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MATHEMATICAL MODEL OF THE TUNNEL LINER FLOATING SYSTEM

МАТЕМАТИЧНА МОДЕЛЬ СИСТЕМИ СПИРАННЯ УСТАНОВКИ ОБКЛАДАННЯ ТУНЕЛЮ ПЛИТАМИ

The paper develops a method for determining the distribution of stresses and deformations of a rail track as a two-layer structure on a semi-infinite height soil base.
***Keywords**: rail track; loads; deformations; stresses.*

Розвиток великих міст при щільній міській забудові неможливий без освоєння підземного простору. Таке освоєння здебільшого пов'язано зі спорудженням тунелів. Їх будівництво поблизу наземних споруд спричиняє додаткові деформації ґрунту під останніми. Прогнозуван-

ня рівня таких деформацій — актуальна задача безпеки використання споруд за будівництва поблизу підземного тунелю. Метою статті є формулювання аналітичного методу визначення прогину рейкової колії та її основи під тиском тунельного укладача плит, що використовується під час спорудження тунелю. Задля реалізації поставленої мети запропоновано аналітичний спосіб визначення напружено-деформованого стану (НДС) ґрунтової основи напівбезмежної висоти як об'єктивно пружного тіла. З використанням методів класичної лінійної теорії пружності, а саме функцій напружень Ері, сформульована комплексна математична модель взаємодії двошарової рейкової колії та ґрунтової основи з урахуванням статичного тиску на колію скатів установки обкладання тунелю плитами. Аналітично розв'язана комплексна математична модель. Наведені вирази показників НДС для загального випадку розподілу механічних властивостей та параметрів елементів системи — «рейка - підрейкова плита - ґрунтова основа» за довільно прикладеного зосередженого тиску з боку установки обкладання тунелю плитами. Аналітичне кількісне визначення деформацій рейки як двошарової конструкції на напівбезмежній по висоті ґрунтовій основі дозволяє визначити напружено-деформований стан (НДС) напівбезмежної по висоті ґрунтової основи в районі рейкової колії укладача плит тунелю. Урахування кількісних значень просідань ґрунтової основи в районі колії в процесі проектування будівництва підземного тунелю дозволяє підвищити рівень достовірності отриманих результатів та обґрунтованості прийнятих при проектуванні тунелю інженерних рішень спрямованих на забезпечення достатнього рівня надійності та безпеки спорудження підземного тунелю. В подальшому доцільно розробити метод розрахунку за навантаження рейки сумісно нормальним та дотичним зусиллям.

***Ключові слова**: тунель; навантаження рейкової колії; напружено-деформований стан ґрунтової основи; функція напружень Ері.*

Problem's Formulation

The development of large cities with dense urban development and an acute shortage of free areas for construction is impossible without the development of underground space. Mining is also associated with the further development of underground space. Such development is mostly associated with the construction of tunnels.

Analysis of recent research and publications

Constructing tunnels within urban areas [8] often introduces several engineering challenges, including the potential lowering of existing buildings' basement levels and changes in the stress-strain state (SSS) of adjacent structures. The vaults of tunnels are lined with slabs during construction using a tunnel paver [11]. The paver in the tunnel moves along rails. Vehicles that remove rock mass from the space of the tunnel being formed move along the rails. The rails, through reinforced concrete slabs, rest on the soil. The vault slabs also rest on it. The movement of the paver is accompanied by deformation of the soil base, which accordingly affects both the vaults and the soil under ground structures. Soil deformations depend on the pressure of the paver, vehicles on the rails, their mechanical interaction with the concrete slabs under them and their soil base. As a result, the level of soil deformation can only be determined by considering the rail, which rests on the soil via concrete slabs, as a unified mechanical system. In the article [10], the feasibility and substantiated possibility of eliminating the deformed state of buildings and structures by controlling the stiffness of their foundations — soils are shown. In the study [5], using the mechanism of load formation on underground structures, the influence of soil properties on the formation of rock pressure on the underground structure and the formation of maximum bending moments in structures is substantiated. In the article [7], the bending of an elastic rectangular three-layer plate on an elastic Winkler base is considered. In the study [4], using the Winkler model, a model of an elastic base is proposed taking into account the friction between the plate and the base. In the work [12], an analytical-numerical approach to solving the problem of bending a rectangular plate on an elastic Winkler-Pasternak base is considered. Thesis [1] is devoted to the study of the SSS of the sub-rail base with intermediate rail fasteners taking into account the forces of interaction of the rolling stock and the railway track. The issue of soil deformation was not considered. In the article [6] it is proposed to consider the rail as a beam loaded with a distributed load that provides equivalent deflection when supported on two supports. In [9] only the stress on the surfaces of discrete sleepers and soil is taken into account.

Formulation of the study purpose

Analysis indicates that current mathematical models for building foundations do not fully account for the soil base as an elastic semi-infinite in height material body, but is simulated by the Winkler-Pasternak model. In this model, the base is modeled by a system of unrelated springs. Their compression stiffness is determined by the bed coefficient. The structure on such a base is modeled by a beam or plate and does not allow taking into account the real values of the mechanical parameters of the rail on the sub-rail plate. In general, in solving the problems of determining the SSS of a rail of limited length, which through the sub-rail plate rests on the soil base, there is no approach to modeling a two-layer track built on the soil base, as a single system that takes into account the external influence on the rail track, semi-infinite in height soil base and the mechanical interaction of the track layers and the soil base, their mechanical properties. The use of numerical methods for solving a problem involving a semi-infinite height of the soil base does not allow obtaining analytical expressions for the SSS indices of the components of the specified system based on their known parameters and mechanical properties. The absence of analytical expressions for determining the specified indices as functions of a significant number of factors does not allow predicting the results of implementing the proposed technical solutions with a high level of reliability. The latter complicates multivariate design aimed at ensuring a sufficient level of reliability and safety of operation of the tunnel slab lining installation. In order to eliminate the specified shortcomings, the following tasks must be solved. 1. To propose an analytical method for determining the SSS of a semi-infinite height soil base as an objectively elastic body. 2. To formulate a complex mathematical model of the interaction of a double-layer rail track and a soil base of semi-infinite height, taking into account the static pressure on the track of the tunnel lining installation. 3. To construct an analytical solution of the formulated complex mathematical model with the definition of each of the indicators of the SST for the general case of the distribution of mechanical properties of all elements of the system — "rail - sub-rail plate - soil base", the heights of the rail and sub-rail plate under an arbitrarily applied concentrated pressure of a unit value from the side of the tunnel lining installation.

Presenting main material

The components of the complex system "rail - sub - rail plate - soil base", its layers, from bottom to top, will be given numbers from zero to two. The numbers of the layers of the system k will be entered in the indices of the quantities that concern them. We will solve the problem within the limits of static, flat, linearly elastic deformation in the *xz* plane. We will direct the x axis along the track. We will place the reference point of the *xz* coordinate system in the middle of the track section on the soil surface. We will combine it with the vertical axis of their symmetry of the foundation and base. We will denote the z coordinate of the rail surface more distant from the base as Z_2, the less distant one as Z_1. The SSS of the system elements will be given by the functions of the Erie stresses of the linear theory of elasticity. This approach was used in models of layered composite materials [2,3]. Its application will provide the possibility of implementing a comprehensive approach to determining the SSS of all components of the layered system "rail - sub - rail plate - soil base" since the SSS indicators (stresses and displacements) of the system components are determined by the same type of dependencies only with different coefficients. The solution of the model will consist in determining these coefficients given by the Erie stress functions. The SSS indicators of the components of the layered system for the given functions have the form.

$$u_x^{(k)} = -\frac{\mu_k+1}{E_k}\frac{\partial^2\varphi^{(k)}}{\partial x\partial z},\quad u_z^{(k)} = \frac{\mu_k+1}{E_k}\left(2(1-\mu_k)\nabla^2-\frac{\partial^2}{\partial z^2}\right)\varphi^{(k)},\quad \sigma_x^{(k)} = \frac{\partial}{\partial z}\left(\mu_k\nabla^2-\frac{\partial^2}{\partial x^2}\right)\varphi^{(k)};$$

$$\sigma_z^{(k)} = \frac{\partial}{\partial z}\left((2-\mu_k)\nabla^2-\frac{\partial^2}{\partial z^2}\right)\varphi^{(k)},\ \tau^{(k)} = \frac{\partial}{\partial x}\left((1-\mu_k)\nabla^2-\frac{\partial^2}{\partial z^2}\right)\varphi^{(k)}, \tag{1}$$

where E_k, μ_k — modulus of elasticity and Poisson's ratio of the material of the k — layer; $\varphi^{(k)}$ — stress functions;

$$\nabla^2(\varphi)=\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}.$$

Additionally, we note that the stress functions can be chosen arbitrarily, but they must satisfy the condition.

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}\right)=0,$$

that is, to be biharmonic. Let us take into account the above condition. Let us assume the following forms of the Erie stress functions for all components of the complex system.

$$\varphi^{(k)}=\sum_{m=1}^{\infty}\left(A_{k,m}e^{\rho_m z}+B_{k,m}e^{-\rho_m z}+C_{k,m}z\ e^{\rho_m z}+D_{k,m}z\ e^{-\rho_m z}\right)\cos(\rho_m x), \tag{2}$$

where $A_{k,m}$, $B_{k,m}$, $C_{k,m}$, $D_{k,m}$ — unknown coefficient vectors; $\rho_m=\pi\frac{2m-1}{2a}$; $2a$ — the length of the rail section.

Substitute function (2) into the expressions for stresses and displacements (1). We obtain.

$$u_z^{(k)}=-\frac{1+\mu_k}{E_k}\sum_{m=1}^{\infty}\left(\begin{array}{c}\left(A_m^{(k)}e^{\rho_m z}+B_m^{(k)}e^{-\rho_m z}\right)\rho_m+C_m^{(k)}e^{\rho_m z}\times\\ \times(4\mu_k+\rho_m z-2)+D_m^{(k)}e^{-\rho_m z}(2+\rho_m z-4\mu_k)\end{array}\right)\rho_m\cos(\rho_m x);$$

$$u_x^{(k)}=\frac{1+\mu_k}{E_k}\sum_{m=1}^{\infty}\left(\begin{array}{c}\left(A_m^{(k)}e^{\rho_m z}-B_m^{(k)}e^{-\rho_m z}\right)\rho_m+C_m^{(k)}e^{\rho_m z}(\rho_m z+1)+\\ +D_m^{(k)}e^{-\rho_m z}(1-\rho_m z)\end{array}\right)\rho_m\sin(\rho_m x);$$

$$Z_z^{(k)}=\sum_{m=1}^{\infty}\left(\begin{array}{c}\left(-A_m^{(k)}e^{\rho_m z}+B_m^{(k)}e^{-\rho_m z}\right)\rho_m-C_m^{(k)}e^{\rho_m z}\times\\ \times(2\mu_k-1+\rho_m z)-D_m^{(k)}e^{-\rho_m z}(2\mu_k-1-\rho_m z)\end{array}\right)\rho_m^2\cos(\rho_m x);$$

$$Z_x^{(k)}=\sum_{m=1}^{\infty}\left(\begin{array}{c}\left(A_m^{(k)}e^{\rho_m z}+B_m^{(k)}e^{-\rho_m z}\right)\rho_m+C_m^{(k)}e^{\rho_m z}\times\\ \times(\rho_m z+2\mu_k)+D_m^{(k)}e^{-\rho_m z}(\rho_m z-2\mu_k)\end{array}\right)\rho_m^2\sin(\rho_m x);$$

$$X_x^{(k)}=\sum_{m=1}^{\infty}\left(\begin{array}{c}\left(-A_m^{(k)}e^{\rho_m z}+B_m^{(k)}e^{-\rho_m z}\right)\rho_m-C_m^{(k)}e^{\rho_m z}\times\\ \times(3+\rho_m z-2\mu_k)-D_m^{(k)}e^{-\rho_m z}(2\mu_k-3+\rho_m z)\end{array}\right)\rho_m^2\cos(\rho_m x). \tag{3}$$

According to the task, press the supporting slopes of the tunnel lining installation with slabs onto the rail. Let's consider the most dangerous case — the installation is located in the middle of the rail. The unit load on the rail is given by Fourier expansions.

$$P=\sum_{m=1}^{\infty}\square P_m\cos(\rho_m x),$$

where $\square P_m=\frac{1}{2,4a}\cos(\rho_m b)$; b — half the distance between the axles of the support wheels of the tunnel lining machine.

Within the limits of the problem, no tangential load is applied to the rail. Two conditions — a given normal load and the absence of tangential loads on the working surface of the rail allow us to formulate two ratios of the vectors of unknown coefficients of the Erie stress function (2), and (3), respectively.

$$A_{2,m}=-\frac{\square P_m e^{\rho_m Z_2}}{2\rho_m^3}-B_{2,m}\frac{2\mu_2-0,5-\rho_m Z_2}{\rho_m}+D_{2,m}\frac{e^{2\rho_m Z_2}}{2\rho_m}; \tag{4}$$

$$B_{2,m} = \frac{\square P_m e^{\rho_m Z_2}}{2\rho_m^3} - B_{2,m}\frac{e^{-2\rho_m Z_2}}{2\rho_m} + D_{2,m}\frac{2\mu_2 - 0{,}5 + \rho_m Z_2}{\rho_m}. \quad (5)$$

Let us take into account the influence of the infinite growth of the coordinate z in the negative direction on the soil base. With the infinite growth of the coordinate in the soil base, an infinite growth of neither displacements nor stresses is possible. To take into account such deformation of the base, we will assume the values of the two vectors of the function (2) to be zero.

$$A_{0,m} = C_{0,m} = 0. \quad (6)$$

Acceptance (6), taking into account the assumption of stress functions (2), corresponds to the solution of the first of the above problems regarding the formulation of the model of the soil base as an elastic body of semi-infinite height.

Within the framework of the formulation of the model of the system "rail - sub - rail plate - soil base", it is necessary to take into account the nature of the interaction of the system elements. At the boundaries of the interaction of its components, the conditions of equality of displacements and forces of interaction of layers must be met.

Namely if

$$z = Z_k \; u_x^{(k)} = u_x^{(k+1)}, \; u_z^{(k)} = u_z^{(k+1)}, \; \sigma_z^{(k)} = \sigma_z^{(k+1)}, \; \tau^{(k)} = \tau^{(k+1)}. \quad (7)$$

The last dependence, expressions (4—6), the adopted stress functions (3) constitute a flat, linear mathematical model of the SSS installed on a semi-infinite in height soil base of a layered rail track, taking into account the pressure of the tunnel lining installation with slabs, corresponds to the solution of the second of the tasks set. The formulated mathematical model is based on the adopted stress functions (2). The latter have the form of sums. The selected structures of the terms for all components are similar. They depend on the value of *m* — the number of the sum term and the layer number. Conditions (7), expressions (4—6) allow for each value of the value m of the function (2) and expressions (3) to form a system of linear algebraic equations of the eighth order. The solutions of the formed systems allow us to determine the elements of the vectors of coefficients of the function (2). According to expressions (3), the m-th terms of the SSS indicators of all components of the complex system "rail - sub - rail slab - soil base". By adding the terms, find the SSS of the components of the complex system "rail - sub - rail plate - soil base" — analytically solve the formulated model in general form, which is the desired result of the last (third) problem set. Let us demonstrate the result of solving the problems set. Let us consider a special case of the support system of the tunnel lining installation with slabs. A rail 12 m long with a rectangular cross-section and a height of 50 mm rests on a rectangular plate 150 mm thick. The rail is loaded by two equal unit forces applied symmetrically to its middle. The elastic modulus of the rail material E=10^5MPa, Poisson's ratio μ=0.25. The corresponding indicators of the plate material and soil are 10^3MPa and 0.1MPa, Poisson's ratios 0.2 and 0.3. According to the above sequence, the SSS indicators of the tunnel lining installation support system were determined.

The pressure distribution is shown in Fig. 1.

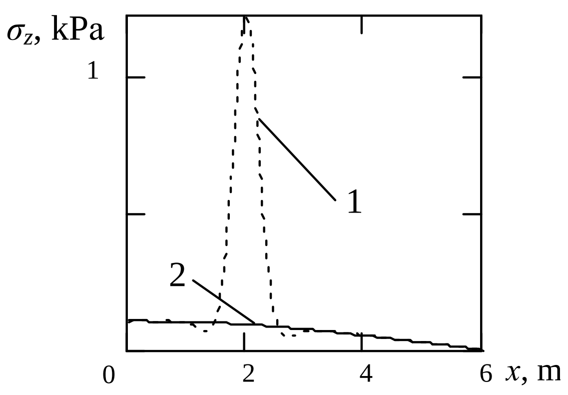

Fig. 1. Distribution of normal stresses *Zz* along the x axis in the foundation and base on the track section *0≤x≤a*: 1 — surface of interaction of the plate and rail, 2 — soil surface

According to the graph, the nature of curve 1 practically corresponds to the external concentrated load reduced by three orders of magnitude, and the pressure on the soil decreases, its extreme value is two orders of magnitude less than the extreme pressure of the rail on the

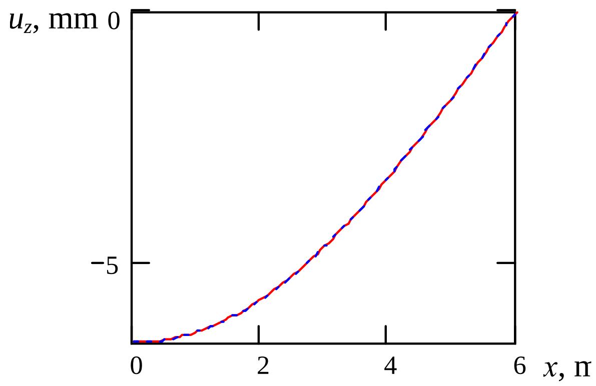

Fig. 2. Movement u_z of the surfaces of the paver support elements on the ground on half of the rail

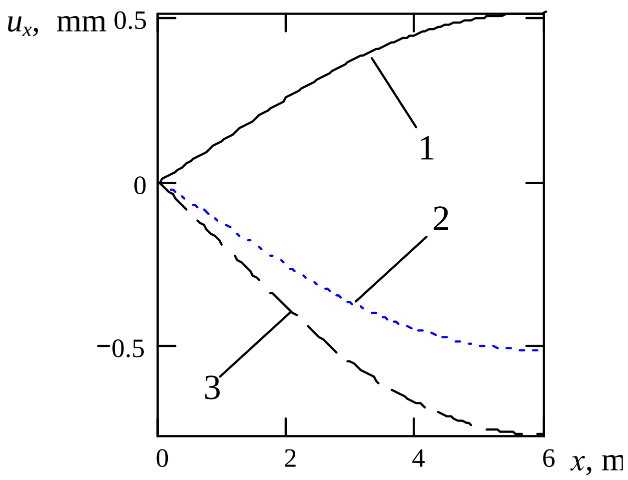

Fig. 3. Displacement *ux* of the surfaces of the foundation and elastic base at the interval $0 \le x \le a$: 1 — loaded surface of the rail, 2 — surface of the rail and slab interaction, 3 — soil surface

slab. Thus, the graph shows the redistribution between the components of the complex system "rail - sub - rail slab - soil base". It is clear that changes in the height of the rail, sub-rail slab, as well as the mechanical characteristics of the materials from which they are made, as well as the mechanical properties of the soil base will be accompanied by changes in the graphs given. The pressure of the paver on the rail leads to the deflection of the entire system of its support (Fig. 2).

According to Fig. 2, the vertical displacements of the surfaces of the rail, plate and soil practically coincide. Their surfaces bend almost equally and acquire similar parabolic shapes. As a result of the bending of the complex system "rail - sub - rail plate - soil base", the movement of the opposite surfaces of the specified paver support system has the opposite direction (Fig. 3).

According to the given graphs, the elongations of the opposite surfaces of the components of the paver support system are not symmetrical. Installing the paver track rails with gaps, on the one hand, minimizes the effect of the deformation of the rail under the paver on other rails, on the other hand, they allow free movement of the ends of the rail and the sub-rail plate. The latter allows us to state that the obtained solution is acceptable, and the limited horizontal movement of the soil surface points in the $x=\pm a$ sections leads to larger deformations. That is, the inaccuracy of the results will go into the margin of safety. We conditionally assumed the paver support base to be 4 m. The analysis performed showed that increasing the distance between the paver axes from 4 m to 6 m leads to a decrease in the deflection of the soil base by 20 %. Such a change reduces the difference in the elongations of the rail and soil surfaces by 10 %.

Conclusions

In the work, a complex mathematical model of the interaction of a two-layer rail track and a semi-infinite height soil base is formulated using the methods of the linear theory of elasticity, taking into account the static pressure on the rail of the tunnel lining installation. At the same time, the indicators of the stress-strain states of all components of the model are given using similar Erie stress functions. The model takes into account the load on the upper part of the rail, the conditions of joint deformation of all components of the support system of the tunnel lining installation and the objective impossibility of unlimited growth of either displacements or stresses of the soil base under the condition of unlimited growth of the absolute negative value of the coordinate *z*. The representation of the SSS indices of all components of the complex model using similar Erie stress functions provided the possibility of analytical solution of the formulated complex mathematical model with the determination of each of the SSS indices for the general case of the distribution of mechanical properties of all elements of the system "rail - sub - rail plate - soil base", the heights of the rail and sub - rail plate under an arbitrarily applied concentrated pressure of a unit value from the side of the tunnel slab laying installation. It is established. The surfaces of the rail, the slab under it and the soil base bend almost equally and take the shape of a parabola. Increasing the distance between the axes of the paver from 4m to 6m leads to a decrease in the deflection of the soil base by 20 % and a decrease in the difference in elongations of the rail and soil surfaces by 10 %. The work was carried out on the initiative of the authors. Its results are based on the methods of the linear theory of elasticity, the mathematical model was built without accepting additional hypotheses, and was solved analytically. Its results, within the limits of static plane linear deformation, considering the soil as an isotropic, not a dispersed medium, can be considered sufficiently reliable and can be used during the development of tunnel construction projects. Analytical calculation of the system with its selected parameters allows, in the process of tunnel design, to make justified engineering decisions aimed at ensuring a sufficient level of reliability and safety of tunnel operation. It can be recommended for use in practice, in particular when choosing the optimal option based on the results of calculations of several options. In the future, it is advisable to develop an algorithm to take into account not only normal, but also tangential load of the rail from the side of the tunnel lining installation with slabs.

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