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STUDY OF SPEED-OPTIMIZED RELAY CONTROL SYSTEMS USING CONTROLLERS WITH HYSTERESIS LOOPS

ДОСЛІДЖЕННЯ ОПТИМІЗОВАНИХ ЗА ШВИДКОДІЄЮ РЕЛЕЙНИХ СИСТЕМ КЕРУВАННЯ ПРИ ЗАСТОСУВАННІ РЕГУЛЯТОРІВ З ПЕТЛЯМИ ГІСТЕРЕЗИСУ

The relevance of the work is due to the need to adapt the results of the application of optimal control theory methods to modern technical implementation capabilities as control algorithms for real dynamic objects. The purpose of the research is to develop a structural solution for relay controllers that combines the ability to limit the frequency of sliding modes with the unhindered implementation of control algorithms, the parametric synthesis of which is aimed at optimizing systems in terms of speed under conditions of intermediate coordinate constraints. To achieve this goal, the work proposes structural modifications of relay controllers that allow the use of relay elements with hysteresis characteristics only under the conditions of sliding modes, performing single switching with the help of ideal relays; mathematical modeling of an improved control system is performed to confirm the effectiveness of the proposed solution to the problem. The result of the work is structural diagrams of mathematical models of improved relay controllers.

***Keywords**: sliding mode control system, hysteresis, transient, N–i switching method, optimality in speed.*

Актуальність роботи зумовлена потребою в адаптації результатів застосування методів теорії оптимального керування до сучасних можливостей технічної реалізації в якості алгоритмів керування реальними динамічними об’єктами. Серед них окремо стоїть метод N–і перемикань, який є альтернативним засобом оптимізації, оскільки не містить явного розв’язання варіаційної задачі. Йому притаманна простота математичного апарату, досягнута завдяки орієнтації на певний клас структур релейних систем керування електроприводами. Цей метод призначений для параметричного синтезу каскадно-підпорядкованих систем на основі розрахункової оптимальної за швидкодією траєкторії.

Метою цього дослідження є розробка структурного рішення для релейних регуляторів, яке дозволяє поєднати засоби обмеження частоти ковзних режимів з безперешкодною реалізацією алгоритмів керування, параметричний синтез яких має за мету оптимізацію систем за швидкодією в умовах обмеження проміжних координат.

Для досягнення цієї мети у роботі розв'язано наступні задачі: виявлено протиріччя між наявністю пульсацій струму, що виникають внаслідок обмеження частоти ковзних режимів, та необхідністю виконання одиничних перемикань регуляторів у точно розрахованих точках простору станів; здійснено дослідження перехідних процесів, що демонструє негативні наслідки цього протиріччя для динаміки позиційних електроприводів; запропоновано структурні модифікації релейних регуляторів, які дозволяють застосовувати релейні елементи з гістерезисними характеристиками лише за умов існування ковзних режимів, виконуючи одиничні перемикання за допомогою ідеальних реле; виконано математичне моделювання вдосконаленої системи керування на підтвердження ефективності запропонованого розв’язку проблеми.

Результатом роботи є структурні схеми математичних моделей релейних регуляторів, статичні характеристики яких є адаптивними до поточних режимів функціонування системи. Використання модифікованих регуляторів дозволяє реалізувати оптимальні за швидкодією алгоритми керування в умовах обмеження частоти ковзання без додаткових заходів параметричної оптимізації.

***Ключові слова**: релейна система керування, гістерезис, перехідний процес, метод N–і перемикань, оптимальність за швидкодією.*

Problem’s Formulation

The switching frequency of relay elements in ideal sliding modes tends to infinity [1], although for the effective technical implementation of relay control systems for electric drives, a limited slip frequency of their controllers is sufficient [2]. Relay controllers used in practice have hysteresis, which allows the formation of current pulsations in a given range, thereby indirectly limiting the switching frequency [3]. At the same time, the presence of hysteresis shifts the actual switching points of the system from the calculated positions in the state space, thereby moving the representative point of the system away from the optimal trajectory [4]. These deviations are relatively small; however, in long-term transient processes, they create a cumulative effect, the result of which can be overshoots unacceptable for position control systems.

Analysis of recent research and publications

Relay controllers are known for their advantages in control problems of electromechanical systems [5]. Sliding modes impart low sensitivity to relay systems to disturbances, the effect of which on coordinates and parameters is typical for any electric drives [6]. Relay systems also have structurally determined prerequisites for speed optimization [7]. The presence in the structure of electric drive control systems of elements that limit intermediate coordinates for reasons of electrical and mechanical strength [8] guarantees the deterministic behavior of systems in dynamic modes. Such elements include, in particular, the internal loops of cascade-subordinate structures [9]. Their sliding modes ensure high accuracy of processing the calculated constraints and, consequently, the possibility of applying speed optimization methods that are based on the prediction of transient trajectories. An example is the $N-i$ switching method [4], designed to optimize the response time of technically simple relay systems of subordinate control with linear slip equations. This method has a very compact mathematical apparatus and establishes an unambiguous relationship between the levels of constraints and the parameters of optimal controllers [10]. However, relay systems synthesized on the basis of trajectory prediction [6] are very sensitive to discrepancies between actual and calculated motion.

Formulation of the study purpose

The hysteresis characteristics of relay elements shift the switching points of controllers by half the width of their loops, which inevitably affects the dynamics of control systems. The goal of this work is to modify the structure of relay controllers to neutralize the negative impact of their hysteresis characteristics on the ability of electric drive control systems to reproduce optimal motion profiles with high accuracy.

Presenting main material

The dynamics of a positional electric drive as a control object is described by the following system of differential equations

$$\left.\begin{aligned} &p\varphi = \omega; \\ &p\omega = \varepsilon = \frac{k_p \cdot c}{J} \cdot (i - i_s) \\ &p\varepsilon = a = \frac{k_p \cdot c}{J} \cdot \frac{u - R \cdot i - c \cdot \omega / k_p}{L} \end{aligned}\right\}, \qquad (1)$$

where $\varphi, \omega, \varepsilon, a$ are the angular position, velocity, acceleration and jerk of the actuator shaft, respectively, i, i_s are the armature current and the static current, u is the converter voltage; $k_p, R, L, J, c = k\Phi$ are the parameters of the electromechanical system.

A relay system with a cascade structure, designed for time-optimal control of an electromechanical system (1), implements the following algorithm:

$$\left.\begin{aligned} &u_{R\varphi} = \omega^* = \omega_{max} \cdot sign\left(\varphi^* - \varphi - K_{\varphi\omega} \cdot \omega - K_{\varphi\varepsilon} \cdot \varepsilon\right) \\ &u_{R\omega} = \varepsilon^* = \varepsilon_{max} \cdot sign\left(\omega^* - \omega - K_{\omega\varepsilon} \cdot \varepsilon\right) \\ &u_{R\varepsilon} = u^* = u_{max} \cdot sign\left(\varepsilon^* - \varepsilon\right) \end{aligned}\right\}. \qquad (2)$$

where u_{max} is the voltage amplitude of the delay-free power converter.

The symbol "*" in the system of equations (2) denotes the given values of variables, one of which (the given position) enters the system from the outside, while the others are generated by the controllers. The problem of calculating the acceleration feedback [10] will be left outside the scope of this work.

For position and velocity controllers, the $N-i$ switching method allows, for given levels of constraints on the canonical coordinates ω_{max}, ε_{max}, a_{max}, to find the feedback coefficients $K_{\varphi\omega}$, $K_{\varphi\varepsilon}$, $K_{\omega\varepsilon}$, which ensure the implementation of single switches at the calculated points of the optimal trajectory in terms of speed, without resorting to the explicit construction of the trajectory as such:

$$K_{\omega\varepsilon} = \frac{\varepsilon_{max}}{2 \cdot a_{max}}; \; K_{\varphi\omega} = \frac{\omega_{max}}{2 \cdot \varepsilon_{max}} + \frac{\varepsilon_{max}}{2 \cdot a_{max}}; \; K_{\varphi\varepsilon} = \frac{\omega_{max}}{4 \cdot a_{max}} + \frac{\varepsilon_{max}^2}{12 \cdot a_{max}^2}. \qquad (3)$$

To minimize losses in an electric motor, one should strive to ensure armature current pulsations with the lowest possible amplitude, which requires the controllers in cascade (2) to slide at the highest technically feasible frequency. However, to avoid unacceptably high losses in the power converter switches, a compromise solution is often used. In cascade (2), ideal relays with a static characteristic $y = sign(x)$ are replaced by relay elements with a static characteristic

$$y = gist(x) = \begin{cases} -1 \text{ if } \; x < -\delta \\ -1 \text{ if } \; -\delta \le x \le \delta = 0 \text{ and } y = -1 \\ +1 \text{ if } \; -\delta \le x \le \delta = 0 \text{ and } y = +1 \\ +1 \text{ if } \; x > \delta \end{cases}. \qquad (4)$$

These relays are configured based on the following considerations. Limiting the switching frequency of the voltage inverter is accompanied by an increase in the oscillation amplitude of the system coordinates. In this case, a significant pulsation amplitude is generally allowed only for the "fastest" coordinate, i.e., the coordinate closest to the input of the controlled object. For a dynamic object (1) controlled by a cascade of relay controllers (2), this coordinate is the current, the pulsation amplitude of which we denote as δi_{max}. The corresponding acceleration pulsation amplitude is $\delta\varepsilon = \frac{k_p \cdot c}{J} \cdot i$ according to (1). This value determines the hysteresis loop width for relay R_ε as $2\delta\varepsilon$. Consequently, the hysteresis loop width of controller R_ω of system (2) should be equal to twice the pulsation amplitude of the acceleration feedback signal $2\delta\varepsilon \cdot K_{\omega\varepsilon}$, and the hysteresis loop width of controller R_φ should be equal to $2\delta\varepsilon \cdot K_{\varphi\varepsilon}$.

We will study the dynamics of system (1), (2), (3) using mathematical modeling. The results of using controllers of type (4), whose loop widths were calculated as described above, are presented in Fig. 1. The time diagrams show a constant amplitude of acceleration pulsations generated by all cascade controllers in sliding modes. However, entering sliding mode for the position and velocity controllers in the 3rd- and 2nd-order subsystems, respectively, occurs with overshoot, and its manifestations become more pronounced with increasing subsystem order. Note that this conclusion is confirmed by studies of higher-order systems, which are not considered in this paper.

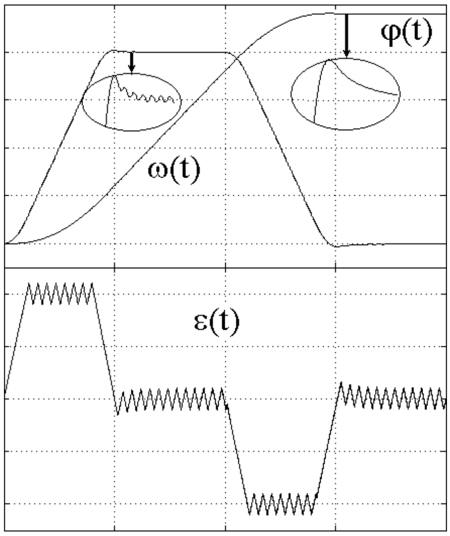

Fig. 1. Transient processes in the presence of hysteresis loops in controllers

The cause of the detected overshoot is the delay of single switching of the controller relative to the calculated switching moments, indirectly incorporated into the controller parameters by linking their switching times to characteristic points of the optimal phase trajectory. Such delays can be interpreted as a reflection in the time domain of deviations between actual switching points and calculated ones, which are caused by the presence of hysteresis loops in the controller characteristics.

The simplest solution to this problem is to use variable-structure controllers. In sliding modes, that is, for small deviations of the controlled coordinate from the setpoint, they implement a hysteresis static characteristic, while in single-switch mode, that is, for large deviations from the equilibrium point, they implement the signature static characteristic of an ideal relay. A schematic diagram of a controller model with this structure is shown in Fig. 2.

The threshold value of the control error of the i-th coordinate Y_i (with numbering from the system input), at which a change in the algorithm of its controller should be implemented, is not strictly regulated by the logic of the system operation; however, it should not exceed the control error of this coordinate at the point of the last single switching before the occurrence of the sliding mode, that is, at the $N-i$-th characteristic point of its switching. The coordinates of all such points are easily determined in the context of this method, since they are intermediate results of parametric synthesis [4]. By linking the moment of the structure change to the achievement of the error of half the contour mismatch at the $N-i$-th characteristic point, for the positional drive

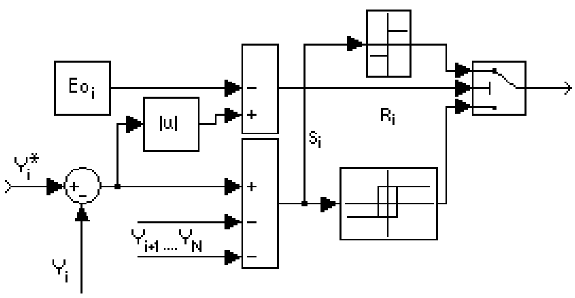

Fig. 2. Variable structure controller

under consideration we obtain:

$$\Delta_1 = \Delta\varphi = \frac{1}{2}\cdot\frac{\varepsilon_{max}^3}{6\cdot a_{max}^2};\ \Delta_2 = \Delta\omega = \frac{1}{2}\cdot\frac{\varepsilon_{max}^2}{2\cdot a_{max}}\ . \quad (5)$$

To confirm the effectiveness of the proposed structural solution, we will study the dynamics of system (1), (2), (3) with controllers of type (4), (5). The results of modeling the transients of the system, whose controllers disable hysteresis during sliding mode breakdowns, presented in Fig. 3, demonstrate an aperiodic nature acceptable for positional drives upon entering the sliding mode and the limitation of acceleration and current pulsations in the sliding mode itself. Note that the microprocessor implementation of this structure requires only one additional condition to be checked for each control loop, and the implementation of ideal relays is simpler than that of controllers with a hysteresis loop.

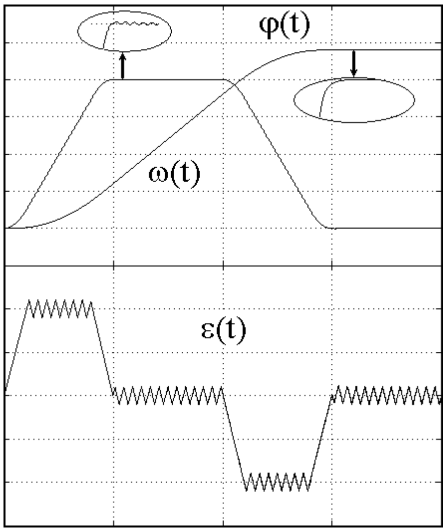

Fig. 3. Transient processes in a system with a variable controller structure

Instead of the above-described creation of parallel channels in the controllers, switched by a logic block, disabling the hysteresis in the event of a sliding mode breakdown can also be implemented by including a nonlinear feedback loop, as shown in Fig. 4. A relay with a dead zone adds a component to the switching function equal to half the loop width with a sign opposite to the current state of the relay controller. The loop closes in cases of saturation of the inertial element with a time constant shorter than the switching period, since saturation of the filtered output signal of the controller indicates a breakdown of the sliding mode. Note that the value of the switching period for adjusting this version of the controller is easily calculated taking into account the time constant of the armature circuit and the specified amplitude of the current pulsations [4].

It should be noted that a structural solution to the problem under consideration is essential. It is quite simple to account for the shift of switching points from sliding hyperplanes by half the width of the hysteresis loops by appropriate increments of the phase coordinates. However, the small increments acquired by the phase coordinates during the periods when the representative point passes through the hysteresis loops are, in real transient processes, subject to the influence of a number of random factors. These include: the changing frequency of the sliding mode, the presence of resistance and interference in the feedback channel, the phase of self-oscillations at the moments of controller switching, etc. Consequently, analytical refinement of the parameters of controllers with hysteresis loops is unjustified due to the impossibility of practical implementation of its results. At the same time, the structural solution is technically simple and does not exclude options for adaptive system tuning depending on the motion profile.

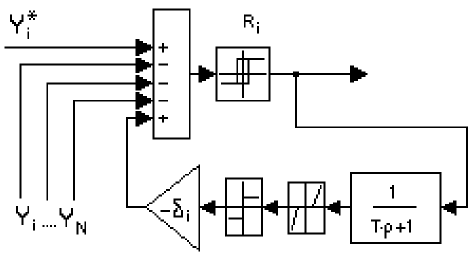

Fig. 4. Nonlinear feedback controller

Conclusions

The proposed modification to the relay controller structure, which involves switching from a relay with a hysteresis characteristic to ideal relay elements upon exiting the low-error control mode, eliminates the discrepancy between the actual and calculated switching points of relay controllers in a cascaded-slave system. This measure ensures limited amplitude of acceleration and current pulsations while accurately predicting the optimal trajectories for transients. A promising direction for further development of the research results is their implementation in fourth-order (and higher) control systems.

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