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DYNAMICS STUDY OF POSITION CONTROL RELAY SYSTEMS WITH VARIOUS TYPES OF FEEDBACK

ДОСЛІДЖЕННЯ ДИНАМІКИ РЕЛЕЙНИХ СИСТЕМ КЕРУВАННЯ ПОЛОЖЕННЯМ З РІЗНИМИ ТИПАМИ ЗВОРОТНИХ ЗВ’ЯЗКІВ

The need to adapt modern methods of automatic control theory to the latest technical implementation of electric drives prompts preliminary mathematical modeling, which determines the relevance of this work. The purpose of this study is to perform a series of numerical experiments to assess the ability of certain structures of control systems to implement properties potentially inherent in them due to parametric optimization, in particular, the ability to reproduce the calculated optimal trajectory in terms of speed under conditions of intermediate coordinate constraints. To achieve the goal, the transient processes of a relay system with a cascade structure when controlling an electric drive with different types of feedback in typical operating modes were calculated in the work. The results of the work consist in a reasonable choice of electric drive structures that optimally correspond to the specific modes of its operation.

Keywords*: relay controller, N–i switching method, position control system, derivatives observer, sliding mode.*

Необхідність адаптації систем, синтезованих у результаті застосування сучасних методів теорії автоматичного керування, до новітніх технічних рішень у сфері апаратної реалізації електроприводів спонукає до попередніх досліджень, виконуваних шляхом математичного моделювання, що зумовлює актуальність цієї роботи.

Окремий напрямок параметричної оптимізації за швидкодією релейних систем керування, оснований на методі N–і переможень, демонструє високу ефективність завдяки орієнтації на порівняно вузький клас структур електроприводів, але допускає певну варіативність структурних рішень для врахування технічних вимог та наявних засобів їх імплементації. Мета цього дослідження полягає у виконанні серії чисельних експериментів для оцінки спроможності певних структур систем керування до реалізації властивостей, потенційно притаманних їм завдяки параметричній оптимізації, зокрема, на здатність до відтворення розрахункової оптимальної за швидкодією траєкторії в умовах обмеження проміжних координат.

Для досягнення поставленої мети у роботі були здійснені такі дослідження: розраховані перехідні процеси релейної системи з каскадною структурою при керуванні електроприводом зі зворотними зв'язками за швидкістю та прискоренням в режимі позиціонування та стабілізації положення за умов постійного впливу сталого та змінюваного у часі активного моменту опору а також в режимі захоплення та відпрацювання синусоїдальної траєкторії руху; розраховані перехідні процеси релейної системи з каскадною структурою при керуванні електроприводом зі зворотними зв'язками за першою та другою похідними похибки керування в режимі захоплення та відпрацювання синусоїдальної траєкторії руху.

Результати роботи полягають у виборі структур електропривода, що оптимально відповідають типовим режимам його роботи. Обґрунтуванням цього вибору є висновки щодо статичних та динамічних властивостей структур, основані на матеріалах виконаного дослідження.

***Ключові слова**: релейний регулятор, метод N–і перемикань, система керування положенням, спостерігач похідних, ковзний режим.*

Problem's Formulation

Control systems of the electric drives with very similar technical implementation and parameters [1] can exhibit fundamentally different static and dynamic characteristics [2] when closed by different types of feedback signals. Therefore, when selecting the structure of electromechanical systems, it is important to find solutions [3] that are balanced in terms of implementation complexity and achievable control quality.

Analysis of recent research and publications

Relay control systems, possessing a number of progressive properties [4], best meet the requirements for electromechanical systems in terms of insensitivity to disturbances, as well as speed and accuracy [5]. Modern automatic control theory convincingly demonstrates the advantages of sliding mode control systems [6], and it has developed a fairly large number of methods for substantiating the structures [7] and synthesis of the parameters of such systems [8], including optimization according to specified criteria [9]. The element base of modern electric drives [1] fully ensures the possibility of applying the principles of discontinuous control [5], since the power section of the overwhelming majority of electromechanical systems is based on high-speed power converters [6]. At the same time, the complexity of calculating feedback [10], which provides relay systems with the highest control quality indicators [9], justifies their use only for solving comparatively complex [6] technical problems.

Formulation of the study purpose

Position control systems are distinguished by a wide range of applications [1] and, accordingly, a wide range of requirements for the accuracy and speed of electric drives, as well as a variety of conditions in which these requirements must be met [2, 6]. The objective of this work is to determine structural solutions for relay control systems for electric drives [9] that satisfy various operating conditions, by comparatively evaluating their dynamics and statics with different methods of calculating feedback on intermediate coordinates [10].

Presenting main material

The equations of dynamics of a positional electromechanical system with a rigid kinematic chain can be written in the following form [9]:

$$\left.\begin{aligned} &p\varphi = \omega \\ &p\omega = \varepsilon = \frac{k_p \cdot c}{J} \cdot (i - i_s) \\ &p\varepsilon = a = \frac{k_p \cdot c}{J} \cdot \frac{u - R \cdot i - c \cdot \omega / k_p}{L} \end{aligned}\right\}, \qquad (1)$$

where u is the converter voltage, i, i_s are the armature current and the static current, $k_p, R, L, J, c = k\Phi$ are the parameters of the electromechanical system, $a, \varepsilon, \omega, \varphi$ are the angular jerk, acceleration, velocity and position of the actuator shaft.

A cascade of relay controllers of a three-loop system, which allows optimizing the electric drive for speed under conditions of velocity and acceleration limitations at levels $\omega_{max}, \varepsilon_{max}$ is described by a system of equations [9]

$$\left.\begin{aligned} &u_{R\varphi} = \omega^* = -\omega_{max} \cdot sign\left(\varphi - \varphi^* + K_{\varphi\omega} \cdot \omega + K_{\varphi\varepsilon} \cdot \varepsilon\right) \\ &u_{R\omega} = \varepsilon^* = -\varepsilon_{max} \cdot sign\left(\omega - \omega^* + K_{\omega\varepsilon} \cdot \varepsilon\right) \\ &u_{R\varepsilon} = u^* = -u_{max} \cdot sign\left(\varepsilon - \varepsilon^*\right) \end{aligned}\right\} \qquad (2)$$

in which the voltage amplitude is designated as u_{max}, and the symbol "*" marks the specified values of the quantities.

The controller (2) combines structural solutions aimed at solving several problems. The cascade structure provides constraints on internal coordinates during the transients, which is the basis for parametric optimization in terms of speed with the *N–i* switching method [9]. The use of sliding mode controllers based on relay elements provides the ability to compensate for parametric and coordinate disturbances. Feedback on the higher derivatives of the controlled coordinate not only gives the system astatic properties, but also ensures the formation of design motion profiles regardless of the form of the setting and disturbing influences. At the same time, the implementation of some structural properties creates conditions for a more effective manifestation of others.

The implementation of a cascade relay controller (2) is a fairly simple technical task. However, the justification of the method of obtaining feedback signals deserves special attention, since the calculation of derivatives of measured values requires significant complication of the control system.

Systems for stabilizing the position of the working body, for which the conditions

$$p\left(\varphi - \varphi^*\right) = \omega, \; p^2\left(\varphi - \varphi^*\right) = \varepsilon,$$

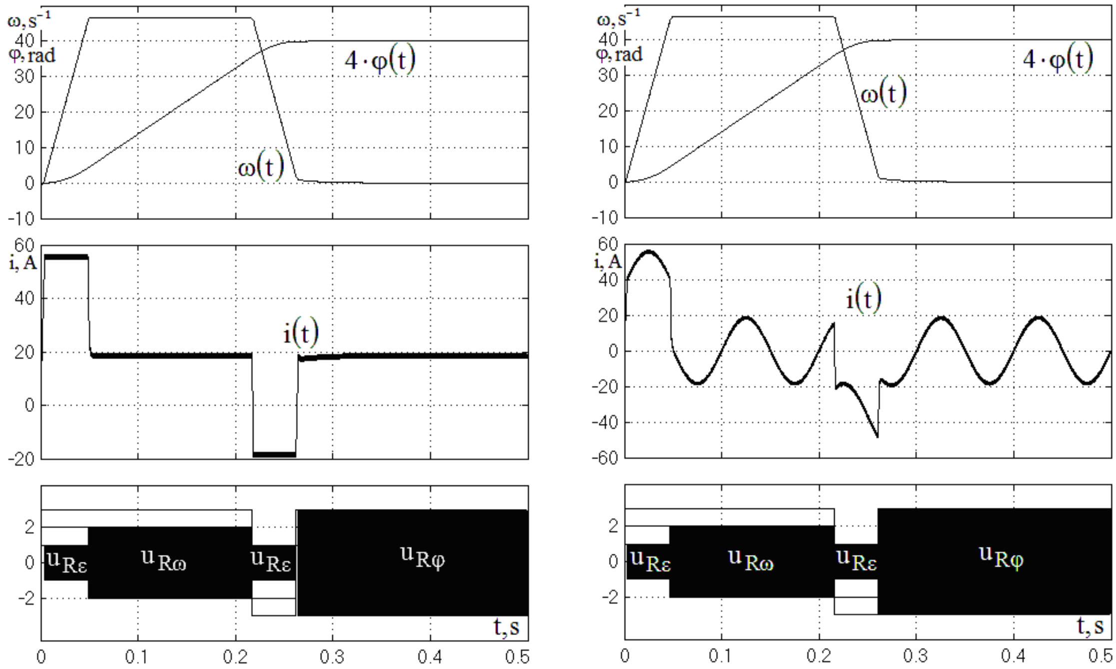
are always fulfilled, require the calculation of only one time derivative of the coordinate ω, which is measured to close the velocity stabilization loop. For such systems, it is possible to replace the variable

$$\varepsilon = \hat{\varepsilon}, \qquad (3)$$

where $\hat{\varepsilon}$ is the acceleration obtained using the first-order derivative observer.

Fig. 1, a shows the transients of the system, described by equations (1), (2), (3), under the conditions of constant action of the active resistance torque. Full compensation of the disturbance becomes possible due to exceeding the given level of current limitation by its instantaneous value. This happens only at certain time intervals, but in order to realize this effect, the power part of the electric drive must have a margin of voltage amplitude and overload capacity. It is important to emphasize that in these conditions the system not only demonstrates astatic properties, but also preserves the correspondence of the shape of its transient to the time-optimal motion profile.

Positioning electric drives, which include acceleration observers, have low sensitivity in relation to both time-invariant moments of resistance and arbitrarily changing ones, if the armature currents compensating for them do not exceed the permissible values in terms of amplitude and growth rate. In confirmation, we will model the reaction of the electric drive to the influence of the active torque, which varies according to the sinusoidal law. Such a test signal is characterized by the difference from zero of the derivatives of all orders, which removes restrictions on the estimated order of



a) $M_c = M_н$ b) $M_c = M_н \, sin(62{,}83t)$

Fig. 1. Dynamics of the positional electric drive with feedback on acceleration

astatism. The graphs of the main values of the system for the process of a large travel occurring under the conditions of action of a sinusoidal moment with a frequency of $f = 10\,\text{Hz}$ (Fig. 1, b) confirm the invariance of the system with acceleration feedback to the form of the disturbance, namely: astatism and compliance with the optimal motion profile in terms of speed are preserved. The additional component of the armature current effectively suppresses all deviations from the prescribed movement. This component automatically acquires a sinusoidal shape, since the controller R_ε purposefully forms uniformly accelerated movements.

Let's consider the transient processes of the relay system of electric drive control (1), (2), (3) under zero initial conditions after applying a harmonic signal $\varphi^*(t) = 10\,cos(4t)$ to the input. Let's set the zero value of the static current $i_s = 0$ in order to ignore the reaction to the disturbance. The tracking mode in such a system must be preceded by the process of capturing the trajectory (Fig. 2). The time diagrams obtained by modeling noticeably deviate from the optimal ones in terms of speed, and the amplitude of the position control error reaches 10% of the input signal. It is obvious that the identified problem is caused by the structural properties of the system under study. Feedback on the velocity and acceleration of the system (1), (2), (3) does not take into account the derivatives of the setting signal, which in the servo drive are different from zero, which makes it natural that such a system cannot implement an arbitrarily changing trajectory. Therefore, the solution to the problem consists in eliminating this structural deficiency by closing with feedback loops that include the missing information, that is, signals of the first and second derivatives of the error $\Delta\varphi = \varphi - \varphi^*$. As a result of such refinement, the cascade controller (2) takes the form

$$\left.\begin{aligned} u_{R\varphi} &= u_{R1} = \omega^* = -\omega_{max} \cdot sign\left(\Delta\varphi + K_{\varphi\omega} \cdot p\Delta\varphi + K_{\varphi\varepsilon} \cdot p^2\Delta\varphi\right) \\ u_{R\omega} &= u_{R2} = \varepsilon^* = -\varepsilon_{max} \cdot sign\left(p\Delta\varphi - \omega^* + K_{\omega\varepsilon} \cdot p^2\Delta\varphi\right) \\ u_{R\varepsilon} &= u_{R3} = u^* = -u_{max} \cdot sign\left(p^2\Delta\varphi - \varepsilon^*\right) \end{aligned}\right\}. \qquad (4)$$

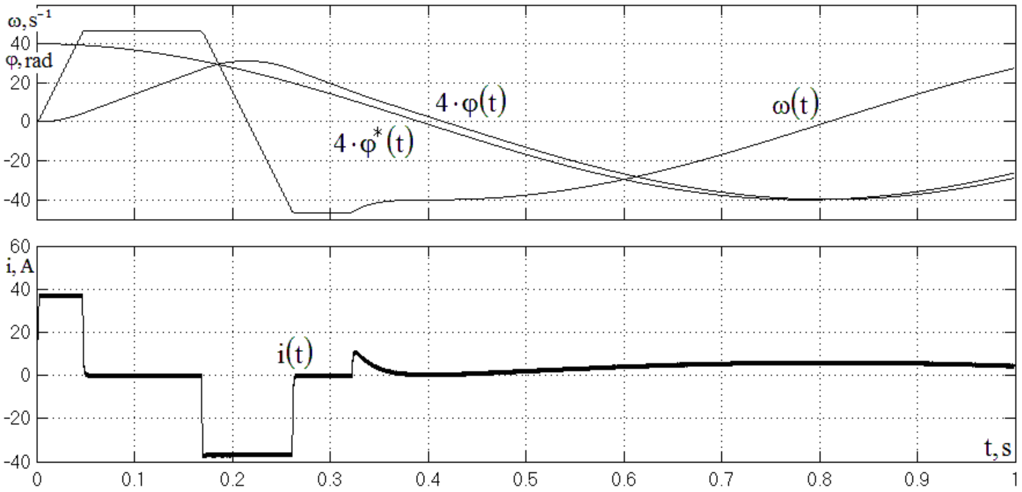


Fig. 2. Transients of the servo drive with feedback on velocity and acceleration

For the position control error $\Delta\varphi$, the derivatives of the first and second orders in the system of equations (4) are defined as

$$p\Delta\varphi = \omega - p\varphi^*,\ p^2\Delta\varphi = \widehat{\varepsilon} - p^2\varphi^*. \tag{5}$$

Note that the use of feedback (5) gives the coordinates ω^*, ε^* in (4) the physical meaning not of the given values of velocity and acceleration, but of the set values of control error derivatives.

Transients presented in Fig. 3, confirm the minimum trajectory capture time by the servo drive (1), (4), (5). The control system, which implements feedback on canonical coordinates, forms a prescribed motion profile in transient processes, regardless of the specifics of the setting and disturbing effects. This makes it possible to optimize its transients in terms of speed under the conditions of limiting the internal coordinates, providing the necessary order of switching the regulators thanks to the application of the N-i switched method. However, such a system meets the highest requirements for tracking accuracy, which is carried out in a quasi-steady state. In Fig. 3, this state corresponds to the

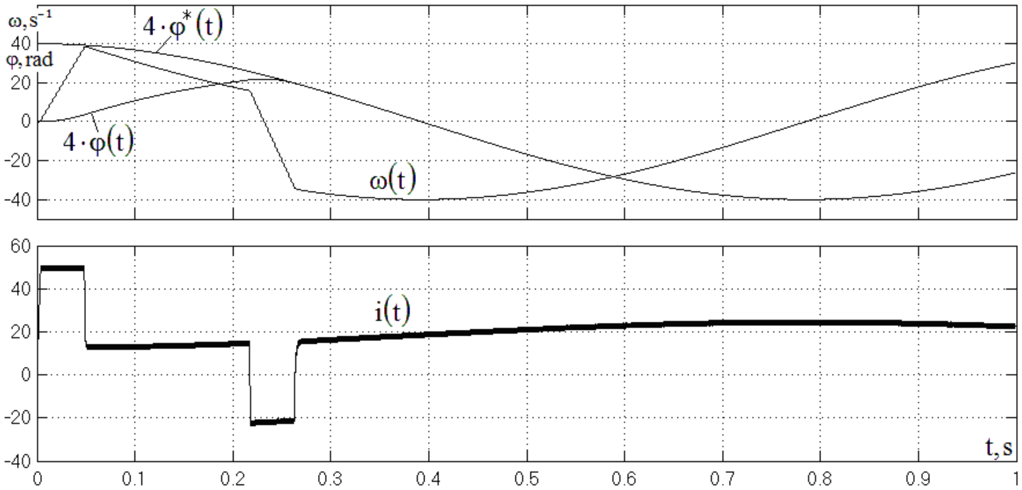
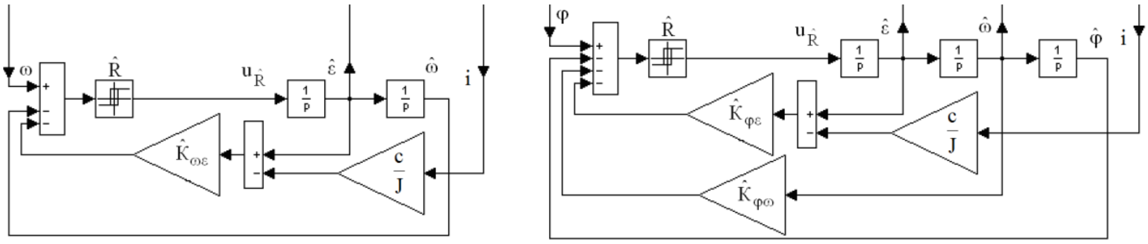


Fig. 3. Trajectory capture and tracking modes of the servo drive with feedback on the first- and second-order derivatives of the control error

merging of the diagrams of the set position $\varphi^{*}(t)=10\,cos(4t)$ and actual position of the working body of the electric drive. That is, feedbacks (5) not only provide the tracking electric drive with the minimization of the position control error, but also contribute to the implementation of time-optimality in dynamics.

There are important requirements that cannot be avoided when developing devices for differentiating the control error in tracking electric drives with relay control systems. The operation of controllers in the sliding mode is the cause of coordinate ripple that occur with high frequency and must be reproduced by the observer. At the same time, the signal for setting the position of the servo drive is not continuous, it can change abruptly. Therefore, the synthesis of observers requires both ensuring certain frequency characteristics and optimizing their own transients [9, 10].

Structural solutions for derivative observers that correspond to the above-mentioned features are shown in Fig. 4. The introduction of the measured current into the switching functions of relays allows to ensure the desired accuracy of differentiation with respect to high-frequency components, since the largest relative amplitude of ripples is inherent in the current as the coordinate closest to the input. Fig. 4,a shows the structure of the relay acceleration observer for implementing feedback (3). To apply formulas (5), the derivative of the controlled coordinate can be calculated using full-order observers (Fig. 4, b). This relay state observer calculates both the acceleration of the electric drive and the velocity. Its use provides compensation for the absence of a directly measured velocity signal.



a *b*

Fig. 4. Structures of relay-controlled derivative observers

The N–i switching method [9] is the basis for the parametric synthesis of these observers, providing the formation of slip equations. Another specific technique for tuning relay differentiators is to ensure a certain ratio of the amplitude of the pulsations of the observer and electric drive coordinates by forming the hysteresis characteristics of their relays. In this way, the desired ratio of the switching frequencies of relay controllers is established, which guarantees the reliability of the observation of derivatives [10]. The synthesis of differentiators is not within the scope of this work.

Conclusions

A comparative study of the structures of positional electric drives controlled by relay systems indicates that the options with acceleration feedback are absolutely sufficient for optimal position stabilization systems, but they have a limited ability to form time-optimal transients and limited accuracy in tracking modes. The results of the study indicate the need to implement feedbacks based on the derivatives of the control error in electric drives for which the set position value is arbitrarily variable in time. This allows, first of all, to ensure high control accuracy, limited only by measurement errors, and, as a result, contributes to the automatic formation of the motion profile in dynamics invariant to the reproduced trajectory, thereby creating the basis for the implementation of speed-optimality. The results of this work can be significantly deepened by a detailed study of the influence of errors in determining feedback signals on the efficiency of high-speed relay systems with different structural implementation options.

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