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# FULFILLMENT OF THE ASSOCIATIVE PROPERTY IN MULTIPLICATION AND THE DUALITY OF ZERO IN THE SET OF VOLUMETRIC NUMBERS

# ВИКОНАННЯ ВЛАСТИВОСТІ АСОЦІАТИВНОСТІ ПРИ МНОЖЕННІ ТА ДУАЛЬНІСТЬ НУЛЯ У МНОЖИНІ ОБ'ЄМНИХ ЧИСЕЛ

*This paper theoretically proves that for the set of volumetric numbers, which are located in a plane passing through the real axis and positioned at an arbitrary angle relative to the complex plane, the associative property holds for multiplication. It is established that any mathematical operations involving the number zero, as an element of the set of volumetric numbers, are valid. Additionally, a «paradox» of the density of the numerical space of volumetric numbers has been discovered, as the cardinality of a finite numerical set is equal to the cardinality of an infinite numerical set.*
***Keywords**: volumetric numbers; associative property; set; zero; coefficient; plane.*

*Теорія об'ємних чисел не тільки істотно розширює можливості, а й спрощує математичне моделювання різних об'єктів навколишнього світу, особливо які характеризуються суперечливими чи двоїстими властивостями, що дозволило розробити інноваційну фізико-математичну модель об'ємного Всесвіту.*

*У роботі проведено розширений аналіз умови виконання асоціативності при множенні об'ємних чисел, а також визначення унікальних властивостей нуля, як числа, що є єдиним елементом, який належить множині дійсних, комплексних та об'ємних чисел одночасно.*

*Властивість асоціативності при множенні об'ємних чисел виконується за умови, що добуток протилежних дійсних коефіцієнтів при уявній та просторово-невизначеній одиниці першого та третього множника рівні. Відповідний добуток являє собою площу деяких прямокутників, з розташуванням дійсних коефіцієнтів уявної та просторово-невизначеної одиниці на гіперболічній кривій. Використовуючи властивості гіперболічної кривої, а також умову, що відношення дійсних коефіцієнтів просторово-невизначеній одиниці до уявної дорівнює тангенсу кута нахилу, було доведено, що асоціативна властивість при множенні виконується для множини об'ємних чисел, які розташовуються в площині, що проходить через дійсну вісь під довільним кутом по відношенню до комплексної площини.*

*На основі властивості, згідно якої, завжди знайдуться такі два довільних об'ємних числа для яких виконується асоціативна властивість при множені розташовані по різні сторони відносно об'ємного числа розташованого на рівнобічній гіперболі, було встановлено, що будь-якому числу нескінченного відрізка відповідає число кінцевого відрізка, тобто має місце взаємно однозначне відображення. Отже, потужність кінцевої числової множини дорівнює потужності нескінченної числової множини, що зумовлює «еквівалентність» нуля нескінченності або парадокс нескінченності та невизначеність нуля.*

*Дії з нулем у множині об'ємних чисел, з урахуванням що нуль є елементом тільки множини об'ємних чисел, показали унікальні властивості нуля (добуток нуля самого на себе дорів-*

*нює одиниці), які можуть бути використані при математичному моделюванні суперечливих або двоїстих аспектів навколишнього світу.*

*Багатоваріантність результатів дії з нулем обумовлена дуальністю нуля та невиконанням умови асоціативності при множенні в даному випадку, так як умовна та просторово-невизначена одиниці розташовані в різних площинах.*

***Ключові слова:*** *об'ємні числа; властивість асоціативності; множина; нуль; коефіцієнт; площина.*

## Problem's Formulation

The theory of volumetric numbers, which is still in its formative stage, not only significantly expands possibilities but also simplifies mathematical modeling of various objects in the surrounding world, especially those characterized by contradictory or even mutually exclusive properties. When studying such objects, it is often necessary to operate with infinitely large or infinitely small quantities, as well as with zero, which introduces an element of uncertainty in the interpretation of final results. Therefore, a deeper investigation of the properties of elements in the set of volumetric numbers for their application in mathematical modeling is of significant interest.

## Analysis of recent research and publications

The expansion of the numerical space by transitioning from a two-dimensional numerical field to a three-dimensional numerical field was achieved through the introduction of the concept of a spatially indeterminate unit $j, (j \notin \{C\})$ [1,2]. According to studies [1,2], the algebraic formula for a volumetric number is expressed as:

$$V = a + bi + cj \,,$$

where: $a, b, c$ — are real numbers; $i$ — is the imaginary unit; $j$ — is the spatially indeterminate unit.

In trigonometric form:

$$V = \rho(\sin\theta\cos\varphi + i\sin\theta\sin\varphi + j\cos\theta),$$

where: $\rho$ — is the length of the radius vector of the corresponding point; $\varphi$ — is the longitude; $\theta$ — is the polar distance.

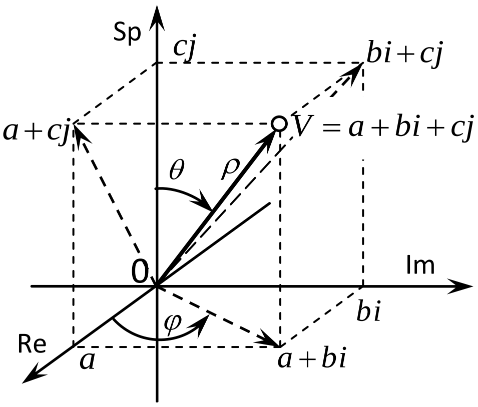


*Fig. 1.* Geometric interpretation of a volumetric number: Re — real axis, Im — imaginary axis, Sp — spatial axis

Thus, volumetric numbers can be represented both as points in space and as vectors (Fig. 1).

The algebraic and trigonometric formulas for representing volumetric numbers allow for operations such as addition, subtraction, multiplication, and division. Additionally, volumetric numbers exhibit all the properties of addition and multiplication except for the associative property in multiplication, which, however, holds under certain conditions $b_1 \times c_3 = b_3 \times c_1$, [2]. A detailed analysis of the condition for the fulfillment of associativity in multiplication is not conducted in this study; instead, the property of reciprocal mirror reflection is examined in greater depth.

In [3], the trigonometric formula for representing volumetric numbers enabled the establishment of a harmonic law governing the variation of the sum of real coefficients of volumetric numbers, depending on their positioning on a sphere. In [4], the nature of the distribution of the set of volumetric numbers with integer coefficients $a, b, c$ on corresponding spheres was determined.

The findings from [1–4] are of significant interest in mathematical modeling and have facilitated the development of an innovative physical-mathematical model of a volumetric universe [5,6], which has been further developed in works [7,8].

## Formulation of the study purpose

The aim of this study is to conduct a deeper investigation into the properties of elements within the set of volumetric numbers. Specifically, it seeks to perform an extended analysis of the conditions under which the associative property holds in multiplication, as well as to determine the unique properties of zero as a number that is the only element belonging simultaneously to the sets of real, complex, and volumetric numbers.

## Presenting main material

According to the axioms of volumetric numbers:

$$i \times i = i^2 = -1;$$
$$j \times j = j^2 = -1;$$
$$i \times j = j \times i = 0,$$

all algebraic operations can be performed on them, similar to ordinary trinomials or complex numbers.

The properties of addition and multiplication hold for all volumetric numbers, except for the associative property of multiplication, which is valid under certain conditions [2].

This study examines the conditions under which the associative property of multiplication holds for volumetric numbers

$$V_1 \times (V_2 \times V_3) = (V_1 \times V_2) \times V_3, \tag{1}$$

where: $V_1 = a_1 + b_1 i + c_1 j$; $V_2 = a_2 + b_2 i + c_2 j$; $V_3 = a_3 + b_3 i + c_3 j$, are arbitrary volumetric numbers.

Accordingly:

$$V_1 \times (V_2 \times V_3) = \left(a_1a_2a_3 - a_1b_2b_3 - a_1c_2c_3 - a_2b_1b_3 - a_3b_1b_2 - a_2c_1c_3 - a_3c_1c_2\right) +$$
$$\left(a_1a_2b_3 + a_1a_3b_2 + a_2a_3b_1 - b_1b_2b_3 - b_1c_2c_3\right)i + \left(a_1a_2c_3 + a_1a_3c_2 + a_2a_3c_1 - b_2b_3c_1 - c_1c_2c_3\right)j$$

and

$$(V_1 \times V_2) \times V_3 = \left(a_1a_2a_3 - a_1b_2b_3 - a_1c_2c_3 - a_2b_1b_3 - a_3b_1b_2 - a_2c_1c_3 - a_3c_1c_2\right) +$$
$$\left(a_1a_2b_3 + a_1a_3b_2 + a_2a_3b_1 - b_1b_2b_3 - b_3c_1c_2\right)i + \left(a_1a_2c_3 + a_1a_3c_2 + a_2a_3c_1 - b_1b_2c_3 - c_1c_2c_3\right)j$$

Substituting these expressions into equation (1) and performing simple transformations, we obtain:

$$-b_1c_2c_3 i - b_2b_3c_1 j \neq -b_3c_1c_2 i - b_1b_2c_3 j.$$

Thus, equation (1) does not hold for arbitrary volumetric numbers.

For equation (1) to hold, it is necessary that:

$$\begin{cases} b_1c_2c_3 \equiv b_3c_1c_2, \\ b_2b_3c_1 \equiv b_1b_2c_3, \end{cases} \Rightarrow$$
$$\begin{cases} b_1 \times c_3 \equiv b_3 \times c_1, \\ b_3 \times c_1 \equiv b_1 \times c_3, \end{cases} \Rightarrow$$
$$b_1 \times c_3 = b_3 \times c_1. \tag{2}$$

Since the coefficient $a$ does not appear in equation (2), for simplicity, we consider the projections of numbers $V_1, V_3$ onto the complex-spatial plane $\mathrm{Im}Sp$, $\left(V_1', V_3'\right)$, (Fig. 2).

The product of the corresponding coefficients $b_1, b_3, c_1, c_3$ represents the area $S$ of certain rectangles (Fig. 2)

$$\begin{cases} b_1 \times c_3 = S, \\ b_3 \times c_1 = S. \end{cases} \tag{3}$$

This system can be rewritten as:

$$\begin{cases} c_1 = \dfrac{S}{b_3}, \\ c_3 = \dfrac{S}{b_1}, \end{cases}$$

which corresponds to the equation of an equilateral hyperbola.

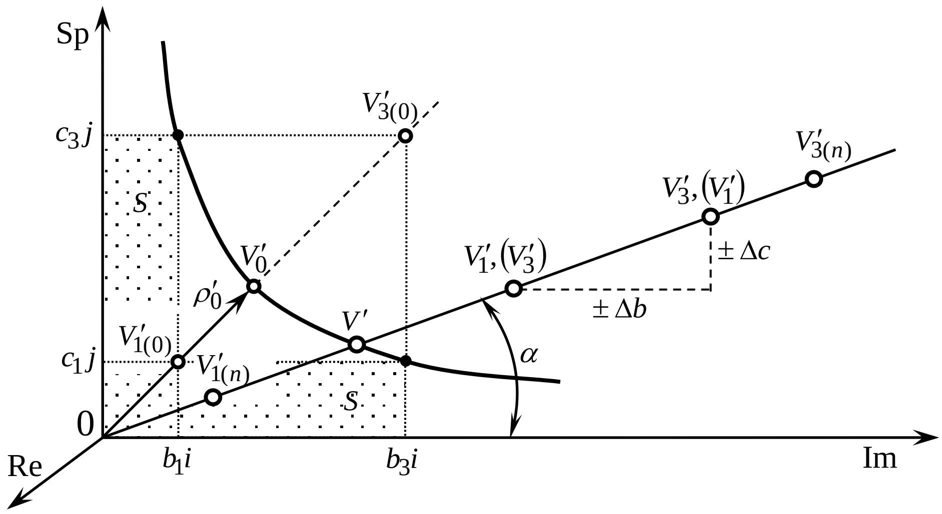


*Fig. 2.* Projections of numbers $V_1, V_3$ onto the complex-space plane $\mathrm{Im}Sp$

Consequently, $S = \frac{1}{2}(\rho'_0)^2 = b \times c$ , (Fig. 2).

Thus, for an arbitrary number $V_1$, there exists a number $V_3$, such that condition (1) holds, since:

$$b_3 = \frac{S}{c_1};\ c_3 = \frac{S}{b_1}.$$

However, for the projections $V'_{1(0)}$ and $V'_{3(0)}$, the following condition holds:

$$b_1 = c_1;\ b_3 = c_3,$$

which ensures that $V_1$ and $V_3$ lie in the same plane, oriented at an angle $\alpha = 45^\circ$ to the complex plane $(\mathrm{Re}\,\mathrm{Im})$.

By dividing the equations in system (3), we obtain:

$$\frac{b_1 \times c_3}{b_3 \times c_1} = 1,$$

which leads to

$$\frac{c_1}{b_1} = \frac{c_3}{b_3} = tg\alpha.$$

Thus, for the set of volumetric numbers located in a plane inclined at an angle $\alpha$ to the complex plane, the associative property of multiplication holds.

To refine this conclusion, we consider an arbitrary plane inclined at an angle $\alpha$ to plane $\mathrm{Re}\,\mathrm{Im}$ (Fig. 2).

Assigning a small increment $V'_1 = b_1 i + c_1 j$ to the projection coefficients of an arbitrary number $\pm\Delta b, \pm\Delta c$ , we obtain a projection of another arbitrary number $V'_3 = (b_1 \pm \Delta b)i + (c_1 \pm \Delta c)j$ (Fig. 2).

For the associative property to hold, it is necessary that,

$$b_1 \times (c_1 \pm \Delta c) = c_1 \times (b_1 \pm \Delta b).$$

Thus:

$$\pm\Delta c \times b_1 = \pm\Delta b \times c_1,$$

which leads to

$$\frac{c_1}{b_1} = \frac{\pm \Delta c}{\pm \Delta b}.$$

Since:

$$\frac{c_1}{b_1} = tg\alpha \;\Rightarrow\; \frac{\pm \Delta c}{\pm \Delta b} = tg\alpha .$$

It follows that number $V_3$ lies in the same plane as $V_1$.

Therefore, for the set of volumetric numbers located in a plane passing through the real axis $\mathrm{Re}$, the associative property of multiplication holds. The complex plane $(\mathrm{Re\,Im})$ is a special case of ($\alpha = 0^\circ$).

From these results, the following property emerges: There always exist numbers $V_1$ and $V_3$ that satisfy the associative property of multiplication and are located on opposite sides of $V$, which lies on an equilateral hyperbola.

To determine the corresponding numbers $V_1$ and $V_3$, we use a proportionality coefficient $n, (n \in \{R\})$. Since the projection of number $V$ is located on the equilateral hyperbola $V' = bi + cj$, then $b \times c = S$. Considering the coefficient $n$ (for instance, $n > 1$), the projection $V'_{3(n)} = b_3 i + c_3 j$ will lie on the same line as $V'$, since $b_3 = b \times n$ and $c_3 = c \times n$ (Fig. 2).

Therefore, for the associative property to hold, the projection coefficients $b_1, c_1$ of $V'_{1(n)}$ are determined by system (3):

$$b_1 = \frac{S}{c_3} = \frac{1}{n} b ;$$

$$c_1 = \frac{S}{b_3} = \frac{1}{n} c .$$

For the considered case ($n > 1$), the following inequalities hold:

$$b_1 < b < b_3 ;$$

$$c_1 < c < c_3 ,$$

which determine the positioning of numbers $V_1$ and $V_3$ relative to $V$.

The positioning of $V'_1$ and $V'_3$ with respect to $V'$ can be determined by the ratio of the radius vectors of their projections,

$$\frac{\rho'_3}{\rho'} = \frac{\rho'}{\rho'_1} = n .$$

Since:

$$\rho' = \sqrt{b^2 + c^2} \; ; \; \rho'_3 = n\sqrt{b^2 + c^2} \; ; \; \rho'_1 = \frac{1}{n}\sqrt{b^2 + c^2} .$$

It follows that any number $V'_3$ from an infinite segment $[V', \infty]$ corresponds to a number $V'_1$ from a finite segment $[0, V']$, for which condition (1) holds. Therefore, the cardinality of a finite numerical set is equal to the cardinality of an infinite numerical set, which, on the one hand, implies the «equivalence» of zero to infinity or the paradox of infinity, and on the other hand, the indeterminacy of zero in the set of volumetric numbers $\{V\}$.This ambiguous or dual conclusion directly supports the theory of the energy-information-time continuum of the volumetric universe, one of whose postulates states: «The infinitely large is the infinitely small, and the infinitely small contains the infinitely large» [6].

From the geometric interpretation of volumetric numbers, zero is the only element that belongs simultaneously to the sets of real, complex, and volumetric numbers, defining its unique properties.

Let us examine operations involving zero within the set of volumetric numbers, considering that zero is an element exclusively of the set of volumetric numbers, leading to a phenomenon akin to «mathematical annihilation» ($i \times j = 0 = j \times i$).

Multiplication and division of zero by itself:

$$0\times 0=(i\times j)\times(i\times j)=(i\times j)^2=i^2\times j^2=(-1)\times(-1)=1,$$

$$\frac{0}{0}=\begin{cases}\dfrac{(i\times j)}{i}\times\dfrac{1}{j}=1,\\[2mm] \dfrac{(i\times j)}{j}\times\dfrac{1}{i}=1.\end{cases}$$

Consequently, in the set of volumetric numbers $\{V\}$

$$0\times 0\equiv\frac{0}{0}=1.$$

Multiplication and division of an arbitrary volumetric number $V=a+bi+cj$ by zero:

$$(a+bi+cj)\times(i\times j)=\begin{cases}{[(a+bi+cj)\times i]\times j}=\begin{cases}(ai-b)\times j=-bj,\\(ai-b+0)\times j=\begin{cases}-bj-i,\\-bj+0\times j=\begin{cases}-bj-i,\\-bj+0\times j=\ldots,\end{cases}\end{cases}\end{cases}\\ {[(a+bi+cj)\times j]\times i}=\begin{cases}(aj-c)\times i=-ci,\\(aj+0-c)\times i=\begin{cases}-j-ci,\\0\times i-ci=\begin{cases}-j-ci,\\0\times i-ci=\ldots,\end{cases}\end{cases}\end{cases}\end{cases}$$

$$\frac{a+bi+cj}{i\times j}=\begin{cases}\dfrac{a+bi+cj}{i}\times\dfrac{1}{j}=\begin{cases}(ai-b)\times j=-bj,\\(ai-b+0)\times j=\begin{cases}-bj-i,\\-bj+0\times j=\begin{cases}-bj-i,\\-bj+0\times j=\ldots,\end{cases}\end{cases}\end{cases}\\ \dfrac{a+bi+cj}{j}\times\dfrac{1}{i}=\begin{cases}(aj-c)\times i=-ci,\\(aj+0-c)\times i=\begin{cases}-j-ci,\\0\times i-ci=\begin{cases}-j-ci,\\0\times i-ci=\ldots.\end{cases}\end{cases}\end{cases}\end{cases}$$

It should be noted that when using the formulas for multiplication and division of volumetric numbers [2]:

$$(a_1+b_1i+c_1j)\times(a_2+b_2i+c_2j)=(a_1a_2-b_1b_2-c_1c_2)+(a_1b_2+a_2b_1)i+(a_1c_2+a_2c_1)j,$$

$$\frac{a_1+b_1i+c_1j}{a_2+b_2i+c_2j}=\frac{a_1a_2+b_1b_2+c_1c_2}{a_2^2+b_2^2+c_2^2}+\frac{a_2b_1-a_1b_2}{a_2^2+b_2^2+c_2^2}i+\frac{a_2c_1-a_1c_2}{a_2^2+b_2^2+c_2^2}j,$$

we obtain the following result:

$$(a+bi+cj)\times 0=(a+bi+cj)\times(i\times j)=\begin{cases}-bj,\\-ci,\end{cases}$$

$$\frac{a+bi+cj}{0}=\frac{a+bi+cj}{i\times j}=\begin{cases}-bj,\\-cj.\end{cases}$$

The variability of the results of operations with zero in the set $\{V\}$ is due to the duality of zero and the failure of the associative property in multiplication since the imaginary unit ($i$) and the spatially indeterminate unit ($j$) are located in different planes.

Thus, the results of multiplication and division by zero, as a number, for all elements of the set

$\{V\}$ can be presented as follows:

$$V\times 0=\begin{cases} V\times 0=0, \ \ if \ \ 0\in\{C\}, 0\notin\{V\}, \\ V\times(i\times j)=\left\{\begin{array}{l}(V\times i)\times j=\begin{cases}-bj,\\ -(i+bj),\end{cases}\\ (V\times j)\times i=\begin{cases}-ci,\\ -(ci+j).\end{cases}\end{array}\right\} \ if \ \ 0\in\{V\}, 0\notin\{C\}, \end{cases}$$

$$\frac{V}{0}=\begin{cases} \frac{V}{0}=Undefined, \ \ if \ \ 0\in\{C\}, 0\notin\{V\}, \\ \frac{V}{i\times j}=\left\{\begin{array}{l}\frac{V}{i}\times\frac{1}{j}=\begin{cases}-bj,\\ -(i+bj),\end{cases}\\ \frac{V}{j}\times\frac{1}{i}=\begin{cases}-ci,\\ -(ci+j).\end{cases}\end{array}\right\} \ if \ \ 0\in\{V\}, 0\notin\{C\}. \end{cases}$$

An analysis of the obtained results indicates the fulfillment of the condition for mutually inverse operations of multiplication and division only for purely imaginary $(bi)$ and purely spatially indeterminate $(cj)$ numbers:

$$bi\times(i\times j)\equiv\frac{bi}{i\times j}=-bj, \ \Rightarrow \ -bj\times(i\times j)\equiv\frac{-bj}{i\times j}=bi,$$

$$cj\times(i\times j)\equiv\frac{cj}{i\times j}=-ci, \ \Rightarrow \ -ci\times(i\times j)\equiv\frac{-ci}{i\times j}=cj.$$

Thus, in the set of volumetric numbers $\{V\}$, zero, both as a number and in operations with it, possesses unique properties that can be used in mathematical modeling of contradictory or dual aspects of the surrounding world. Since the parameters of such objects are characterized by infinitely small or infinitely large values, this leads to certain uncertainties in the research process, such as the collapse of the wave function in quantum physics. It should also be noted that according to the physical and mathematical model of the volumetric Universe [6], the synthesis of energy parameters, which is the basis for the self-realization of the «Beginning» beyond the point of singular void, can be expanded by considering the multiplication of zero by itself. That is, $0\times 0\equiv 1$, consequently, entire worlds arise from «nothingness».

**Conclusions**

1. The associative property of multiplication holds for the set of volumetric numbers that are located in a plane passing through the real axis and positioned at an arbitrary angle relative to the complex plane.
2. The «paradox» of the density of the numerical space of volumetric numbers has been discovered, which implies the «equivalence» of zero to infinity or the paradox of infinity and the indeterminacy of zero.
3. In the set of volumetric numbers $\{V\}$, zero possesses unique properties that can be used in mathematical modeling of contradictory or dual aspects of the surrounding world, as it has been established that any mathematical operations with the number zero are valid.

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