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MATHEMATICAL MODEL OF MECHANICAL OSCILLATIONS AND PROPAGATION OF ACOUSTIC WAVES

МАТЕМАТИЧНА МОДЕЛЬ МЕХАНІЧНИХ КОЛИВАНЬ ТА РОЗПОВСЮДЖЕННЯ АКУСТИЧНИХ ХВИЛЬ

The article presents the results of the study of the influence of external forces, resonance and damping on the amplitude of oscillations, as well as the relationship between oscillation and the level of generated sound pressure for mechanical systems with one and two degrees of freedom. An analysis of the Lagrange equation, the frequency of natural oscillations, the transmission of dynamic forces through vibrations and their influence on the noise intensity and the level of the generated sound pressure is carried out. The paper proposes a method for evaluating vibrations and the effectiveness of isolation of the propagation of sound waves in space.
***Keywords**: mathematical model of mechanical vibrations, acoustics, acoustic waves, sound insulation, vibrations.*

Актуальність статті обумовлена необхідністю дослідження механічних вібрацій з метою більш досконального розуміння механізмів розповсюдження звукових хвиль від джерел їх генерації, що дасть змогу розробити більш ефективні методи їх ізоляції та ослаблення при вирішенні питань оптимізації розміщення звукових джерел екстреного оповіщення населення. Також вивчення різних аспектів вібраційних явищ та їх впливу на конструкційні елементи дозволить оптимізувати конструкцію відповідних пристроїв для підвищення їхньої надійності та довговічності.

Метою дослідження є аналіз механічних вібрацій у системах з одним та двома степенями свободи, розгляд впливу амортизації та резонансу на амплітуду коливань, а також визначення взаємозв’язку між вібраціями та рівнем звукового тиску спеціалізованих пристроїв.

Проведено дослідження основних аспектів механічних вібрацій, зокрема, їх виникнення, розглянуто особливості вимушених та вільних коливань, а також методи розрахунку параметрів таких систем. Було розглянуто математичні моделі на основі диференціальних рівнянь руху для систем з одним та двома степенями свободи, що дозволяє проводити розрахунки рівня вібрацій для пружних систем, зокрема для електричних машини та трансформаторів. Також встановлено функціональний зв’язок частот і амплітуд коливань, сформульовані умови резонансу та його впливу на ефективність і безпеку роботи обладнання.

Проведено аналіз питань передачі вібрацій на несучі конструкції спеціалізованих електронних пристроїв, наведено формули для визначення коефіцієнта амортизації та передачі, що дозволить ефективніше оцінити вплив їх вібрації на відповідні споруди, що надасть можливості оптимізувати місця їх розміщення.

***Ключові слова**: математична модель механічних коливань, акустика, акустичні хвилі, звукоізоляція, вібрації.*

Problem's Formulation

The propagation of sound waves, both information and noise, has an adverse effect on humans. Prolonged exposure to noise leads to the development of chronic fatigue, decreased performance, and symptoms such as poor sleep, drowsiness, hearing loss, and impaired thermoregulation. In the case of a constant noise background of up to 70 dB, endocrine and nervous systems are affected, up to 90 dB — hearing is affected, up to 120 dB — physical pain, which can be unbearable. It should be noted that the normalized noise parameters are sound pressure levels in octave bands with geometric mean frequencies of 63, 125, 250, 500, 1000, 2000, 4000 and 8000 Hz, and the energy equivalent sound level in decibels on the A scale should not exceed the permissible levels of 110, 94, 87, 81, 78, 75, 73 dB, respectively. Noise not only worsens human health, but also reduces labor productivity by 15—25 %, so the improvement and development of methods for studying the mechanisms of its formation and regulation is extremely important.

Mechanical vibrations also have a significant impact on the reliability and efficiency of various machines and mechanisms. Forced oscillations caused by external excitatory forces can cause significant mechanical stresses, reducing the performance of equipment. In addition, the interaction between vibrations with different numbers of degrees of freedom and damping also generates complex effects, including resonance, which leads to a significant increase in the amplitude of vibrations. All of this makes it difficult to effectively isolate vibrations and noise, which is a key task in the design and installation of emergency information systems for the public and industrial equipment.

Analysis of recent research and publications

A number of scientific papers have been devoted to the study of various aspects of mechanical vibrations and acoustic wave propagation. For example, in [1], you can learn more about vibrations and mechanical oscillations. In particular, you can see examples of research and discussions on vibration damping control using particles in vacuum packaging. It also describes the effect of material properties on vibration frequency. The same topic was further developed in [2], where, after considering the problem, it is recommended to develop an acoustic-mechanical meta-surface with combined vibration reduction and sound absorption functions.

In a more general way, the basic basis and problems of vibrations, waves and acoustics can be found in [3]. Paper [4] discusses the various possibilities of using mechanical vibrations for technological processes and gives directions for further possible vibrating equipment. The behavior of mechanical vibrations and acoustic waves in continuous mechanical systems is described in [5]. For a more experimental approach to the topic, where the results of real experiments in various aspects of elasticity, acoustics, and vibration are used, see [6].

Formulation of the study purpose

The purpose of the study is to analyze mechanical vibrations in systems with one and two degrees of freedom, to consider the effect of damping and resonance on the amplitude of vibrations, and to determine the relationship between vibrations and noise in specialized equipment.

Presenting main material

***Mechanical vibrations*.** A noise source is any physical process of change in sound pressure or mechanical vibrations in solid, liquid or gaseous media. In most cases, the causes of vibrations lead to acoustic vibrations within the frequency spectrum of audible noise and beyond, so it is possible to study both phenomena at once, as they are very similar and the differences can only be established conditionally, although some of the results of the analysis may be quantitatively different.

It is known that under the influence of a short shock or impact, an elastic system performs oscillations, which are called natural or free oscillations, because they occur after the shock without the participation of external forces. In sound signal generators and other electrical machines, external forces (excitatory forces) are always acting, and therefore this leads to forced oscillations, which are a special case of free oscillations. It should also be noted that most often the study of machine vibration isolation and attenuation is associated with the study of systems with one or two degrees of freedom.

One of the simplest oscillatory systems with one degree of freedom can be represented by a mass *m* suspended on a spring with constant elasticity that can move vertically in the *x-direction*.

We can assume that this mass is subject to a certain external periodic force $F_0 sin\omega f$ and that there is a cushioning (damping) mechanism with an attenuation coefficient *r* between the mass and the suspension point. In this case, the law of motion of the above system is determined by formula [7]:

$$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} = F_0\, sin\, \omega t. \quad (1)$$

The general solution of the differential equation (1) is written as follows:

$$x = e^{-\frac{r}{2m}t} X_1 \cos(\omega_1 t + \varphi_1) + Xsin(\omega t - \varphi). \quad (2)$$

where X_1 and φ_1 — are integration constants that are a function of the initial conditions, and the oscillation parameters ω_1, X and φ are determined by the formulas:

$$\omega_1 = \sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}; \quad (3)$$

$$X = \frac{F_0}{\sqrt{(r\omega)^2 + (k - m\omega^2)^2}}; \quad (4)$$

$$\varphi = arctg\,\frac{r\omega}{k - m\omega^2}. \quad (5)$$

Each of the two terms in equation (1) represents a specific vibration motion: the first represents free damped oscillations, and the second represents forced oscillations. After a certain time after the start of the motion, the free oscillations completely dampen, and the system performs only forced oscillations as a result.

The amplitude of forced oscillations is determined by the following formula:

$$X = \frac{X_{st}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 + \left(2\frac{r}{r_{cR}}\frac{\omega}{\omega_0}\right)^2}}, \quad (6)$$

where ω_0 is the natural frequency,

$$\omega_0 = \sqrt{k/m}; \quad (7)$$

X_{st} — static deflection,

$$X_{st} = F_0/k; \quad (8)$$

r_{cR} — critical attenuation,

$$r_{cR} = 2m\upsilon_0 = 2\sqrt{mk}. \quad (9)$$

It follows from formula (6) that the amplitude of oscillations reaches its maximum value at $\omega = \omega_0$ and is defined as follows

$$X_{max} = F/\omega r\,. \quad (10)$$

In this case, in the case of forced vibrations that do not have damping (r = 0), the amplitude increases infinitely, i.e., the system resonates. Also, with the increase of the damping properties of the system, namely, the greater the value of the coefficient *r*, the maximum value of the vibration amplitude will decrease. It should also be noted that low-frequency vibration dampens more slowly than high-frequency vibration.

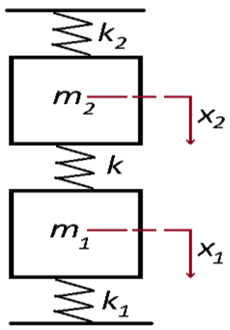

Fig. 1. Diagram of a mechanical system with two degrees of freedom

In some cases, when no external disturbing force acts on the system (F = 0) and there is no damping (r = 0), the system will perform harmonic oscillations according to the sinusoidal law of motion, as follows from equation (1).

Now let us consider the oscillations of a mechanical system with two degrees of freedom, whose model is shown in Fig. 1. The system consists of masses m_1 and m_2, suspended on springs with constant elasticity coefficients k_1 and k_2 and connected by a spring with a constant elasticity coefficient, *k*. The positions of the masses relative to the fixed mark are determined by the independent displacements x_1 and x_2 in the vertical direction.

Taking into account the absence of damping, such a system can be classified as conservative with the potential energy defined as follows:

$$W_p = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 + \frac{1}{2}k(x_1 - x_2)^2. \tag{11}$$

Since the external forces acting on the system are derivatives of the potential energy for the corresponding parameter with the opposite sign, the Lagrange equations in this case are written as follows:

$$\begin{cases} m\frac{d^2x_1}{dt^2} = -\frac{dW_\text{п}}{dx_1} = -k_1x_1 - \square(x_1 - x_2); \\ m\frac{d^2x_2}{dt^2} = -\frac{dW_\text{п}}{dx_2} = -k_2x_2 - k(x_1 - x_2). \end{cases} \tag{12}$$

Assuming that the masses m_1 and m_2 perform harmonic motions with different amplitudes but the same frequency ω, from the system of equations (12) we obtain expressions for determining the natural frequencies of the system:

$$\begin{matrix}\omega_0' \\ \omega_0''\end{matrix} = \sqrt{\frac{1}{2}\left[(\omega_1^2 + \omega_2^2) \pm \sqrt{(\omega_1^2 - \omega_2^2)^2 + 4\mu^2\omega_1^2\omega_2^2}\right]}, \tag{13}$$

where $\omega_1$and ω_2 — are the frequencies of natural oscillations,

$$\omega_1^2 = \frac{k_1+k}{m_1};\ \omega_2^2 = \frac{k_2+k}{m_2}; \tag{14}$$

μ — is the coefficient of connection, which is defined as follows:

$$\mu = \frac{k}{\sqrt{(k_1+k)(k_2+k)}}. \tag{15}$$

Analysis of the solution to (13) shows that:

1) A mechanical system consisting of two interconnected oscillating bodies has two natural frequencies ω'_0 and ω''_0.

2) The frequency of natural oscillations of the system depends on the frequencies of natural oscillations of the bodies taken separately and on the coefficient of coupling between the bodies μ.

3) If the coupling coefficient between the bodies $\mu = 0$then the frequencies of natural oscillations ω'_0 and ω''_0 coincide with $\omega_1$and ω_2 which means that each system oscillates independently of the other.

4) If $\mu = 1$, i.e., the masses of the system are rigidly coupled, then one of the natural oscillation frequencies is equal to ($\omega'_0 = 0$), and the other has the maximum value equal to the $\left(\omega''_0 = \sqrt{\omega_1^2 + \omega_2^2}\right)$.

The two-degree-of-freedom system under consideration is most often found in installations with electrical machines and transformers.

When installing machines, it is usually a requirement that the force transmitted from the machine to the supporting structure should be as small as possible. This force can be determined by the transmission coefficient C_{trans} , which is the ratio of the maximum elastic force F_y transmitted by the system to the base to the maximum applied force F. In the case of an unbalanced machine with mass m mounted on a base whose equation of motion is defined by formula (1), the transmission coefficient is equal to:

$$C_\text{trans} = \frac{|F_y|}{|F|} = \sqrt{\frac{1+\left(2\frac{r}{r_\text{cR}}\frac{\omega}{\omega_0}\right)^2}{\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]^2 + \left(2\frac{r}{r_\text{cR}}\frac{\omega}{\omega_0}\right)^2}} = \frac{\sqrt{1+4\beta^2\gamma^2}}{\sqrt{(1-\gamma^2)^2+4\beta^2\gamma^2}}, \tag{16}$$

where β is the relative transmission coefficient, γ is the frequency ratio,

$$\beta = \frac{r}{r_{cR}},\ \gamma = \frac{\omega}{\omega_0}. \tag{17}$$

The angle between the forces F and F_y is determined by the expression

$$tg\varphi = \frac{r\omega}{k-m\omega^2} = \frac{2\beta\gamma}{1-\gamma^2}. \tag{18}$$

The analysis of the relations (17) shows that at the ratio of frequencies $\gamma = \sqrt{2}$ for any value of the relative transmission coefficient β the transmission coefficient $C_{trans} = 1$. At $\beta = 0$ the value of the transmission coefficient will be determined as follows:

$$C_{trans} = \frac{1}{\gamma^2 - 1}. \quad (19)$$

The ratio of frequencies at which the transmission will have the maximum value:

$$\gamma = \sqrt{\frac{-1\,\sqrt{1+8\beta^2}}{4\beta^2}}. \quad (20)$$

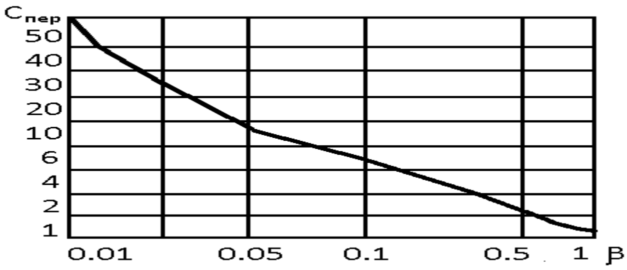

Fig. 2. Dependence of the maximum transmission ratio on the values of the relative depreciation ratio

It follows from formulas (16) and (20) that the maximum transmission value depends only on the value of the coefficient β. The dependence of the values of the maximum transmission coefficient C_{trans} on the values of the relative depreciation coefficient β is shown in Fig. 2. With a slight damping of the system oscillations, the transmission coefficient is calculated as follows:

$$C_{trans.max} = \frac{1}{2}\beta. \quad (21)$$

Note that the oscillation power level is calculated by the formula:

$$L_v = 10\lg\frac{P}{P_0}, \quad (22)$$

where P is the average value of oscillation power for a quarter of a period; $P_0 = 10^{-5}$ m /s^{23} = 10^{-5} W/kg is the reference power for systems with a mass equal to one.

In this case, the reference power is calculated for a frequency of 1 Hz, and the oscillation power level has the same dimension as the intensity level. Accordingly, the oscillation power of a unit mass for a quarter of a period is determined by the ratio:

$$P = 8\pi^2 X^2 f^3 , \quad (23)$$

where X is the amplitude of the harmonic oscillation with frequency f.

The presented algorithm for calculating the vibration power allows classifying and determining the most effective ways of isolating foundations and silenced chambers in order to select the optimal method of installing the relevant equipment to reduce the level of noise generation and transmission.

It is important to note that vibrations of bodies excite sounds, provided they are in the frequency range perceived by humans. During the process of oscillations, the amplitude of movements determines their intensity only partially. Based on the results of oscillation studies, it can be concluded that the corresponding noise level is determined by the amplitude of the surface vibration velocity.

During vibrations of large surfaces, the pressure of the radiated acoustic wave is proportional to the vibration velocity υ:

$$\upsilon = \frac{P}{Z_c}, \quad (24)$$

where Z_c — is the specific acoustic impedance.

The direct relationship between acoustic pressure and vibration velocity allows you to establish a correlation between the level of vibration velocity and the level of acoustic pressure, expressed in decibels:

$$L = 20\lg\frac{P}{P_0} = 20\lg\frac{\upsilon}{\upsilon_0}, \quad (25)$$

where 9 = 5· 10-8 m/s is the reference value of the air vibration velocity corresponding to the threshold value of the acoustic pressure and calculated by formula (23). Therefore, the level of acoustic pressure can be obtained from the level of surface vibration velocity without measuring it.

The radiation can be attenuated if the generating surfaces are small compared to the wavelength of the emitted waves, as strong vibrations of small parts of the mechanism will not be able to cause much noise. This is why small-sized equipment is good at generating high-frequency vibrations (short-wave vibrations), and large-sized equipment, in turn, is good at emitting high and low sounds. Therefore, it is obvious that in order to determine the noise frequency spectrum, not only the amplitude of the surface vibration rate is required, but also the parameters of the generating mechanism, which converts vibrations into acoustic energy.

An important characteristic of noise is its frequency composition. If the noise composition is dominated by sounds with a frequency of oscillation up to 400 Hz, such noise is called low-frequency noise, if it is dominated by sounds with a frequency of 400—1000 Hz, it is called medium-frequency noise, and if it is above 1000 Hz, it is called high-frequency noise. Low-frequency noise with an intensity of up to 100 dB does not cause a noticeable adverse effect on the hearing organ; for medium-frequency noise, this standard is 85—90 dB; for high-frequency noise, 75—85 dB. Normalized parameters of sound pressure levels in octave bands for geometric mean frequencies are given in Tabl. 1. It should be noted that high-frequency noise has a particularly dangerous effect on the human body.

Table 1. Normalized sound pressure levels for different frequency values

Oscillation parameters	Numerical values of sound vibration parameters							
Average frequencies of octave bands, Hz	63	125	250	500	1000	2000	4000	8000
Maximum acoustic pressure level, dB	110	94	87	81	78	75	73	71

Reducing the intensity of low-frequency vibrations of the equipment reduces the magnitude of dynamic forces and, therefore, reduces the high-frequency vibrations of the parts, i.e. the sound pressure level.

Conclusions

The results of the study of the impact of mechanical vibrations on the generated sound pressure level revealed significant effects of external excitatory forces on the operation of equipment. Forced oscillations occurring in systems with one and two degrees of freedom can significantly increase the level of mechanical loads, especially in cases of resonance. The correct assessment of the amplitude and frequency of these vibrations, as well as the application of effective vibration isolation methods, can be key factors in ensuring reliability and reducing sound pressure levels in the operation of the equipment concerned. The development and implementation of sound measures during the installation and operation of equipment can significantly improve its efficiency and extend its service life.

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