

DOI: 10.31319/2519-8106.1(54)2026.351034

UDC 656.13:519.87

Averyanov Volodymyr, Candidate of Technical Sciences, Associate Professor, Department of automobiles and transport and logistics systems

Авер'янов В.С., кандидат технічних наук, доцент, кафедра автомобілів та транспортно-логістичних систем

ORCID: 0000-0001-6594-3964

e-mail: averyanov@ukr.net

Serdyuk Dmytro, undergraduate student, Department of automobiles and transport and logistics systems

Сердюк Д.О., здобувач першого (бакалаврського) рівня вищої освіти, кафедра автомобілів та транспортно-логістичних систем

e-mail: serduk@ukr.net

Dniprovsky State Technical University, Kamianske

Дніпровський державний технічний університет, м. Кам'янське

MODELING OF CAR SERVICE CUSTOMER FLOW BASED ON THE THEORY OF MASS SERVICE

МОДЕЛЮВАННЯ ПОТОКУ КЛІЄНТІВ АВТОСЕРВІСУ НА ОСНОВІ ТЕОРІЇ МАСОВОГО ОБСЛУГОВУВАННЯ

The article presents a theoretical analysis and mathematical modeling of the flow of customers in a car service enterprise. The work of the car service is presented in the form of a queueing system of the M/M/c type. The main analytical dependencies are given for determining the probabilistic and time characteristics of the system, in particular the load factor, the probability of queue formation, the average number of applications in the system and the waiting time of customers. The results obtained can be used to optimize the parameters of the car service.

Keywords: car service, customer flow, queueing theory, mathematical model, queue, optimization.

Швидке зростання світового автопарку та зростаюча складність сучасних автомобілів значно посилили попит на високоякісні операції з обслуговування автомобілів. Станції технічного обслуговування автомобілів (СТО) працюють у висококонкурентному середовищі, де задоволеність клієнтів, надійність обслуговування та операційна ефективність є критичними показниками ефективності. Одним із найскладніших аспектів управління сервісом автомобілів є стохастичний характер прибуття клієнтів та процесів обслуговування, що часто призводить до перевантажень, надмірного часу очікування та неефективного використання ресурсів обслуговування.

Традиційні детерміновані підходи до планування недостатні для врахування цих невизначеностей і часто призводять до неоптимальних рішень щодо планування потужностей, розподілу робочої сили та використання сервісних майданчиків. Отже, існує велика потреба в суворох математичних інструментах, здатних формально описувати ймовірнісну динаміку процесів сервісного обслуговування автомобілів.

Існуючі дослідження операцій автомобільного обслуговування в основному зосереджені на емпіричному аналізі, підходах на основі моделювання або евристичних методах планування. Хоча ці методи пропонують практичні висновки, їм часто бракує аналітичної придатності та вони не забезпечують показників ефективності в закритій формі, необхідних для теоретичного аналізу та оптимізації. Крім того, багато досліджень нехтують явним включенням струк-

тур витрат, пов'язаних з пропускнуою здатністю обслуговування та очікуванням клієнтів, що є важливим для прийняття управлінських рішень.

У роботі здійснено теоретичний аналіз та математичне моделювання потоку клієнтів автосервісного підприємства. Роботу автосервісу представлено у вигляді системи масового обслуговування типу $M/M/c$. Наведено основні аналітичні залежності для визначення імовірнісних та часових характеристик системи, зокрема коефіцієнта завантаження, імовірності утворення черги, середньої кількості заявок у системі та часу очікування клієнтів. Отримані результати можуть бути використані для оптимізації параметрів автосервісу.

Ключові слова: автосервіс, потік клієнтів, теорія масового обслуговування, математична модель, черга, оптимізація.

Problem's Formulation

Car service enterprises belong to complex service systems, the functioning of which is determined by the random nature of the arrival of customers and the stochastic duration of service processes. In such conditions, traditional deterministic planning methods do not always allow an adequate assessment of the efficiency of the enterprise. One of the most justified approaches to the analysis of such systems is the use of the apparatus of the theory of mass service, which allows describing a car service as a stochastic system and determining its main operational indicators.

The rapid growth of the global vehicle fleet and the increasing complexity of modern vehicles have significantly increased the demand for high-quality automotive service operations. Automotive service stations (ATS) operate in a highly competitive environment where customer satisfaction, service reliability, and operational efficiency are critical performance indicators [1]. One of the most challenging aspects of automotive service management is the stochastic nature of customer arrivals and service processes, which often leads to congestion, excessive waiting times, and inefficient use of service resources. From an operational perspective, automotive service systems exhibit inherent randomness caused by unpredictable customer behavior, varying service requirements, and heterogeneous repair durations. Traditional deterministic planning approaches are inadequate to account for these uncertainties and often lead to suboptimal decisions regarding capacity planning, workforce allocation, and service site utilization. Consequently, there is a great need for rigorous mathematical tools that can formally describe the probabilistic dynamics of automotive service processes.

Analysis of recent research and publications

In works [2—3] the authors proposed a method of optimizing the production structure of a car service and options for solving the problem of working with customers. They considered a functional model of choosing a strategy for the form of production organization for high-quality performance of services at a car service enterprise. But this method applies to the organization of work with trucks.

In the work [4—5], the authors considered the optimization of the queue system in automobile workshops using queue theory, simulations, and probabilistic methods. This work contains mathematical formulas and optimization dependencies suitable for comparison with the $M/M/c$ model, substantiation of modern models of optimization of automobile maintenance at service stations.

In the works [4—5], the authors propose a model for assessing service quality based on queueing theory, with experimental verification and analytical formulas for service and waiting times. We considered expanding the literature review on service quality and performance indicators. We proposed a systematic review of queueing networks models, which are widely used for the analysis and optimization of complex service and logistics systems. Although not specifically dedicated to car services, the methodology is well suited to extended queueing models in multi-channel SMOs.

Formulation of the study purpose

The purpose of the study is to develop theoretical foundations for modeling the flow of car service customers and obtain analytical dependencies for assessing the effectiveness of its work.

To achieve the goal, the following tasks were set:

- to formalize the operation of the car service as a mass service system;
- to derive the main calculation formulas;
- to investigate the influence of customer flow parameters on the performance of the system.

Presenting main material

There is a wide class of discrete dynamic systems that have received the general name of "Queuing Systems" (QS). Processes in QS are usually complex and most often random. But the need for their modeling is constantly growing, since computer network servers, without which the modern information society is unthinkable, are typical representatives of QS. Queueing systems are such systems that receive requests (requests, requirements) for service at random moments of time, while the received requests are serviced using the service channels available to the system [3].

From the perspective of modeling the queuing process, situations when queues of requests (requirements) for service are formed arise as follows. Having entered the service system, the request joins the queue of other (previously received) requirements. The service channel selects a request from those in the queue in order to start servicing it. After completing the procedure for servicing the next request, the service channel starts servicing the next request, if there is one in the queue. The cycle of operation of a mass service system of this kind is repeated many times during the entire period of operation of the service system. It is assumed that the system switches to servicing the next request after completing servicing the previous request instantly, at random moments in time.

A flow is a set of objects that are perceived as a single whole, and exists as a process at a certain time interval and is measured in absolute units for a certain period. Flow parameters are parameters that characterize the process. The main flow parameters are: initial and final points of movement, movement trajectory, path length (measure of the trajectory), speed and time of movement, intermediate points, intensity [3].

Flows are classified according to various characteristics:

1) in relation to a given system (internal flows — circulate within the system, external flows — are outside the system);

2) by the degree of continuity (continuous flows — at each moment of time, a certain number of objects move along the flow trajectory; discrete flows — are formed by objects that move with intervals);

3) by the degree of stability (stable flows — characterized by the constancy of parameter values over a certain period of time, unstable flows — characterized by changes in flow parameters).

They are also distinguished by the nature of movement, periodicity, synchronicity, and so on.

The incoming flow of customers to a car service is generally random and can be described by a Poisson process. The probability of the arrival of k cars in a time interval t is given by the formula:

$$P(k, t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \quad (1)$$

where λ — is the average intensity of customer arrivals.

The service time of one car at a car service station is considered to be an exponentially distributed random variable:

$$f(t) = \mu e^{-\lambda t}, \quad (2)$$

where μ — is the intensity of service.

The car service is modeled as a mass-service system of type M/M/c with an unlimited queue. Loading coefficient, provided that the ergodicity criterion is met, is given by the formula

$$\rho = \frac{\lambda}{c\mu} \leq 1. \quad (3)$$

This value characterizes the degree of utilization of car service posts. At $\rho \geq 1$, the system is unstable, and the queue length grows indefinitely.

Stationary probabilities of the distribution are determined by the formula

$$P_n = \begin{cases} \frac{(\lambda / \mu)^n}{n!} P_0, & 0 \leq n \leq c, \\ \frac{(\lambda / \mu)^c}{c!} \rho^{n-c} P_0, & n \geq c. \end{cases} \quad (4)$$

The probability that there is no customer in the system:

$$P_0 = \left[\sum \frac{(\lambda / \mu)^n}{n!} + \frac{(\lambda / \mu)^c}{c!} \cdot \frac{1}{1 - \rho} \right]^{-1}. \quad (5)$$

The performance indicators obtained from the selected model will be calculated according to the following dependencies.

The average number of customers in the queue is determined by the formula

$$L_q = \frac{(\lambda / \mu)^c \cdot \rho}{c!(1 - \rho)^2} P_0. \quad (6)$$

The average number of customers in the system is determined by the formula

$$L_s = L_q + \frac{\lambda}{\mu}. \quad (7)$$

The average waiting time of a customer in the queue is determined by the formula

$$W_q = \frac{L_q}{\lambda}. \quad (8)$$

The average time a car stays in the car service is determined by the formula

$$W = W_q + \frac{1}{\mu}. \quad (9)$$

To determine the optimal number of service posts, we introduce the function of total expected costs:

$$J(c) = C_s \cdot c + C_w \cdot L_q(c), \quad (10)$$

where C_s — is the average operating cost of one post per unit of time; C_w — is the specific loss associated with customer waiting; $L_q(c)$ — is the average queue length as a function of the number of posts.

Since the function $J(c)$ is discrete and unimodal, the optimal value of c^* can be found by brute force or by limit analysis:

$$J(c+1) - J(c) \geq 0. \quad (11)$$

The optimal value of c^* is reached at the point where the marginal costs of opening an additional post are equal to the marginal losses from reducing the queue:

$$C_s \approx C_w \cdot [L_q(c) - L_q(c+1)]. \quad (12)$$

This ensures a balance between the level of service and the operating costs of the car service.

Conclusions

The obtained analytical dependencies allow us to study the influence of the intensity of the flow of customers and the number of service posts on the key performance indicators of the car service. In particular, an increase in the number of posts reduces the waiting time, but leads to an increase in costs, which requires finding an optimal compromise.

The article deepens the theoretical principles of modeling the flow of customers in a car service. The proposed mathematical model of the M/M/c type allows us to analytically evaluate the probabilistic and time characteristics of the mass service system. The obtained results can be used to optimize the structure of car service enterprises and increase the efficiency of their operation.

References

- [1] Blyznyuk O. (2024). *Marketing in Ukraine, problems of formation and development [Marketynh v Ukraini, problemy stanovlennia ta rozvytku]*. K.: IVC "Polytechnica". [in Ukrainian].
- [2] Markov O., Kovalev A., Skyba A. (2016). Optimization of the production structure of a car service [Optymizatsiia vyrobnychoi struktury avtoservisu]. *Scientific and technical collection Bulletin of the National Transport University. Series "Technical Sciences"*. Kyiv, NTU. Vol. 1(34). P. 247–254. [in Ukrainian].

- [3] Tarandushka L., Kostyan N. (2018). Functional model of choosing strategies and forms of production organization for high-quality service provision at a car service enterprise [Funktionalna model vyboru stratehii formy orhanizatsii vyrobnytstva dlia yakisnoho vykonannia posluh na avtoservisnomu pidpriemstvi]. *Modern Technologies in Mechanical Engineering and Transport*. Vol. 1 (10), P. 131–136. [in Ukrainian].
- [4] Royer-Torney M., Mennenga M. (2020). A Reference Model for Analyzing Automotive Service Formats. *Herrmann Institute of Machine Tools and Production Technology*. Brunswick, Germany. P. 355–362.
- [5] Sugianto W., Abdul Haq R., Abdul Rahman M. (2024). Automobile workshop queue system optimization using response surface methodology. *International Journal of Industrial Engineering & Production Research*. Vol. 35(1). P. 1–20.
- [6] Zhao X., Gilbert K. (2025). A queuing theory–based approach to evaluate service delivery quality. *International Journal of Quality and Service Sciences*. Vol. 17(2), P. 155–167.
- [7] Kerbache L. (2022). Queueing network models for the analysis and optimisation of material handling systems: A literature review. *Flexible Services and Manufacturing Journal*. Vol. 55(1). P. 60–70.
- [8] Markov O.D. (2018). *Organization of a car service [Orhanizatsiia avtoservisu]*. Lviv: Oriyana-Nova. [in Ukrainian].
- [9] Kudenko N.V. (2022). *Marketing strategies of car service firms [Marketynhovi stratehii firm avtoservisu]*. Kyiv: KNEU. [in Ukrainian].

Список використаної літератури

1. Близнюк О.В. Маркетинг в Україні, проблеми становлення та розвитку. 2-ге вид., випр. і допов. К.:ІВЦ "Політехніка", 2024. 400 с.
2. Марков О.Д., Ковальов А.В., Скиба А.П. Оптимізація виробничої структури автосервісу. *Науковотехнічний збірник Вісник національного транспортного університету. Серія «Технічні науки»*. К.: НТУ, 2016. № 1(34). С. 247–254.
3. Тарандушка Л.А., Костьян Н.Л. Функціональна модель вибору стратегії форми організації виробництва для якісного виконання послуг на автосервісному підприємстві. *Сучасні технології в машинобудуванні та транспорті*. 2018. № 1 (10), С. 131–136.
4. Royer-Torney M., Mennenga M. A Reference Model for Analyzing Automotive Service Formats. *Herrmann Institute of Machine Tools and Production Technology, Brunswick, Germany*. 2020, P. 355–362.
5. Sugianto W., Abdul Haq R., Abdul Rahman M. Automobile workshop queue system optimization using response surface methodology. *International Journal of Industrial Engineering & Production Research*. 2024. Vol. 35(1). P. 1–20.
6. Zhao X., Gilbert, K. A queuing theory–based approach to evaluate service delivery quality. *International Journal of Quality and Service Sciences*. 2025. Vol. 17(2). P. 155–167.
7. Kerbache L. Queueing network models for the analysis and optimisation of material handling systems: A literature review. *Flexible Services and Manufacturing Journal*. 2022. Vol. 55(1). P. 60–70.
8. Марков О.Д. Організація автосервісу. Львів: Оріяна-Нова, 2018. 332 с.
9. Куденко Н.В. Маркетингові стратегії фірм автосервісу. К.: КНЕУ, 2022. 245 с.

Надійшла до редколегії 06.01.2026

Прийнята після рецензування 12.01.2026

Опублікована 22.01.2026