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METHODS OF CONTROLLING THE DYNAMICS OF COMPETITIVE SYSTEMS WITH A DELAY

МЕТОДИ УПРАВЛІННЯ ДИНАМІКОЮ КОНКУРЕНТНИХ СИСТЕМ ІЗ ЗАПІЗНЕННЯМ

The article discusses modern methods and approaches to managing the dynamics of competitive systems with delays.

Traditional approaches to the analysis and compensation of delays and the latest methods that have emerged at the intersection of control theory and machine learning are investigated. The emphasis is placed on mathematical modeling of systems where the interaction between agents and subsystems occurs with a time lag. The use of machine learning as a tool for identifying parameters and developing control strategies is proposed.

It is proved that the combination of classical methods of the theory of differential equations with a time lag and artificial intelligence algorithms can increase the efficiency of management decisions in complex competitive environments.

***Keywords**: delay, machine learning, adaptive control, system dynamics, competitive systems.*

У статті розглянуто сучасні методи та підходи до управління динамікою конкурентних систем із запізненням. Досліджено традиційні підходи до аналізу та компенсації запізнень, а також новітні методи, що виникли на перетині теорії керування та машинного навчання. Наголошено на математичному моделюванні систем, у яких взаємодія між агентами та підсистемами відбувається із часовим зсувом. Запропоновано використання методів машинного навчання як інструменту для ідентифікації параметрів і розроблення стратегій керування.

Доведено, що поєднання класичних методів теорії диференціальних рівнянь із часовим запізненням та алгоритмів штучного інтелекту може підвищити ефективність управлінських рішень у складних конкурентних середовищах.

Метою статті є аналіз і узагальнення методів управління динамікою конкурентних систем із запізненням, зокрема поєднання класичних математичних підходів і сучасних методів машинного навчання для розроблення ефективних стратегій компенсації часових затримок.

Основними завданнями дослідження є:

- проаналізувати традиційні методи аналізу систем із запізненнями;

- дослідити сучасні підходи, що базуються на машинному навчанні та адаптивному керуванні;
- проаналізувати можливості інтеграції класичних і нейромережевих підходів.

Запропонований у дослідженні комбінований підхід до управління динамікою конкурентних систем із запізненням поєднує аналітичні інструменти та методи штучного інтелекту. У роботі доведено, що машинне навчання може бути ефективно використане для ідентифікації параметрів, прогнозування динаміки та оптимізації стратегій управління в умовах складних конкурентних взаємодій. Такий підхід забезпечує адаптивність і стійкість систем в умовах невизначеності та змін зовнішнього середовища.

Подальші дослідження можуть бути спрямовані на розроблення гібридних моделей керування із використанням пояснюваного штучного інтелекту (XAI) для підвищення інтерпретованості рішень, а також на розширення сфери застосування в біоінженерії, логістиці та системах ухвалення рішень у реальному часі.

***Ключові слова**: запізнення, машинне навчання, адаптивне керування, динаміка систем, конкурентні системи.*

Problem's Formulation

In today's world, where markets evolve dynamically and interactions between entities are becoming increasingly complex, competitive systems constitute a key object of study. At the same time, economic markets function as dynamic systems in which participants constantly compete for resources, influence, and other advantages. Under such conditions, the factor of delay (time lag) becomes particularly significant. Delays may arise from the time required to collect and process information, make decisions, implement strategies, or as a result of market inertia. If these delays are underestimated or poorly managed, the system may experience instability, cyclical fluctuations, ineffective managerial decisions, or even chaotic behaviour.

Classical approaches are based on the theory of differential equations with delays, which rely on assumptions about model structure and parameters that do not always correspond to real-world conditions. Competition in modern environments unfolds in complex and highly dynamic contexts, creating a growing demand for flexible and adaptive management methods capable of accounting for time lags. Consequently, the integration of machine learning methods—particularly deep learning—with classical mathematical modelling appears to be a promising direction. This approach not only enables the automation of parameter identification, but also supports the development of adaptive management strategies under uncertainty. Moreover, such integration calls for new approaches to interpreting models and results, especially in tasks where the transparency and predictability of management decisions are critical.

Analysis of recent research and publications

The problem of delays in dynamic systems has been widely discussed in the fields of applied mathematics, control theory, and artificial intelligence. Among the classical contributions, Hale's work (Hale, 1993) stands out, as it laid the foundation for the formal stability analysis of such systems. Subsequent studies have focused on numerical methods, stabilization, and optimal control of systems with time delays (Gu et al., 2003; Kolmanovskii & Myshkis, 2012).

In recent years, the scientific community has shown increasing interest in stochastic models with delays, which account for random perturbations and Markovian switching (Liu et al., 2018), as well as in hybrid systems. Wang et al. (2020) demonstrated the integration of classical models with machine learning techniques, while Zhang et al. (2021) explored the challenges of adaptive control and the combination of reinforcement learning with optimal control theory.

Another promising direction is explainable artificial intelligence (XAI), which provides insights into the logic underlying models in complex systems, thereby ensuring transparency and trust in management decisions (Doshi-Velez & Kim, 2017). Despite significant progress, the integration of formal control methods with the flexibility of learning algorithms capable of adapting to changing environments remains an urgent and relevant research task.

Formulation of the study purpose

The objective of this article is to examine contemporary methods for controlling the dynamics of competitive systems with delays by integrating the theoretical foundations of delay differential equations with the tools of machine learning.

Presenting main material

Dynamic competitive systems describe the processes of interaction between elements that compete for resources, influence, or advantages. In real-world conditions, such interactions are not instantaneous but are accompanied by delays, which complicate both the analysis and the management of system dynamics. The factors and causes of delays may include physical limitations, information transmission time, agents' responses to changes, and other constraints. Under these circumstances, it becomes essential to develop new approaches to modelling and managing competitive systems that explicitly account for delays and the complex structure of competitive interactions.

Delays are a common phenomenon that can be classified into the following categories:

Information delay — the time required to obtain and analyse data on competitors' actions, market trends, or consumer behaviour.

Operational delay — the time needed to implement new strategies, production processes, or structural changes.

Decision-making delay — the time interval between recognising the necessity of addressing a problem and initiating an appropriate management decision.

Market reaction delay — the extended period during which the actions of one market participant provoke a noticeable response from competitors or consumers [1].

Among the most significant consequences of delays, instability should be emphasized, as both minor and substantial delays exert a destabilizing influence on the system and may lead to uncontrolled fluctuations. Another negative consequence is the deterioration of management quality, since decisions based on outdated information can prove ineffective or even harmful. In addition, reduced competitiveness may result in a loss of market share. Delays also hinder the accurate prediction of a system's future state, thereby making planning less reliable [2].

Effective management of the dynamics of competitive systems with delays requires a comprehensive approach that combines both analytical and strategic methods [5]. The main components of this approach can be outlined as follows.

1. **Modelling and analysis of delays.**

The first step in managing such systems is to understand the nature of delays. Delay differential equations (DDEs) represent a fundamental mathematical tool for describing systems in which the current state depends not only on present values of variables but also on their past values. By explicitly incorporating time lags, DDEs make it possible to model the dynamics of competitive interactions under delayed responses [1].

$$\frac{dx(t)}{dt} = f(x(t), x(t-\tau)), \tag{1}$$

where $\tau>0$ — delay parameter representing the time lag in the system's response.

Stability theory.

A critical aspect of analysing dynamic systems with delays is the study of their stability, which determines whether the system tends toward an equilibrium state, exhibits oscillatory dynamics, or develops chaotic behaviour. Stability analysis is typically carried out using methods such as the root distribution of the characteristic equation or the Lyapunov-Krasovskii approach.

Lyapunov–Krasovskii functionals

$$V(t) = x^2(t) + \int_{t-\tau}^{t} q x^2(s) ds, \tag{2}$$

where $q > 0$ — is a constant.

For stability, it is necessary that the derivative of the functional satisfies the condition

$$\frac{dV(t)}{dt} \le -cx^2(t), \tag{3}$$

$q > 0$ — constant

This condition guarantees that the solutions approach equilibrium with a decrease in the energy of the system.

Critical delay

There is a critical delay value $\tau > \tau_{crit}$, in case of exceeding which the system loses stability

$$\tau_{crit}\frac{1}{\omega}\arccos(\frac{a}{b}),\omega=\sqrt{b^2-a^2}, \tag{4}$$

At $\tau > \tau_{crit}$ the equilibrium will be unstable and harmonic oscillations may occur (Hopf bifurcation).

Identification and evaluation of delays

Model with delay under conditions of unknown quantities and nature of delays.

$$y(t)=G(s)e^{-s\tau}u(t)+v(t), \tag{5}$$

y(t) — system output; *u(t)* — input signal; *G(s)* — transfer function without delay; *τ* — delay time (delay); *v(t)* — noise or measurement error.

Parameter identification

The identification task consists in finding parameters θ (parameters *G(s)* and delay *τ*, that minimise the error function between measured *y(t)* and model inputs $\bar{y}(t,\theta)$ [5].

$$J(\theta)=\int_0^T \left|y(t)-\dot{y}(t,\theta)\right|^2 dt, \tag{6}$$

Search for the optimum $\bar{\theta}$

$$\dot{\theta}=\arg\min_{\theta} J(\theta) \tag{7}$$

Estimating delay time

Delay time *τ* is identified by maximising the correlation between the input signal *u(t- τ)* and the output *y(t)*:

$$\dot{\tau}=\arg\max_{\tau\ge 0}\int_0^T u(t-\tau)y(t)dt, \tag{8}$$

Thus, using estimates $\bar{\tau}$ and $\bar{\theta}$ є it is possible to construct an accurate model of the system with delay

$$\frac{dx(t)}{dt}=f(x(t),x(t-\dot{\tau});\dot{\theta}), \tag{9}$$

This model is critical for further analysis, forecasting, and management.

2. **Compensatory and Predictive Management Strategies**

Immediately after identifying the delay, a strategy to compensate for it is developed. Predictive controllers, such as Smith's predictor, act proactively, mitigating the effects of delays. The principles of their operation can also be adapted for strategic planning, including anticipating the reactions of market competitors. This approach relies on a system model to predict behavior [1].

Delay is defined as the time interval between the initiation of an event at one point in the system and the corresponding response or output at another point [7]. It is also referred to as transport delay, dead time, or time lag. An example of such a delay in a heat exchange system is shown in Fig. 1.

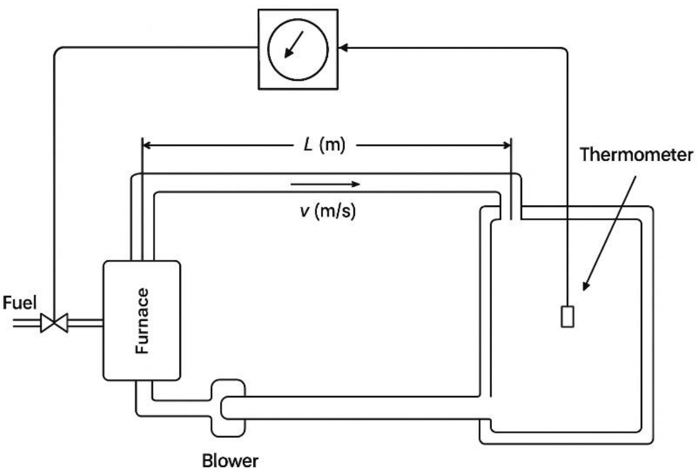

Fig. 1. Example of delay in a heat exchange system due to mass flow [6]

Delay is a common phenomenon observed in physical, chemical, biological, economic, measurement, and computing systems [1]. The causes of delay may include transport or communication delays, the time required to generate feedback in sensor systems with sampling and analysis, delays in forming the control signal, and the simplification of system parameters to a lower order.

Delays always reduce the stability of minimum-phase systems (i.e., systems that have no poles or zeros in the right half-plane of the s-plane or other delay components) [3], making it important to analyze system stability in the presence of delays.

In control systems, delays are typically described in the frequency domain of a complex variable.

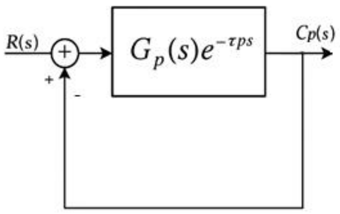

Fig. 2 Closed-loop control system with delay component [6]

The Smith predictor is a widely used method for delay compensation in control systems. This approach is effective for both small and large delays. The main idea of the Smith predictor is to eliminate the delay component from the closed loop of the system (see Fig. 2). This is important because it is the delay in the closed loop that affects system stability. Therefore, removing the delay from the loop improves stability.

Fig. 2 shows an automatic control system in which there is a delay within the closed loop. To eliminate this delay component, a compensation element can be added to the system *C*(s)*, as shown in Fig 3.

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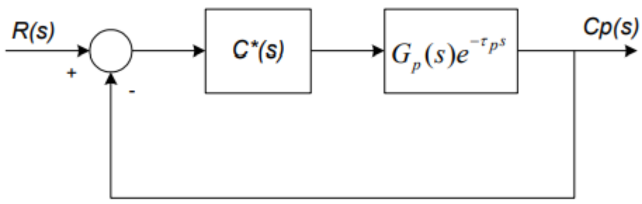

Fig. 3. Adding a compensation element to the system [6]

Fig. 4 shows the structure of the control system after the introduction of the compensation element *C*(s)*.

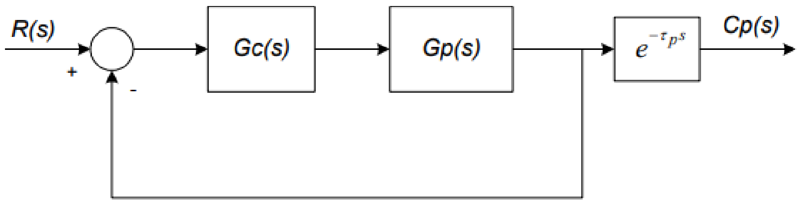

Fig. 4. Updated system structure due to the addition of a compensation element *C*(s)* [6]

Thus, Smith's predictor is a powerful mathematical model for compensating delays to improve system stability. Its main principle is to remove the delay component from the feedback loop, allowing the system to be analyzed and compensated as if no delay were present. The use of Smith's predictor contributes significantly to enhancing the stability of systems with time delays [6].

The MPC model is an approach based on optimizing management decisions over a given time horizon. It uses a dynamic system model to calculate optimal actions and continuously recalculates them based on updated data, enabling the system to respond adaptively to changes and uncertainties.

Adaptive and Robust Control

Adaptive controllers can modify their parameters over time, adapting to delays and other system variations. Robust control, on the other hand, focuses on developing strategies that are resilient to model uncertainties or variations in delay parameters [3].

3. **Strategic Approaches to Delay Management**

Reducing the source of delays is the most effective way to mitigate them. This involves accelerating data collection and analysis (e.g., using Big Data and AI for fast data processing), optimizing the decision-making process, and implementing flexible operational procedures (such as flexible production systems or Just-In-Time logistics) to minimize time costs [7].

Adaptability of strategies involves the rapid adjustment of management actions in response to changes in the competitive environment. The management process includes systematic assessment and mitigation of risks arising from delays. Coordination with partners helps synchronize processes and reduce the cumulative effects of delays [10].

Delay Management with Machine Learning

The integration of machine learning (ML) opens new and powerful possibilities for managing systems with delays. ML models, such as LSTM and GRU neural networks, are trained on historical data to predict the magnitude and dynamics of delays, thereby increasing the accuracy of controllers [4].

ML algorithms can detect complex and nonlinear dependencies, including hidden delays that are very difficult to identify using traditional methods. Reinforcement learning (RL) enables the discovery of optimal control strategies in dynamic and competitive environments with delays and helps identify factors that negatively impact delays [3].

Neural networks are used to create adaptive controllers capable of dynamically adjusting parameters, compensating for variable delays, and handling nonlinear system behavior [7]. With the aid of AI, including Explainable AI (XAI), it is possible to build accurate models that analyze the nature and impact of delays on the system and propose appropriate management decisions.

In many real dynamic systems—such as ecosystems, economic models, and biological populations — delays are integral: signals affect the system with a time lag. Formally, a system with delay can be described by delay differential equations (DDEs) (see Formula (1)).

The competition model with delay can be expressed as follows. One of the classic examples is the modified Lotka–Volterra model with time delay:

$$\frac{dx_i(t)}{dt} = x_i(t)\left[r_i - \sum_{j=1}^{n} a_{ij} x_j(t-\tau_{ij}) \right], i = 1, 2, ..., n, \tag{10}$$

where $x_i(t)$ — population size of species I; r_i — internal growth rate; a_{ij} — intensity of competition between species *i* and *j*; τ_{ij} — individual delay in interaction.

Explainable AI (XAI) allows interpreting 'black boxes' (neural networks) that learn from data with delays.

Forecast model

$$x(t) \text{ based on } x(t-\tau_1), x(t-\tau_2), ... \tag{11}$$

With the help of SHAP (SHapley Additive exPlanations), the contribution of each delay to the result is determined.

$$f(x) = \phi_0 + \sum_{i=1}^{n} \phi_i, \tag{12}$$

where ϕ_i — contribution of a feature $x(t-\tau_i)$ in the final prediction.

This approach allows visualization of which delays have the greatest impact on changes in the system state. Using SHAP or LIME explanations, it is possible to identify the critical time windows of delay impact, determine which system parameters should be adjusted (e.g., to reduce or compensate for a specific τ), and compare the importance of the current state $x(t)$ with delayed states $x(t-\tau)$ [9].

Among the advantages of XAI in the context of delays, explainability is particularly important, as it clarifies why the model produced a given prediction. This allows verification of whether the model accounts for key aspects of dynamics with delays. At the same time, experts can see how and which delays influence the system, facilitating informed decision-making.

Intelligent Methods (Machine Learning)

Machine learning (ML) is widely recognized as one of the key areas of artificial intelligence, focused on creating adaptive algorithms capable of learning and improving independently through data analysis. ML enables computer systems to detect hidden patterns in large datasets and make informed decisions. Unlike classical algorithms with fixed logic, ML models can gradually improve their performance during operation. Their accuracy depends directly on the quantity and quality of the available training data [3].

Numerous studies demonstrate the high efficiency of machine learning algorithms for modeling and controlling systems with delays [10]. Recurrent neural networks (RNNs), particularly Long Short-Term Memory (LSTM) networks, are designed to capture temporal dependencies in input data with delays. These architectures are well suited for modeling time series in which the value of the system depends on both the current state and past values, thereby explicitly incorporating delay effects.

$$h_t = \sigma(W_{xh}x_t + W_{hh}h_{t-1} + b_h)$$
$$y_t = W_{hy}h_t + b_y, \quad (13)$$

where $x_t \in R^n$ — input data at time t; $h_t \in R^d$ — hidden state (which accumulates information from the past); $\bar{y}_t \in R^d$ — prediction; W — weight matrices; σ — nonlinear activation function.

LSTM also adds input, output, and forget gates - a mechanism for controlling information.

$$f_t=\sigma\ (W_f\, x_t+U_f\, h_{t-1}+b_f)$$
$$i_t=\sigma\ (W_i\, x_t+U_i\, h_{t-1}+b_i)$$
$$o_t=\sigma\ (W_o\, x_t+U_o\, h_{t-1}+b_o)$$
$$c_t=f_t \odot c_{t-1}+i_t \odot tanh((W_c\, x_t+U_c\, h_{t-1}+b_c)$$
$$h_t=o_t \odot tanh(c_t), \quad (14)$$

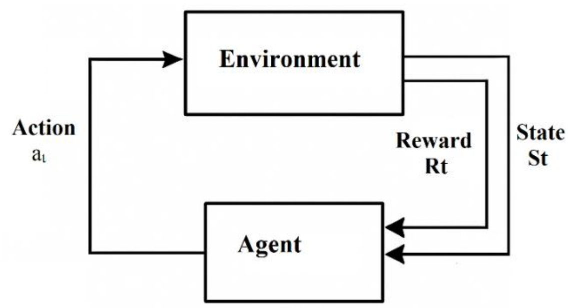

Fig. 5. Block diagram of the interaction cycle [8]

Reinforcement learning (RL) is used to develop management policies under conditions of incomplete or delayed information. RL is one of the most actively researched areas of machine learning, aimed at creating algorithms capable of learning to make decisions through direct interaction with the environment (see Fig. 5).

The basic principle of RL is that an agent gradually learns from experience: it receives rewards for successful actions and penalties for mistakes. Through this process, the agent incrementally develops and refines its behavior strategy (policy) to maximize the total accumulated reward.

Key Concepts of Reinforcement Learning

Agent and Environment. An agent operates within a given environment, receives observations, and performs actions. In response, the environment changes its state and provides the agent with a reward signal that reflects the outcome of the action.

Reward. A numerical signal indicating the effectiveness of the agent's action. The primary goal of learning is for the agent to develop the ability to make decisions that maximize cumulative rewards in the long run.

Policy. A strategy that defines which actions the agent will take in different situations. Policies may be deterministic (rigidly defined) or stochastic (random).

Value Function. A function that represents the expected amount of future rewards in a given state or for a given action. Two main types are distinguished: the state-value function (V-function) and the action-value function (Q-function) [8].

Model as a Delayed Markov Decision Process (Delayed MDP): s_t — state of the environment at time t; a_t — agent action; r_{t+d} — reward delayed by d steps.

Calculation of expected reward

$$R_t = \sum_{k=0}^{\infty} \Upsilon^k r_{t+k+d}, \quad (15)$$

where Y ∈ [0,1] — discount factor; r_{t+k+d} — delayed reward.

In order to account for delays in input data and rewards, RL reinforcement learning is combined with LSTM.

The state **st** → is replaced by the history of observations

$$h_t = LSTM(x_{t-k,\dots,}x_t), \tag{16}$$

$$a_t \sim \pi\,(a \mid h_t), \tag{17}$$

Hybrid models (ML + DDE) are a combination of analytical equations with machine learning for real-time parameter adaptation.

Hybrid ML + DDE approach

The essence of the hybrid model is that part of the function *f* is determined non-analytically; it is learned using a neural network or other ML algorithm.

$$\frac{dx(t)}{dt} = f_{phys}(x(t), x(t-\tau), u(t)) + f_{ML}(x(t), x(t-\tau), u(t), \theta), \tag{18}$$

where f_{phys} — physically derived (analytical) part of the model; f_{ML} — component obtained using machine learning; θ — ML model parameters (neural network weights, etc.).

Adaptation of coefficients through ML

Let the system have the form:

$$\frac{dx(t)}{dt} = ax(t) + bx(t-\tau) + u(t),$$

Instead of fixed coefficients *a*,*b* we use the model

$$a = \hat{a}(t) = ML_a(x_{|t-T,t|}),$$

$$b = \hat{b}(t) = ML_b(x_{|t-T,t|}), \tag{19}$$

ML_a and ML_b — models (RNN, LSTM) that learn from the history of states.

Therefore, as shown in Tabl. 1, recurrent neural networks can effectively model temporal dependencies with delays in input data, making them suitable for predicting the behavior of dynamic systems. Reinforcement learning, in turn, enables the development of control strategies even under conditions of delayed or incomplete information.

Table 1. Methods for processing delays in dynamic systems using machine learning

Method	Application	Delay Handling
RNN/LSTM	State prediction with temporal dependencies	Memory-based modeling
Reinforcement Learning	Control under delayed actions or rewards	Optimization through delayed rewards
RNN + RL	Control in partially observable environments	Integration of historical information

The combination of these methods ensures adaptability, accuracy, and stability in modeling and controlling systems with delays, which is particularly important in engineering and medical applications.

Conclusions

Managing the dynamics of competitive systems with delays is a complex but critically important task for ensuring both stability and long-term success. The key factor in achieving the desired outcomes lies in understanding the nature of delays, accurately modeling them, and applying appropriate management strategies.

Analytical capabilities are significantly enhanced by the integration of advanced machine learning methods, which make it possible to process large volumes of data and capture complex nonlinearities. This integration paves the way for the development of a new generation of adaptive and flexible management systems.

Future research in this area will focus on constructing hybrid models (analytical approaches combined with machine learning), testing these models on empirical data in economics, ecology, and logistics, as well as developing stabilization techniques for systems that display unstable behavior under long delays.

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