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THREE-DIMENSIONAL MODELING OF SOIL DISPLACEMENTS CONSIDERING THE SPATIAL POSITION OF THE SEISMIC SOURCE

ТРИВИМІРНЕ МОДЕЛЮВАННЯ ЗСУВІВ ҐРУНТУ З УРАХУВАННЯМ ПРОСТОРОВОГО ПОЛОЖЕННЯ ДЖЕРЕЛА СЕЙСМІЧНОЇ АКТИВНОСТІ

This article presents a three-dimensional mathematical model for analyzing soil displacements caused by seismic events. The model integrates classical equations of motion for soil using the finite difference method and takes into account complex geological and mechanical properties of the soil. A distinctive feature of this work is the consideration of the azimuthal and zenith angles of the earthquake source, which significantly increases the accuracy of predicting the impact of seismic displacements on engineering structures, as well as the use of the Perfectly Matched Layer (PML) method, which allows to prevent wave reflections at the boundaries of the computational domain.

The model offers an innovative approach to assessing potential seismic impacts and has the potential for further improvement and integration with other dynamic analysis systems.

***Keywords**: seismic engineering, soil displacement modeling, finite difference method, soil dynamic analysis, PML boundary conditions, seismogram.*

Стаття представляє розробку та валідацію тривимірної математичної моделі для аналізу зсувів ґрунту, викликаних сейсмічними подіями. Модель інтегрує класичні рівняння руху для ґрунту з використанням методу скінченних різниць і враховує комплексні геологічні та механічні властивості ґрунту. Особливістю цієї роботи є урахування азимутального та зенітного кутів джерела землетрусу, що дозволяє значно збільшити точність прогнозування впливу сейсмічних зсувів на інженерні споруди, а також використання методу PML(PerfectlyMatchedLayer), який демонструє ефективність у запобіганні відображення хвиль на межах обчислювальної зони.

Модель пропонує інноваційний підхід до оцінки можливих сейсмічних впливів та має потенціал для подальшого вдосконалення та інтеграції з іншими системами динамічного аналізу.

У статті було проведено короткий аналіз актуальних публікацій на тему дослідження, сформульовано ціль роботи та обґрунтування обраних методів та припущень. У основній частиніроботи описані основні теоретичні засади моделювання зсувів, зокрема застосування рівнянь Нав'є-Ламе для опису внутрішніх напружень і деформацій у лінійно-еластичному ґрунті під впливом зовнішніх сил. Далі подано методику налаштування граничних умов з використанням PML, що забезпечує ефективне "поглинання" хвиль. Розглянуто структуру моделі, що дозволяє аналізувати вплив різних сейсмічних сценаріїв, зокрема за різними напрямками дії джерела.

У чисельних експериментах виконано моделювання зсувів для різних конфігурацій джерела сейсмічної активності, що дало змогу оцінити якісну адекватність моделі. Отримані результати показують відповідність обчислень реальним умовам і ефективність використаних методів. У заключній частині статті наведено аналіз результатів та обґрунтовано можливості подальшого вдосконалення моделі для інженерних застосувань, включаючи інтеграцію з іншими системами динамічного аналізу.

Ключові слова: *сейсмічна інженерія, моделювання зсувів ґрунту, метод скінченних різниць, динамічний аналіз ґрунту, граничні умови PML, сейсмограма.*

Problem's Formulation

The consideration of possible seismic impacts has always been a highly relevant topic in the theory of buildings and structures. Over time, both globally and particularly for Ukraine, the relevance of this topic has only intensified, which is explained by the increasing necessity to construct technological facilities that are subject to high seismic resistance requirements, regardless of the regions in which they are built [1].

Analysis of recent research and publications

The complexity of the objects themselves and the specific tasks related to ensuring their normal functioning during seismic impacts primarily demand the development of the most accurate methods for modeling soil displacements under such conditions. Confirmation of this is the significant number of studies published on this topic, including very recent ones [2—5]. This encompasses both linear-elastic [2] and nonlinear [3] soil models, as well as various methods of approximating differential equations, particularly the finite difference method [2] and the finite element method [3—5].

Formulation of the study purpose

The goal of this study is the development and validation of a model capable of adequately modeling soil displacements resulting from seismic impacts. The object of the research is the shallow soil layers directly beneath the foundation or structure, which are subjected to seismic activity, with a particular focus on the azimuthal and zenith angles of the seismic source. The results of the calculations will subsequently be used to compute the impact of the earthquake on steel vertical cylindrical tanks containing liquid.

This article presents a three-dimensional mathematical model of soil displacements induced by seismic activity. Developed based on the equation of motion for soil and utilizing the finite difference method, this model stands out among similar studies due to its ability to incorporate the azimuthal and zenith angles of the earthquake source. This provides more detailed understanding of the impact of seismic events on the soil.

Moreover, one of the key advantages of the proposed model is the use of boundary conditions with absorbing layers (PML — Perfectly Matched Layer), which significantly reduces the reflection of seismic waves from the edges of the modeling area. This ensures a more accurate reproduction of real conditions compared to well-known software packages like ANSYS, which may not always effectively handle such boundary conditions.

The main purpose of using this model is to assess risks for engineering structures located in areas with increased seismic activity, which is extremely important for ensuring their stability and safety. The next step in the development of this model is the simulation of movements of various foundations under the influence of the calculated soil displacements. This will allow for a detailed assessment of the dynamic impacts on engineering structures, which is critically important for ensuring their stability and safety in areas with increased seismic activity.

Presenting main material

The methodology of this study is based on a comprehensive approach to the mathematical modeling of soil displacements due to seismic activity.

At the core of the mathematical model lies the Navier-Lamé equation, which describes internal stresses and deformations in elastic bodies under the action of external forces:

$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = \nabla \cdot \boldsymbol{\sigma} + \boldsymbol{f}, \quad (1)$$

where:

- ρ — soil density;
- $\mathbf{u}$ — vector of soil particle displacements;
- $\boldsymbol{\sigma}$ — stress tensor;
- $\mathbf{f}$ — external forces acting on the soil.

For the stress tensor $\boldsymbol{\sigma}$, the standard model of linear elasticity is used, which takes into account both the deformation of the soil and the impact of external loads. In the case of an isotropic, linear-elastic material, it is expressed as follows:

$$\boldsymbol{\sigma} = \lambda(\nabla \cdot \mathbf{u})\mathbf{I} + 2\mu\varepsilon(\mathbf{u}), \quad (2)$$

where:

- λ, μ — Lamé parameters, that define elastic properties of soil;
- $\mathbf{I}$ — identity tensor;
- $\varepsilon(\mathbf{u})$ — deformation tensor, which is defined as $(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}})\ /\ 2$.

Data from seismograms are utilized for the external forces $\mathbf{f}$, providing information about the temporal sequence and magnitude of accelerations that occur during an earthquake. These forces can be applied to the boundaries of the computational domain depending on the location of the earthquake source, or they may represent point sources of seismic loading. In addition to determining the locations where external forces are applied, their directions were also taken into account by decomposing the seismogram data into the corresponding components:

$$\begin{aligned} a_x(t) &= a(t) \cdot \sin(\theta) \cdot \cos(\phi) \\ a_y(t) &= a(t) \cdot \sin(\theta) \cdot \sin(\phi) \\ a_z(t) &= a(t) \cdot \cos(\theta), \end{aligned} \quad (3)$$

where θ — zenith angle between earthquake source and computational domain; ϕ — azimuthal angle-between earthquake source and computational domain; $a(t)$ — the acceleration values from the seismogram at time t.

Great attention was given to the boundary conditions. From the perspective of the model, the soil is considered virtually infinite; however, we are calculating a limited area. Therefore, the Perfectly Matched Layer (PML) [6] was utilized. This type of boundary condition effectively prevents wave reflections at the edges of the computational domain, which is critically important for the accuracy of seismic displacement simulations. The PMLefficiently "absorbs" outgoing waves, mimicking an infinite medium without introducing artificial reflections that could distort the simulation results.

The main idea of the PML is to extend the modeled area with "layers" in which waves attenuate before reaching the physical boundaries of the model. These layers are positioned at the outer edges of the modeling zone.

For seismic modeling, the equation of motion (1) within the PML can be represented as follows:

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot (\boldsymbol{\sigma} + \boldsymbol{\sigma_{PML}}) + \mathbf{f}, \quad (4)$$

where $\boldsymbol{\sigma_{PML}}$is a damping term that depends on the position within the PML and increases linearly from zero at the entrance of the PML to its maximum value at the outer edge of the PML. The additional term $\boldsymbol{\sigma_{PML}}$acts only within the PML region, that is, in the boundary cells where waves are absorbed. Inside the main computational domain, where there is no PML, its value is zero, and within the PML region, it is defined by the following formula:

$$\boldsymbol{\sigma_{PML}} = -\,\mathbf{D}(\boldsymbol{x})\frac{\partial \mathbf{u}}{\partial t}, \quad (5)$$

where $\mathbf{D}(\boldsymbol{x})$ — damping function, that depends on the coordinate $\boldsymbol{x}$. A quadratic function was chosen to ensure a smooth increase in attenuation within the PML zone:

$$\mathbf{D}(\boldsymbol{x}) = \boldsymbol{d_0}\left(\frac{x - x_0}{L}\right)^{\mathbf{2}}, \quad (5)$$

where:

- $\boldsymbol{d_0}$ — maximum attenuation coefficient,
- $\boldsymbol{x_0}$ — coordinate of the beginning of the PML zone,
- $\boldsymbol{L}$ — width of the PML zone.

In the presented model, the parameters of the function $\mathbf{D}(\boldsymbol{x})$ were chosen such that the width of the PML zone constitutes one quarter of the largest side of the computational domain, and the maximum attenuation coefficient $\boldsymbol{d_0}$ was determined experimentally to ensure optimal wave absorption without significant reflections.

For the numerical analysis, the study area was initially discretized in three-dimensional space using Cartesian coordinates. In the next step, to solve the equation of motion, which includes second derivatives with respect to time and space, the finite difference method was applied: the derivatives were approximated using central difference schemes. The time derivative in $\boldsymbol{\sigma_{PML}}$was approximated using an explicit backward difference scheme.

Numerical Experiments

To verify the qualitative adequacy of the developed mathematical model, a series of numerical experiments were conducted, aimed at analyzing soil displacements under the influence of seismic events from various directions. For this purpose, the model was implemented as a software package in the F# programming language. The computational domain was selected as a rectangular parallelepiped with dimensions 80 × 80 × 20 meters. In the context of soil condition modeling, characteristics corresponding to clayey soil were chosen, specifically: density = 2000 kg/m³, Young's modulus = 40 MPa, Poisson's ratio = 0.3.

To simulate seismic impact, a standardized seismogram with a 1-second interval was applied in all experiments, defined as

$$a(t) = [0.5, 1.1, 0.7, 0.4, 0.2, 0.1, 0, -0.1, -0.2] \text{m/s}^2.$$

These scenarios are considered for several key moments in time: during maximum impact (2.0 s) and after the cessation of seismic activity (9.0 s), noting that the state of the soil quickly stabilizes after the wave action ceases.

Fig. 1 illustrates the distribution of values on the surface of the computational domain when the seismogram is applied to one of the lateral faces. For a more comprehensive view, the same calculation in three dimensions is depicted in Fig. 2.

Fig. 3 and 4 similarly illustrate the displacement distributions of the upper soil layer and overall, for the scenario where the seismogram is applied to two lateral faces simultaneously.

Next, Fig. 5 and 6 depict the seismogram acting on the lower and one lateral face.

Fig. 7 and 8 — the seismogram is applied simultaneously to three faces: two lateral and the lower face.

The developed model and software package allow for the specification of any seismic activity sources, including point sources. Fig. 9 and 10 illustrate the results of applying the same seismogram within a specific 3 × 3 × 3-meter zone in a vertically upward direction.

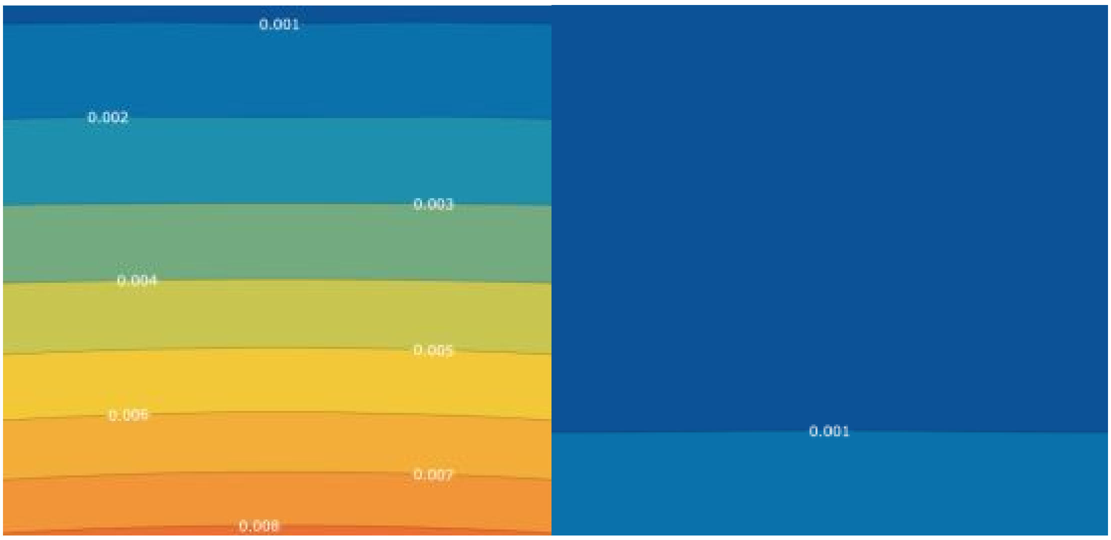

Fig. 1. Displacement Magnitudes (m) of the Upper Soil Layer with the Source on One of the Lateral Faces($\theta = 60°, \phi = 0°$). 2 and 9 s

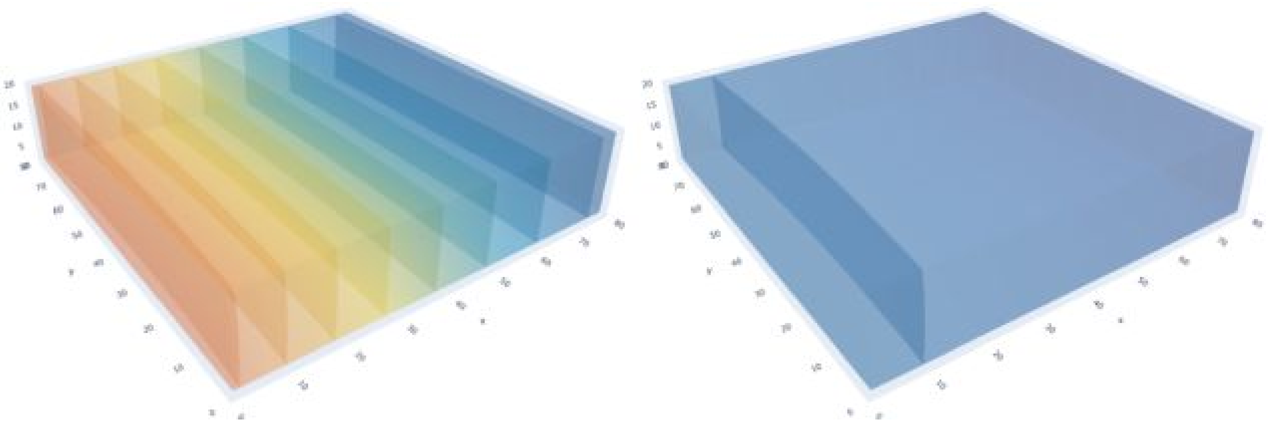

Fig. 2. Displacement Magnitudes (m) in Three Dimensions ($\theta = 60°, \phi = 0°$). 2 and 9 s

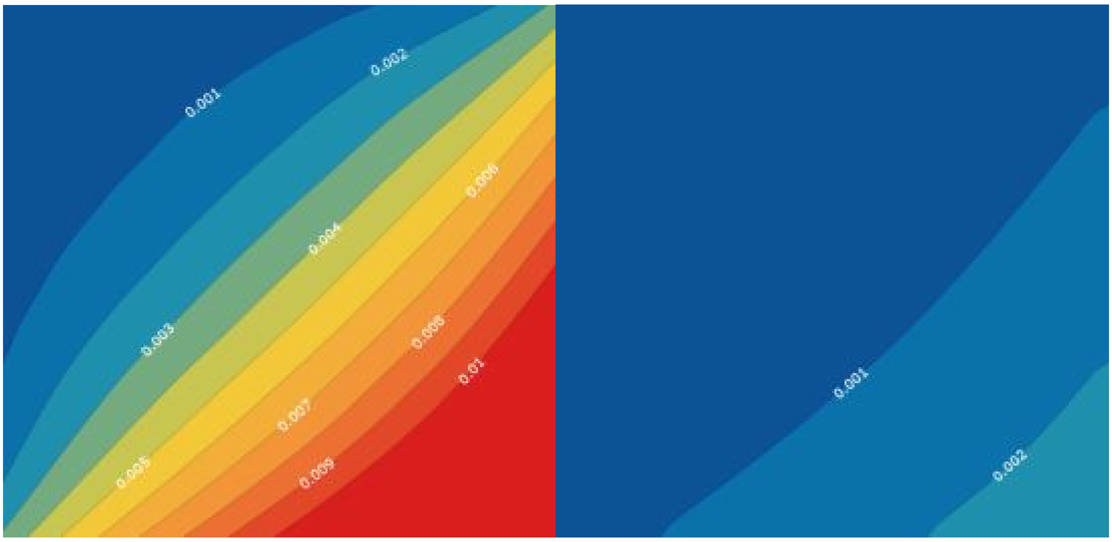

Fig. 3. Displacement Magnitudes (m) of the Upper Soil Layer with the Source on Two Lateral Faces($\theta = 60°, \phi = 45°$). 2 and 9 s

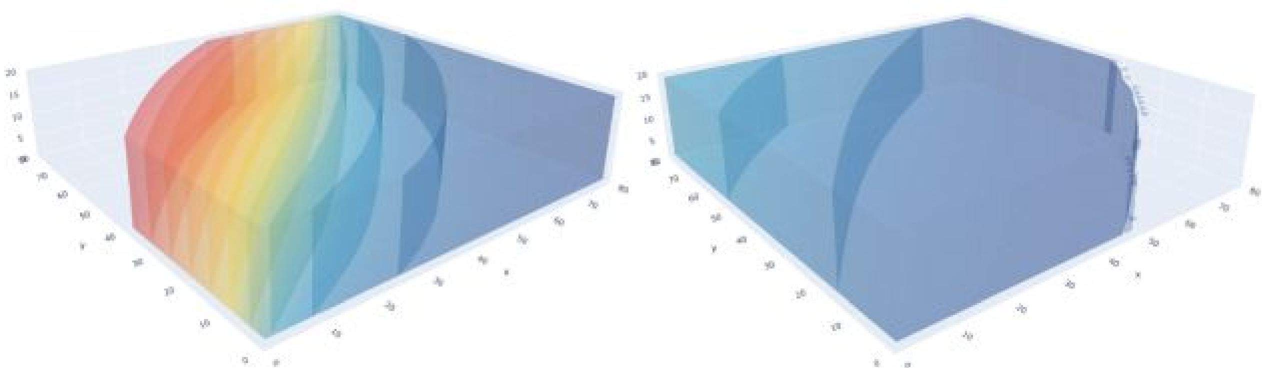

Fig. 4. Displacement Magnitudes (m) in Three Dimensions ($\theta = 60°, \phi = 45°$). 2 and 9 s

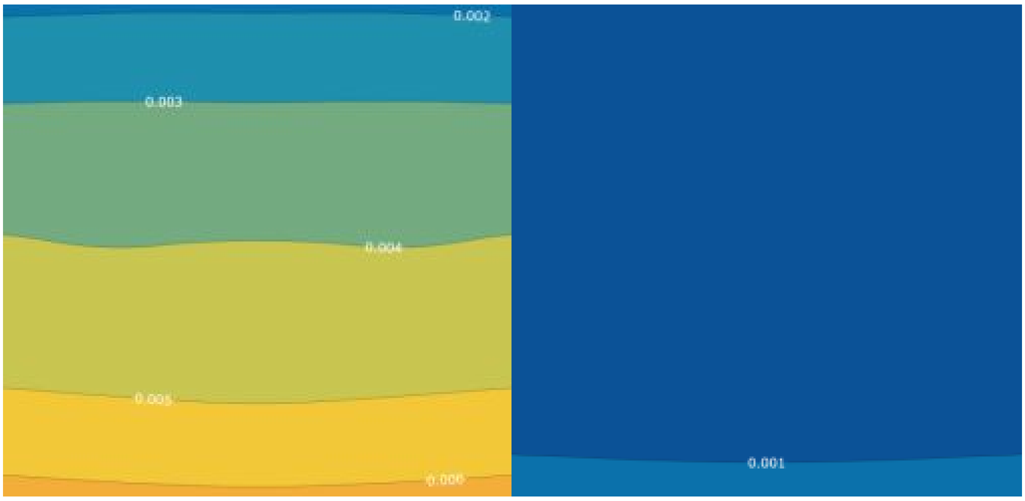

Fig. 5. Displacement Magnitudes (m) of the Upper Soil Layer with the Source on one of Lateral and the Lower Face ($\theta = 45°, \phi = 0°$). 2 and 9 s

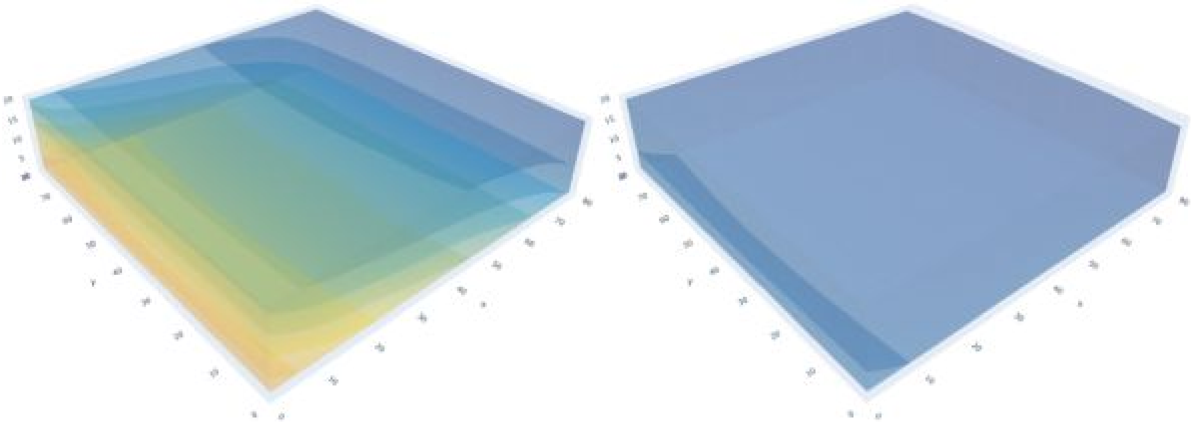

Fig. 6. Displacement Magnitudes (m) in Three Dimensions ($\theta = 45°, \phi = 0°$). 2 and 9 s

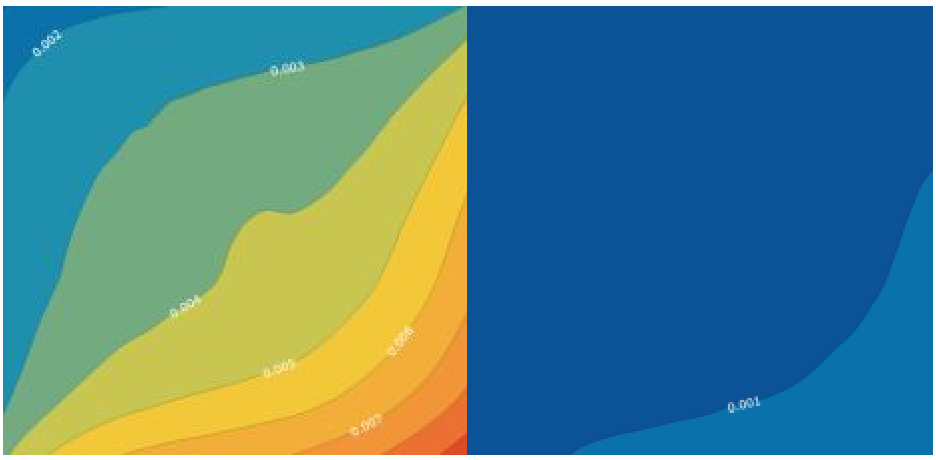

Fig. 7. Displacement Magnitudes (m) of the Upper Soil Layer with the Source on two of Lateral and the Lower Face ($\theta = 45°, \phi = 45°$). 2 and 9 s

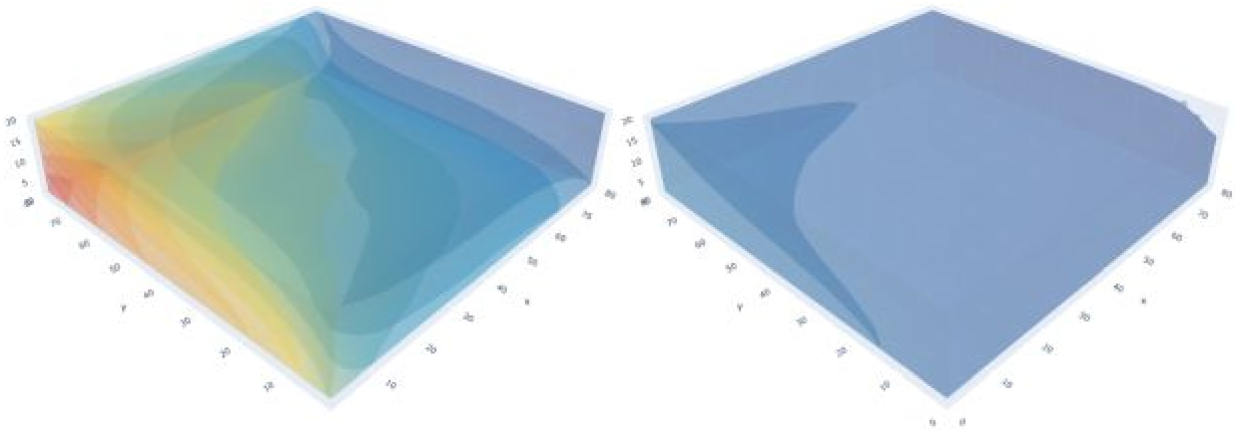

Fig. 8. Displacement Magnitudes (m) in Three Dimensions ($\theta = 45°, \phi = 45°$). 2 and 9 s

Fig. 9. Displacement Magnitudes (m) of the Upper Soil Layer with a Point Source ($\theta = 0°, \phi = 0°$). 2 and 9 s

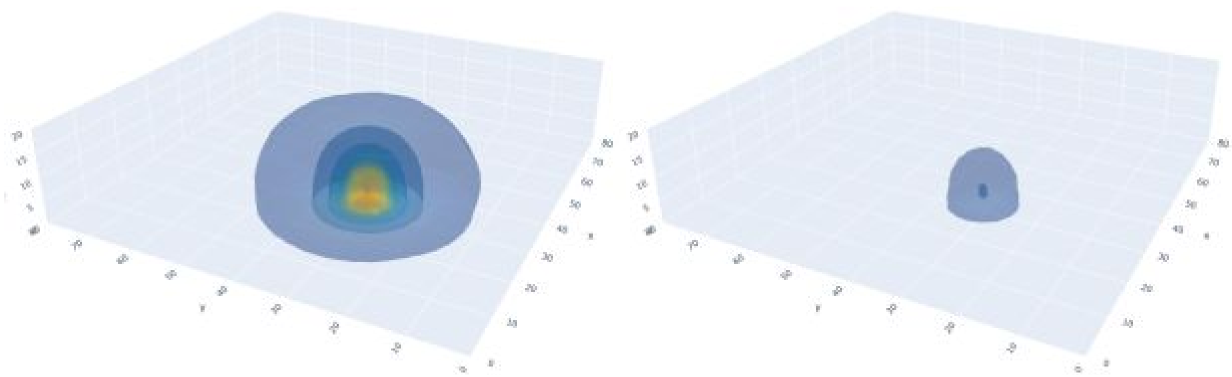

Fig. 10. Displacement Magnitudes (m) in Three Dimensions with a Point Source ($\theta = 0°, \phi = 0°$). 2 and 9 s

For constructing such isosurfaces and contours, the Plotly.NET library [7] was utilized.

The effectiveness and qualitative adequacy of the model were confirmed through its validation, taking into account changes in soil conditions such as density, Young's modulus, and Poisson's ratio.

Conclusions

1. A mathematical model of soil displacements under seismic activity has been developed. The main enhancements compared to existing models are the fully three-dimensional formulation, the consideration of the earthquake source angles and PML for boundary conditions.
2. Given appropriate accelerograms for a design earthquake, the proposed model enables the determination of displacement magnitude, direction, and acceleration over time at each point within the soil computational grid. This is crucial for effectively assessing the seismic resilience of various buildings and structures.
3. The model has been implemented as a software package, allowing for computations across different geometric dimensions, soil types, and accelerograms. Calculations have been performed and results visualized using these tools.
4. The analysis of the obtained results validates the qualitative adequacy of the model.

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