МАТЕМАТИЧНІ МЕТОДИ В СУСПІЛЬНИХ І ГУМАНІТАРНИХ НАУКАХ

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BOUNDARY ELEMENT METHOD IN PROBLEMS OF MODELING A STRESS-STRAIN STATE OF A RANGE-MODULE SOLID AROUND THE HIGH EXTRACTION CHAMBERS

МЕТОД ГРАНИЧНИХ ЕЛЕМЕНТІВ В ЗАДАЧАХ МОДЕЛЮВАННЯ НАПРУЖЕНО-ДЕФОРМОВАНОГО СТАНУ РІЗНОМОДУЛЬНОГО МАСИВУ НАВКОЛО ВИСОКИХ ОЧИСНИХ КАМЕР

Basing on the boundary element method, a numerical algorithm for determining the stress-strain state in an inhomogeneous mass consisting of two linear-elastic subareas was developed. The obtained algorithm was applied to model mineral mining processes based on the analysis of the stress-strain state of the rock mass around high chambers. Different stages of ore mining were studied. The chambers, surrounded by the filling mass under Pivdenno-Bilozirske iron ore deposit conditions, were examined. The chamber stability was evaluated by the maximum “equivalent” stresses defined ac-

cording to the Coulomb-Mohr type criterion. As a result of the numerical studies, the main areas of stress concentration within the ore mass and the goaf filling were defined; the empirical dependence of the "equivalent" stresses on the distance to the chamber contour were obtained; and the requirements for the filling material strength depending on the mining depth were formulated.

***Keywords**: boundary elements method, inhomogeneous rock mass, high extraction chambers, hardening filling, stress-strain state.*

На основі метода граничних елементів розроблено чисельний алгоритм визначення напружено-деформованого стану в неоднорідному масиві, що складається з двох лінійно-пружних підобластей. Отриманий алгоритм застосовано для моделювання процесів видобутку корисних копалин на основі аналізу напружено-деформованого стану області рудного масиву та закладеного простору навколо високих камер. Досліджено різні етапи відпрацювання запасів руди очисними камерами, які знаходяться в умовах Південно-Білозерського родовища залізних руд. Оцінка стійкості камер здійснена за максимальними "еквівалентними" напруженнями, визначеними за критерієм типу Кулона-Мора. В результаті чисельного дослідження встановлені області концентрації напружень в масиві руди та в закладенні, отримані емпіричні залежності "еквівалентних" напружень від відстані до контура камери вглиб масиву, сформульовані вимоги до міцності закладного матеріалу в залежності від глибини ведення очисних робіт.

***Ключові слова**: метод граничних елементів, неоднорідний масив, високі очисні камери, твердіюче закладення, напружено-деформований стан.*

Problem's formulation

Modern mining industry is facing constant challenges related to the efficiency and safety of mining processes. Nowadays, mining systems with hardening goaf filling have become widespread both in Ukrainian and foreign mines; that makes it possible to ensure stability of underground mine workings and minimize risks for workers and the environment.

At the same time, along with the deepening mining operations and changes in mining and geological conditions, the implementation and operation of these systems results in the number of problems requiring further research and improvement. These include issues related primarily to geomechanical stability since the stressed state of the surrounding rock mass can lead to deformation and breaking of the surface of extraction chambers as well as changes in their shape. According to various studies, when stopes at great depths are surrounded by filling mass, dilution of the ore mass with the filling material can reach 5.5—7.6 %, being the reason of significant deterioration of ore quality. At the transition stage, with all changes in the development system parameters, there is a need to consider different options for locating development mine workings for servicing mining operations. This is due to a fact that development operations around the chambers affects stability of fringe drifts driven for the preparatory works and results in support distortion, and rock delamination and caving in the drifts.

Thus, development of a numerical algorithm for studying stress-strain state around the irregular-shaped chambers at great depths to determine rational technological parameters of development systems with hardening filling is an important scientific and technical task.

Analysis of recent research and publications

Both national [1] and foreign scientists [2—3] have contributed much to the studies of rock pressure manifestations while chamber mining with hardening filling. Despite extensive research, changes in physical and mechanical properties of ore and enclosing rocks as well as varying technological parameters of mining operations lead to the new tasks of increasing mining efficiency and solving the topical problems concerning stability of rock and artificial mass outcropping.

Due to the complexity and diversity of mining-geological and mining-engineering conditions, various methods were used to determine the stress state of rocks. Thus, in works [4—5], analytical studies of the formation of the filling massif during intensive working of cleaning chambers were carried out on the basis of a comprehensive analysis of the actual data of the parameters of filling works: the productivity of the complex, as well as the composition and strength of the filling mixture.

The relationship between the technological parameters of the formation of the filling array and its quality characteristics was studied. The negative influence of the layering of the filling array on its stability and quality has been established. To evaluate the strength characteristics of the filling massif it is proposed to use the coefficient of change of the strength of the laying according to the height of the chamber. The dependence of the filling material's strength and the amount of ore depletion was obtain. The use of analytical calculation methods while modelling the problems of contact interactions to determine wearing processes of the structures, taking into account various material properties, is considered in [6].

It should be noted that analytical solutions of the boundary value problems in geomechanics most often fail to be constructed. In this case, using numerical methods, an approximate solution is found. A boundary element method (BEM) helps transform partial differential equations, describing the behaviour of an unknown function inside and at the boundary of some area, into an integral equation.

The problem solution is reduced to solving a system of linear or nonlinear equations obtained from discretization of the integral equation at the area boundary. Unlike a finite element method (FEM) [7—9], the boundary element method (BEM) is distinguished by its ability to model interaction between different materials and structures, focusing on the interface between these materials. This makes it possible to achieve high accuracy in describing the system behaviour without the need to discretize the entire system area. The method easily takes into account nonlinear boundary conditions such as contact conditions or inhomogeneous boundary conditions; it adapts quite easily to changing boundary conditions or object geometry [10—11]. These features make BEM quite attractive for the analysis of complex geomechanical systems.

Formulation of the study purpose

The purpose of this study is as follows: basing on the boundary element method, to develop an algorithm for determining stresses in a multi-modulus rock mass and apply it to the specific case of a level and chamber mining system with a hardening filling of Pivdenno-Bilozerske iron ore deposit to assess the stress-strain state of the stopes and its influence on the stability of both ore mass and filling at the depths of 740—1040 m.

Presenting main material

As an example, we considered a rock mass including an extended mine working in the form of an irregular-shaped chamber (Fig. 1). The development stage was simulated, in which the mine workings surrounding the chamber on the level, as well as those located on the overlying level, are stowed with filling material. It was assumed that the area of the filling mass under study is considered elastic in a state of plane deformation. The assumptions made it possible to use an algorithm developed on the basis of BEM to determine the stress-strain state of the studied mass area. The analytical solution of the Kelvin problem on the action of normal and tangential forces uniformly distributed within a segment arbitrarily oriented in an infinite elastic medium was taken as the basic one.

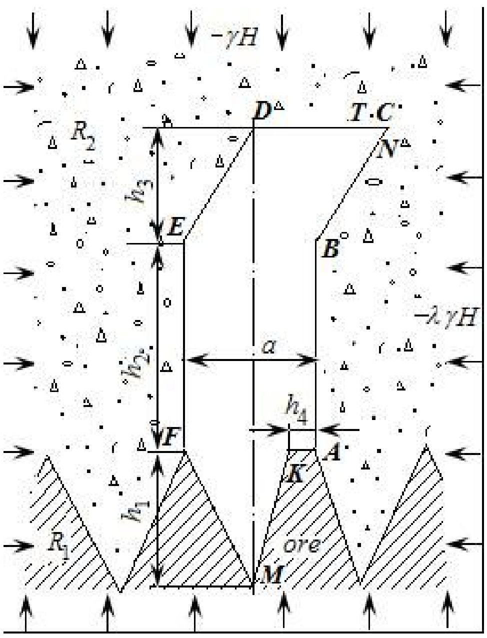

Fig. 1. Calculation scheme for determining the stability of high chambers

The boundaries C_1 and C_2 of two isotropic linear-elastic subareas with different physico-mechanical properties, representing ore (R_1) and filling (R_2), were approximated $N = N_1 + N_2$ by the boundary elements, where N_1 and N_2 are the number of boundary elements of the contours C_1 and C_2, respectively.

The boundary value problem was formulated by specifying boundary conditions in the elements of free part of contours C_1 and C_2 in the form

$$(\sigma_s^i)_0 = 0, \quad (\sigma_n^i)_0 = 0, \tag{1}$$

and conditions of continuity on the contact surface in the form of equality of tangential and normal stresses

$$\sigma_s^{i[1]} = \sigma_s^{i^*[2]}, \qquad \sigma_n^{i[1]} = \sigma_n^{i^*[2]}, \tag{2}$$

or tangential and normal displacements

$$u_s^{i^*[1]} = -u_s^{i[2]}, \quad u_n^{i^*[1]} = -u_n^{i[2]}. \tag{3}$$

Here i and i^* belong to the elements of subareas R_1 and R_2 located on two sides of the contact and coinciding completely with each other.

The unknowns in the method were not real displacements and forces but some fictitious normal P_n and tangential P_s forces applied at the center of each boundary element.

Stresses and displacements at the i^{th} point of the subarea boundary were found using the principle of superposition in the form of total impact of fictitious loads P_n and P_s in all elements of the corresponding boundary.

The relationships for displacements towards the normal to the i^{th} boundary element u_n^i and towards its tangent u_s^i were written as follows:

$$\left.\begin{aligned} u_n^i &= \sum_{j=1}^{N} B_{ns}^{ij} P_s^j + \sum_{j=1}^{N} B_{nn}^{ij} P_n^j \\ u_s^i &= \sum_{j=1}^{N} B_{ss}^{ij} P_s^j + \sum_{j=1}^{N} B_{sn}^{ij} P_n^j \end{aligned}\right\} \quad i = \overline{1, N} \tag{4}$$

and for corresponding stresses in the i^{th} element, they were as follows:

$$\left.\begin{aligned} \sigma_n^i &= \sum_{j=1}^{N} A_{ns}^{ij} P_s^j + \sum_{j=1}^{N} A_{nn}^{ij} P_n^j, \\ \sigma_s^i &= \sum_{j=1}^{N} A_{ss}^{ij} P_s^j + \sum_{j=1}^{N} A_{sn}^{ij} P_n^j. \end{aligned}\right\} \quad i = \overline{1, N}. \tag{5}$$

In expressions (4), $B_{ss}^{ij}, B_{sn}^{ij}, B_{ns}^{ij}, B_{nn}^{ij}$ are boundary coefficients of displacement effects. Coefficient B_{sn}^{ij} is displacement of the i^{th} element of the boundary towards tangential s caused by unit force P_n^j applied within the j^{th} section of the boundary towards its normal. In ratios (5), $A_{ss}^{ij}, A_{sn}^{ij}, A_{ns}^{ij}, A_{nn}^{ij}$ are boundary coefficients of stress influences, in which coefficient A_{sn}^{ij} gives tangential stress within the i^{th} element caused by constant unit normal load P_n^j applied within the j^{th} section of the boundary. The rest of the influence coefficients have the same meaning.

Values of the influence coefficients were determined according to the formulas:

$$B_{ss}^{ij} = \frac{1}{2G}[(3-4\nu)\cos\gamma F_1 - \bar{y}(\sin\gamma F_2 - \cos\gamma F_3)],$$
$$B_{sn}^{ij} = \frac{1}{2G}[(3-4\nu)\sin\gamma F_1 - \bar{y}(\cos\gamma F_2 + \sin\gamma F_3)],$$
$$B_{ns}^{ij} = \frac{1}{2G}[-(3-4\nu)\sin\gamma F_1 - \bar{y}(\cos\gamma F_2 + \sin\gamma F_3)],$$
$$B_{nn}^{ij} = \frac{1}{2G}[(3-4\nu)\cos\gamma F_1 + \bar{y}(\sin\gamma F_2 - \cos\gamma F_3)]. \tag{6}$$

$$A_{ss}^{ij} = -2(1-\nu)(\sin 2\gamma F_2 - \cos 2\gamma F_3) - \bar{y}(\sin 2\gamma F_4 + \cos 2\gamma F_5),$$
$$A_{sn}^{ij} = (1-2\nu)(\cos 2\gamma F_2 + \sin 2\gamma F_3) - \bar{y}(\cos 2\gamma F_4 - \sin 2\gamma F_5),$$
$$A_{ns}^{ij} = F_2 - 2(1-\nu)(\cos 2\gamma F_2 + \sin 2\gamma F_3) - \bar{y}(\cos 2\gamma F_4 - \sin 2\gamma F_5),$$
$$A_{nn}^{ij} = F_3 - (1-2\nu)(\sin 2\gamma F_2 - \cos 2\gamma F_3) + \bar{y}(\sin 2\gamma F_4 + \cos 2\gamma F_5). \tag{7}$$

In formulas (6) and (7), G is shear modulus, ν is Poisson's ratio, $(\bar{x};\bar{y})$ are local coordinates of point $(x^i; y^i)$ relative to the center of the j^{th} element of length $2a^j$.

$$F_1(\bar{x};\bar{y}) = f(\bar{x};\bar{y}), \quad f(\bar{x};\bar{y}) = -\frac{1}{4\pi(1-\nu)}\left[\bar{y}\left(\operatorname{arctg}\frac{\bar{y}}{\bar{x}-a^j} - \operatorname{arctg}\frac{\bar{y}}{\bar{x}+a^j}\right) - \right.$$
$$\left. -(\bar{x}-a^j)\ln\sqrt{(\bar{x}-a^j)^2+\bar{y}^2} + (\bar{x}+a^j)\ln\sqrt{(\bar{x}+a^j)^2+\bar{y}^2}\right],$$

$$F_2(\bar{x};\bar{y}) = \frac{\partial f}{\partial \bar{x}}, \quad F_3(\bar{x};\bar{y}) = \frac{\partial f}{\partial \bar{y}}, \quad F_4(\bar{x};\bar{y}) = \frac{\partial^2 f}{\partial \bar{x}\partial \bar{y}}, \quad F_5(\bar{x};\bar{y}) = \frac{\partial^2 f}{\partial \bar{x}^2},$$

$\gamma = \beta^i - \beta^j$ is inclination angle of the i^{th} element relative to the j^{th} element,

$$\bar{x} = (x^i - x^j)\cos\beta^j + (y^i - y^j)\sin\beta^j, \quad \bar{y} = -(x^i - x^j)\sin\beta^j + (y^i - y^j)\cos\beta^j.$$

Diagonal elements of the matrix of influence coefficients, characterizing the effect of fictitious loads P_s^i and P_n^i of the i^{th} element on the displacement of the i^{th} element itself, were specified by the expression:

$$B_{sn}^{ii} = B_{ns}^{ii} = 0, \; B_{ss}^{ii} = B_{nn}^{ii} = -\frac{3-4\nu}{4\pi G(1-\nu)}a^i \ln a^i, \tag{8}$$

$$A_{sn}^{ii} = A_{ns}^{ii} = 0, \quad A_{ss}^{ii} = A_{nn}^{ii} = \mp\frac{1}{2}, \quad A_{ts}^{ii} = 0, \quad A_{tn}^{ii} = \frac{1}{2}\frac{\nu}{1-\nu}, \quad \bar{y} = 0_{\pm}. \tag{9}$$

The obtained system of equations of order $2N$ with unknown components of fictitious loads was as follows:

$$\left.\begin{aligned} b_s^i &= \sum_{j=1}^{N} C_{ss}^{ij}P_s^j + \sum_{j=1}^{N} C_{sn}^{ij}P_n^j \\ b_n^i &= \sum_{j=1}^{N} C_{ns}^{ij}P_s^j + \sum_{j=1}^{N} C_{nn}^{ij}P_n^j \end{aligned}\right\} \quad i = \overline{1,N}, \tag{10}$$

where $C_{ss}^{ij}, C_{sn}^{ij}, C_{ns}^{ij}, C_{nn}^{ij}$ are influence coefficients, which, depending on the conditions specified at the boundaries, were determined using formulas (6) and (8), or formulas (7) and (9). The left sides of system (10) were written using conditions (1)—(3). Displacements and stresses at the chamber boundary and at an arbitrary internal point of the mass were determined through the identified fictitious loads P_n^i and P_s^i.

To assess stability, a criterion of the Coulomb-Mohr type was used, according to which the "equivalent" stresses σ_e within the mine working contour, as well as in the ore mass and filling, should not exceed the maximum permissible values, determined according to [12]:

$$\max\sigma_e = \frac{1}{2\psi}\left\{\sqrt{(1-\psi)^2(\sigma_{xx}+\sigma_{yy})^2 + 4\psi[(\sigma_{xx}-\sigma_{yy})^2 + 4\tau_{xy}^2]} - (1-\psi)(\sigma_{xx}-\sigma_{yy})\right\} \le \sigma_c , \quad (11)$$

where $\psi = \sigma_t / \sigma_c$ ($\sigma_t - \sigma_{tensil}$, $\sigma_c - \sigma_{compression}$).

Formation of stress fields around mine workings located at the depth of $740-1040\,\text{m}$ was considered. Following geometrical parameters of the stope were taken: chamber height is $h=100\,\text{m}$, chamber width is $a=30\,\text{m}$, $h_1=30\,\text{m}$, $h_2=45\,\text{m}$, $h_3=25\,\text{m}$ and $h_4=7\,\text{m}$.

Elasticity modulus of ore is $E_1 = 0,6\cdot10^4$ MPa; ultimate compressive strength is $\sigma_c = 100$ MPa; ultimate tensile strength is $\sigma_t = 0,1\sigma_c$; Poison's ratio is $\nu_1 = 0,15$ and bulk weight is $\gamma = 1,9\ \text{t}/\text{m}^3$. Physical and mechanical properties of the filling material are as follows: $E_1 = 0,1\cdot10^4$ MPa, $\nu_2 = 0,12$, ultimate compressive strength for the depth of $740\,\text{m}$ was $(\sigma_c)_1 = 45$ MPa.

Fig. 2 shows isolines of the "equivalent" stresses σ_e around the chamber obtained for the effecting load $(\sigma_{yy})_0 = -\gamma H = -14,06$ MPa, $(\sigma_{xx})_0 = -\lambda\gamma H = -7,03$ MPa, which corresponds to the mining depth of $740\,\text{m}$ and lateral pressure ratio of $\lambda = 0,5$.

The calculations have shown that at the analyzed depth, the irregular-shaped chamber preserves its stability remaining in its elastic state. First area of concentration of "equivalent" stresses $(k_\sigma = 0,65)$ is within the lateral part of the chamber BC (Fig. 1) forming a sharp angle with the ceiling *CD*.

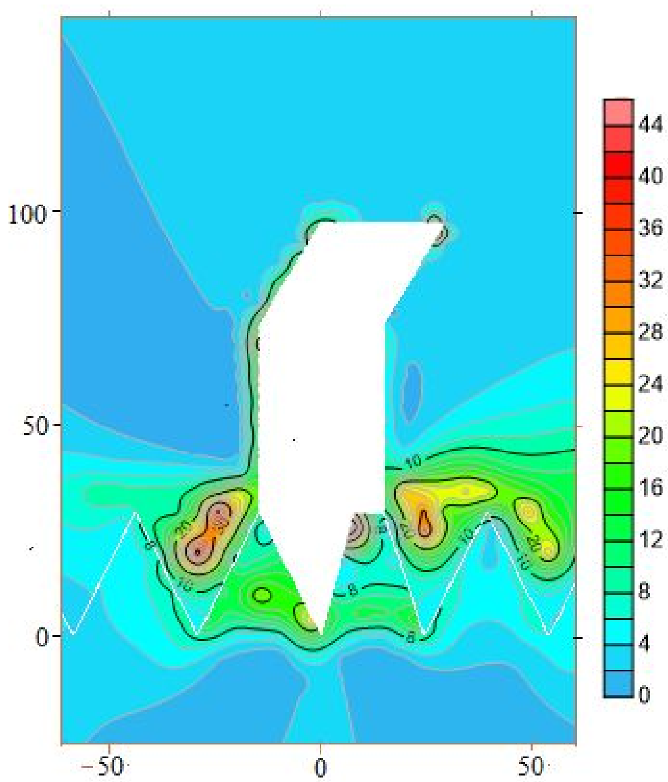

Fig. 2. Isolines of "equivalent" stresses σ_e around high chambers, MPa

Within the boundary element *NC* adjoining the ceiling, the stresses are equal to 29,20 MPa; within the ceiling element *CT* the value is 17,64 MPa $(k_\sigma = 0,39)$.

Vertically upward towards the filling mass, stresses attenuate rapidly, gaining the value of 3,16 MPa at the distance of $5\,\text{m}$ from the contour.

In the chamber's lateral contour, being convex to the filling massive, within its vertical part, the stresses experience slight changes ranging from 18.98 to 22,49 MPa. Within the inclined part of the bottom located in the ore mass, the concentration coefficient is $k_\sigma = 0,49$, which is a favourable condition for mining the underlying level ore.

When modelling the stress-strain state, it is of significant interest to consider the development of stress fields within the filling mass and determine the degree of their influence on the stability of field mine workings. Fig. 2 demonstrates that the second and third areas of stress concentration are located within the filling on

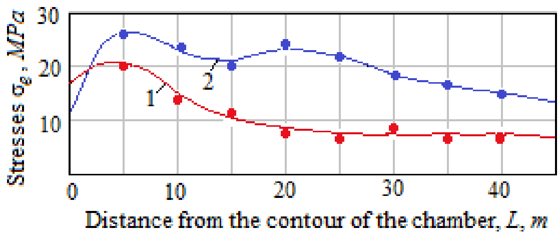

Fig. 3. Changes in stress σ_e along with distancing from the side contours of a chamber, MPa: 1 — from the contour *EF* into the filling mass; 2 — from the contour *AB* into the mined out area

both sides of the chamber. The length of these areas is 13,5 m and 2,8 m; the concentration coefficient of "equivalent" stresses in them reaches the values of 0,94 and 0,75.

The pattern of changes in the "equivalent" stresses along with distancing from the lower part of the lateral boundaries *EF* (1) and *AB* (2) of the chamber into the filling mass is shown in Fig. 3. At 0,35 h height from the peak of the chamber bottom, with the increasing distance *L* from the chamber contour, the "equivalent" stresses increase to 21,9 MPa and 27,2 MPa respectively with further monotonic attenuation.

The empirical dependences of "equivalent" stresses on distance L to the chamber contour obtained for the cases 1 and 2 using the least square method are as follows:

$$\sigma_e = 19,0151 + 2,1596L - 0,4245L^2 + 0,0237L^3 - 0,0005L^4, \text{ MPa;}$$

$$\sigma_e = 11,6982 + 8,6149L - 1,6555L^2 + 0,1378L^3 - 0,00056L^4 + 0,0001L^5, \text{ MPa.}$$

Correlation coefficient in both cases is $0,9989$.

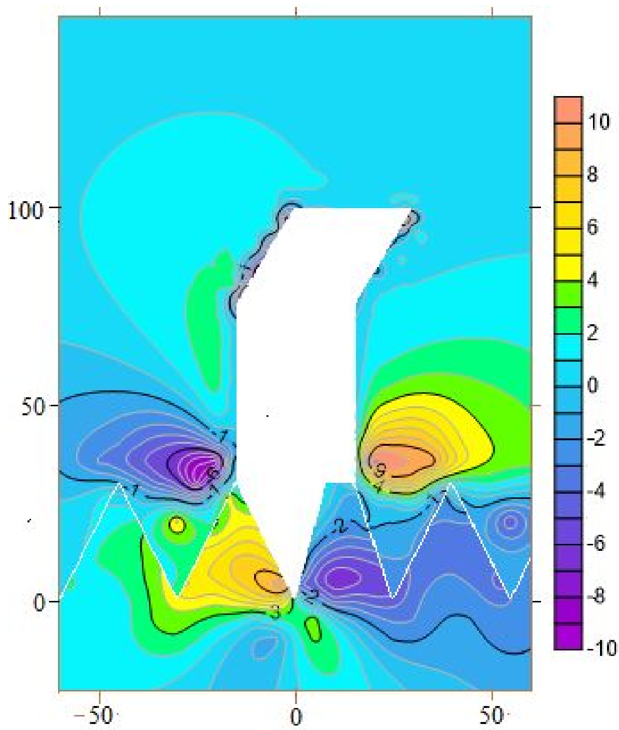

Fig. 4. Isolines of stresses τ_{xy}, MPa

In the areas of concentration of "equivalent" stresses, tangential stresses τ_{xy} reach following values. Within the first area in the boundary element *NC* adjoining the ceiling, the value is $-12,88$ MPa; within the second and third areas, the highest stress concentrations are observed at the distances of 10,2 m and 5,8 m from the mine working contour, having the values of $-9,53$ MPa and $+11,50$ MPa respectively (Fig. 4).

The numerical experiment showed that along with the increasing development depth, to ensure stability of the chambers and create conditions for safe mining operations at the depths of 840 m, 940 m and 1040 m, it is required to use filling materials with ultimate tensile strengths of 50 MPa, 55 MPa and 60 MPa, respectively. In this case, the "equivalent" stresses both within the chamber contour and at some distances in the ore and filling mass do not exceed the maximum permissible ones according to criterion (11).

Conclusions

An algorithm for studying the stress-strain state of an inhomogeneous mass, developed on the basis of the boundary element method, was applied to simulate the processes occurring in ore and filling masses during the mining of high irregular-shaped chambers.

Regularities of the development of stress fields around a chamber, surrounded by filling mass, were studied; the main zones of stress concentration were identified. Polynomial dependences of changes in the magnitude of “equivalent” stresses on the distance to the lateral contours of chamber convex into the mass and into the mined-out space were obtained. Studies were carried out to analyze the stress-strain state of the ore mass and filling at the depths of 740—1040 m. The research made it possible:

– to predict contour stability of a high stope surrounded by the filling mass;
– to formulate recommendations for strengthening the filling material that ensures this stability.

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