieved by conducting the technological process at the maximum allowable heating and cooling rates.

The algorithm is implemented in the form of a program in the PascalABC.NET programming system, which was used to conduct a computational experiment. The results of the experiment confirmed the possibility of applying the developed algorithm to control the rational modes of local heat treatment of plate structures.

### References

- [1] Kryvov, H.O., & Zvorykin, K.O. (2012). *Vyrobnytstvo zvarnykh konstruktsii: pidruchnyk* [Production of welded construction]. Kyiv: KVITs [in Ukrainian].
- [2] Kompaniiets, R.A. (2023). *Mistseva termoobrobka velykohabarynykh vyrobiv: zastosuvannya ta problemy proektuvannya tekhnolohii* [Local heat treatment of large-sized products: application and design problems of the technology]. Proceedings of the 1st International Scientific and Practical Internet Conference: *Achievements of 21st Century Scientific Community*. (pp. 223-225). Dnipro: FOP Marenichenko V.V. [in Ukrainian].
- [3] Dobrianskyi, I.M., & Bozhydarnik, V.V. (2012). *Chyselno-analitychni skhemy rozrakhunku zvarnykh plastynchatykh system* [Numerical-analytical calculation schemes of welded plate systems]. *Interuniversity collection "Scientific Notes"*, 37, 31-37 [in Ukrainian].
- [4] Tsotsko, V.I., Spirydonova, I.M., & Peleshenko, B.H. (2008). *Kinetyka nahrivannia ta plavlennia poverkhni zrazkiv nyzkovuhletsevoi stali pry mistsevii termoobrobtsi* [Kinetics of heating and melting of the surface of low-carbon steel samples during local heat treatment]. *Solid state physics and chemistry*, Vol. 9, 3, 639-643 [in Ukrainian].
- [5] Manusov, I.N., & Karimov, I.K. (1995). *Modeliuvannya mistsevoho nahrivu plynykh konstruktsii u rezhymi nepriamoho teploobminu* [Modeling of local heating of plate construction in the mode of indirect heat exchange]. *Jubilee collection of scientific and technical works*, 322-328 [in Ukrainian].
- [6] Karimov, I.K., Karimov, H.I., Kompaniiets, R.A., & Bulai, O.Iu. (2024). *Kompiuterno oriientovanyi alhorytm modeliuvannya mistsevoho nahrivu plynykh konstruktsii* [Computer-oriented algorithm for modeling local heating of plate construction]. *Science and technology today*, №1(29), 689-700 [in Ukrainian].
- [7] Karimov, I.K. (2015). *Kompiuterno oriientovanyi alhorytm keruvannya protsesom mistsevoi termoobrobky velykohabarynykh detalei* [A computer-oriented algorithm for controlling the process of local heat treatment of large-sized parts]. *Mathematical modeling*, 1(32), 45-48 [in Ukrainian].
- [8] Vernygora, D.V. (2023). *Modyfikovanyi metod elementarnykh teplovykh balansiv v upravlinni protsesamy mistsevoi termoobrobky* [The modified method of elementary heat balances in the management of local heat treatment processes]. Proceedings of the XXXVI International science and practice conference: *Suchasni aspekty modernizatsii nauky: stan, problemy, tendentsii rozvytku - Modern aspects of the modernization of science: state, problems, development trends*. (pp. 188-191). Kyiv: HO "VADND" [in Ukrainian].

### Список використаної літератури

1. Кривов Г.О., Зворикін К.О. *Виробництво зварних конструкцій: підручник для студентів вищ. навч. закладів*. К.: КВІЦ, 2012. 896 с.
2. Компанієць Р. А. *Місцева термообробка великогабаритних виробів: застосування та проблеми проектування технології / Achievements of 21st Century Scientific Community: Proceedings of the 1st International Scientific and Practical Internet Conference, September 14–15, 2023*. Dnipro: FOP Marenichenko V.V., 2023. P. 223-225.
3. Добрянський І.М., Божидарнік В.В. *Чисельно-аналітичні схеми розрахунку зварних*

- пластинчатих систем. *Міжвузівський збірник "Наукові нотатки"*. Луцьк, 2012. Вип. 37. С. 31-37.
4. Цоцко В.І., Спиридонова І.М., Пелешенко Б.Г. Кінетика нагрівання та плавлення поверхні зразків низьковуглецевої сталі при місцевій термообробці. *Фізика і хімія твердого тіла*. Івано-Франківськ, 2008. Т.9, №3. С. 639-643.
  5. Манусов І.Н., Карімов І.К. Моделювання місцевого нагріву плитних конструкцій у режимі непрямого теплообміну. *Ювілейний збірник науково-технічних праць*. Дніпродзержинськ: ДДТУ, 1995. С. 322-328.
  6. Карімов І.К., Карімов Г.І., Компанієць Р.А., Булай О.Ю. Комп'ютерно орієнтований алгоритм моделювання місцевого нагріву плитних конструкцій. *Наука і техніка сьогодні*. 2024. №1(29). С. 689-700.
  7. Карімов І.К. Комп'ютерно орієнтований алгоритм керування процесом місцевої термообробки великогабаритних деталей. *Математичне моделювання*. 2015. № 1(32). С. 45-48.
  8. Вернигора Д.В. Модифікований метод елементарних теплових балансів в управлінні процесами місцевої термообробки. Сучасні аспекти модернізації науки: стан, проблеми, тенденції розвитку: матеріали XXXVI Міжнар. наук.-практ. конф., 7 вер. 2023 р. Софія (Болгарія). К.: ГО «ВАДНД», 2023. С. 188-191.

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