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MULTICRITERIA MATHEMATICAL MODELING OF TRANSPORT FLOWS IN INDUSTRIAL REGIONS UNDER UNCERTAINTY WITH CONSIDERATION OF SUSTAINABLE DEVELOPMENT GOALS

БАГАТОКРИТЕРІАЛЬНЕ МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ТРАНСПОРТНИХ ПОТОКІВ ПРОМИСЛОВИХ РЕГІОНІВ В УМОВАХ НЕВИЗНАЧЕНОСТІ З УРАХУВАННЯМ ЦІЛЕЙ СТАЛОГО РОЗВИТКУ

The paper develops a methodological framework for multicriteria mathematical modeling of transport flows in city-logistics systems of industrial regions under conditions of uncertainty. The transport system is formalized as a directed network, for which a vector-valued objective function is constructed, integrating economic, energy-related, and environmental management criteria in accordance with the principles of sustainable development. A universal formulation of the transport flow optimization problem is proposed, along with methods for transforming the multicriteria model into a scalar optimization problem using scalarization procedures. Particular attention is devoted to methodological approaches for incorporating uncertainty in transport system parameters. The results obtained are of a theoretical and methodological nature and may serve as a foundation for further applied research in the field of transport flow management and city-logistics systems in industrial regions.

***Keywords**: multicriteria optimization, transport flows, mathematical modeling, uncertainty, sustainable development, city logistics.*

У статті сформовано цілісну методологічну концепцію багатокритеріального математичного моделювання транспортних потоків у сіті-логістичних системах промислових регіонів за умов наявності невизначеності. Транспортна система розглядається як складна ієра-

рхічна структура, формалізована у вигляді орієнтованої мережі, що відображає просторово-функціональні взаємозв'язки між елементами регіональної логістичної інфраструктури. У межах запропонованого підходу побудовано векторну функцію мети, яка інтегрує сукупність взаємопов'язаних критеріїв управління, зокрема економічні витрати, енергетичну ефективність та екологічні обмеження, узгоджені з концептуальними засадами сталого розвитку.

Запропоновано узагальнену математичну постановку задачі оптимізації транспортних потоків, що не залежить від конкретних параметрів регіону або виду транспорту та може бути адаптована до різних умов функціонування сіті-логістичних систем. Значну увагу приділено методам приведення багатокритеріальної задачі до еквівалентної скалярної форми з використанням процедур скаляризації, що дозволяє формально описати компромісні рішення між конфліктними цілями управління.

Окремий розділ присвячено методологічним аспектам урахування невизначеності параметрів транспортної системи, яка виникає внаслідок мінливості попиту, нестабільності часових характеристик та неповноти інформації. Показано, що включення невизначеності в математичну модель підвищує її адекватність і розширює можливості застосування в задачах стратегічного планування. Отримані результати мають узагальнений теоретико-методологічний характер і можуть бути використані як наукова основа для подальших прикладних досліджень та розвитку інструментарію управління транспортними і сіті-логістичними системами промислових регіонів.

***Ключові слова**: багатокритеріальна оптимізація, транспортні потоки, математичне моделювання, невизначеність, сталий розвиток, сіті-логістичні системи.*

Problem's Formulation

The operation of transport systems in industrial regions is marked by a complex network of interrelated flows, high transportation intensity, and a strong dependence on both external and internal factors. These transport flows are shaped by production processes, logistics chains, infrastructure constraints, and socio-economic conditions, highlighting the need for formalized methods to analyze and manage them.

The contemporary challenges in the development of industrial regions are associated not only with improving the efficiency of transport processes but also with the necessity of adhering to the principles of sustainable development [3, 4, 9]. Reducing energy consumption, cutting pollutant emissions, and optimizing the use of transport infrastructure have become goals of equal importance alongside economic efficiency [3, 9].

An additional complicating factor is the presence of uncertainty, manifested in fluctuations in transport demand, variability in traffic parameters, instability of energy characteristics, and the influence of random events [1, 2, 7]. Under such conditions, traditional single-objective deterministic models do not provide a sufficiently accurate representation of transport systems [1, 5].

In this context, advancing methodological approaches for multi-criteria mathematical modeling of transport flows—considering both uncertainty and sustainable development objectives—becomes particularly relevant. This article is dedicated to the development of such an approach.

The aim of this work is to develop a universal methodology for multi-criteria mathematical modeling of transport flows in industrial regions under uncertainty, with explicit consideration of sustainable development goals.

Analysis of recent research and publications

Mathematical modeling of transport flows has traditionally relied on network-based representations, in which a transport system is described as a set of nodes and the connections between them. Classical deterministic network models enable efficient solutions to problems such as cost minimization, capacity maximization, or shortest-path determination. However, these models assume complete certainty of parameters, which limits their applicability in real industrial regions characterized by significant fluctuations in demand, travel times, and network capacity.

To overcome these limitations, stochastic and interval-based approaches to modeling transport flows have been proposed [1, 2, 10]. Stochastic models account for random fluctuations in traffic volumes or travel times, while interval models describe parameters as ranges, allowing for the incorporation of incomplete or imprecise information. These approaches enhance the realism and flexibility of modeling but simultaneously complicate optimization procedures and the interpretation of results. Accurate formulation of objective functions and constraints becomes particularly critical to ensure robust and reliable solutions [5, 7, 8].

In the context of sustainable development, transport systems should be analyzed from a multi-criteria perspective, as the objectives of economic efficiency, energy conservation, and environmental protection often conflict [3, 4]. Optimization based solely on costs or capacity may lead to increased emissions or energy consumption, while prioritizing environmental criteria can negatively impact overall system performance. This inherent trade-off justifies the use of vector-based management models, which enable the simultaneous consideration of multiple performance criteria [3, 4, 9].

Consequently, the management of transport flows is formulated as a multi-criteria optimization problem under conditions of system parameter uncertainty.

Formulation of research purpose

The aim of this study is to develop a comprehensive methodological framework for multi-criteria modeling of transport flows within city-logistics systems of industrial regions, explicitly accounting for uncertainty in system parameters. The research focuses on the formalization of transport networks as directed graphs, where nodes represent industrial, logistics, or distribution centers, and arcs correspond to transport links characterized by flow intensity, capacity, economic costs, energy consumption, and environmental impact [1, 6].

Within this framework, the transport system of an industrial region is represented as a directed graph, which is a standard approach for analyzing transport and logistics networks. The management of transport flows is formulated as a multi-criteria optimization problem under uncertainty, allowing the identification of Pareto-optimal solutions and a systematic analysis of trade-offs between conflicting objectives [5, 7, 8].

Thus, the proposed methodological approach provides a formalized basis for further applied research and for the development of tools to manage transport and city-logistics systems in industrial regions in accordance with the principles of sustainable development.

Presenting main material

Математичне представлення транспортної системи.

The transport system of the industrial region is represented as a directed graph:

$$G = (V, E), \tag{1}$$

where $V = \{1, 2, \dots n\}$ — the set of nodes (transport districts, logistics centers, industrial zones); $E \subseteq V \times V$ — the set of arcs corresponding to transport connections. The control variable is the intensity of the transport flow x_{ij} on the arc $(i, j) \in E$.

For each arc$(i, j) \in E$, the following variables and parameters are introduced: x_{ij} — transport flow intensity; c_{ij} — unit economic cost; e_{ij} — unit emissions (CO_2 equivalent); τ_{ij} — travel time; u_{ij} — arc capacity.

The uncertainty of the parameters is taken into account by representing them as intervals or random variables: $c_{ij} \in [\underline{c}_{ij}, \overline{c}_{ij}]$, $e_{ij} \in [\underline{e}_{ij}, \overline{e}_{ij}]$, $\tau_{ij} \in [\underline{\tau}_{ij}, \overline{\tau}_{ij}]$.

Система обмежень транспортного балансу.

For each node $i \in V$, the flow balance equation is satisfied:

$$\sum_{j:(i,j)\in E} x_{i,j} - \sum_{j:(j,i)\in E} x_{j,i} = d_i, \tag{2}$$

where $d_i -$ denotes the net demand (or supply)of transport flows at node i.

The capacity constraints are given by:

$$0 \le x_{ij} \le u_{ij}, \quad \forall (i, j) \in E. \tag{3}$$

Construction of the vector objective function.

Considering the goals of sustainable development, the problem is formulated as a multi-objective optimization task.

Criterion 1. Minimization of economic costs:

$$F_1(x) = \sum_{(i,j)\in E} c_{ij}\, x_{ij}. \tag{4}$$

Criterion 2. Minimization of environmental impact:

$$F_2(x) = \sum_{(i,j)\in E} e_{ij}\, x_{ij}. \tag{5}$$

Criterion 3. Minimization of transportation time.

$$F_2(x) = \sum_{(i,j)\in E} \tau_{ij}\, x_{ij}. \tag{6}$$

Thus, the multicriteria optimization problem takes the form:

$$F(x) = (F_1(x), F_2(x), F_3(x)) \longrightarrow \min \tag{7}$$

Methods for Scalarization of the Multi-Criteria Problem.

Linear Combination of Criteria.

A weight vector is defined:

$$\alpha = (\alpha_1, \alpha_2, \alpha_3),\ \ \alpha_k \geq 0,\ \sum_{k=1}^{3} \alpha_k = 1. \tag{8}$$

Scalarized Objective Function:

$$F(x) = \alpha_1 F_1(x) + \alpha_2 F_2(x) + \alpha_3 F_3(x). \tag{9}$$

Minimax (Chebyshev) Scalarization:

The ideal values for each criterion are defined as:

$$F_k^* = \min_{x\in X} F_k(x), \qquad k = 1,2,3. \tag{10}$$

Problem Formulation:

$$\min_{x\in X}\ \min_{k=1,2,3} \{\alpha_k |F_k(x) - F_k^*|\}. \tag{11}$$

ε- constraints:

$$\min F_1(x), \tag{12}$$

$$F_2(x) \leq \varepsilon_2, \tag{13}$$

$$F_3(x) \leq \varepsilon_3. \tag{14}$$

Solution Method and Algorithmic Implementation.

Accounting for Parameter Uncertainty.

For interval-based parameters, a robust optimization formulation is applied [1, 5, 7]:

$$\min_{x\in X}\ \max_{\varepsilon\in\Xi} F(x,\varepsilon), \tag{15}$$

where ε – vector of uncertain parameters; Ξ – set of their admissible values.

In the stochastic form [2, 10]:

$$\min_{x\in X} \mathrm{E}[F(x,\varepsilon)]. \tag{16}$$

General Solution Algorithm.

Step 1. Construct the transportation system graph and define the set of feasible solutions X.

Step 2. Select a scalarization method and determine the associated weights or threshold values.

Step 3. Develop a scalar robust or stochastic objective function.

Step 4. Execute the optimization procedure to obtain the solution - $\min_{x\in X} F(x)$ using the following methods: linear programming; robust optimization; Pareto-based multi-criteria analysis.

Крок 5. Аналіз структури множини Парето-оптимальних рішень.

Методологічна інтерпретація результатів.

Отримана множина рішень:

Step 5. Analyze the structure of the Pareto-optimal solution set.

Methodological interpretation of the results.

Resulting solution set:

$$X^P = \{x \in X \mid \nexists\, y \in X : F(y) \prec F(x)\}. \tag{17}$$

Enables managerial decision-making in alignment with strategic sustainable development goals.

Ensures the correctness of the multi-criteria formulation.

Let the set of feasible solutions $X \subset R^{|E|}$ be defined by a system of linear constraints representing transport balance and capacity limits. Since all constraints are linear, the set X is convex and closed.

The vector objective function:

$$F(x) = (F_1(x), F_2(x), F_3(x)). \tag{18}$$

is continuous on X, since each criterion is a linear form.

Proposition 1.

The set of Pareto-optimal solutions for the multi-criteria optimization problem is non-empty.

Justification.

The continuity of $F(x)$ and the compactness of X ensure the existence of minimal elements with respect to the Pareto partial order.

Properties of Scalarized Problems.

Linear Aggregation of Criteria.

Consider the scalarized problem:

$$\min_{x \in X} F(x) = \sum_{k=1}^{3} \alpha_k F_k(x), \qquad \alpha_k > 0. \tag{19}$$

Proposition 2.

Any optimal solution of the linear aggregation problem is Pareto-optimal for the original multi-criteria problem.

Proof (sketch).

Assume that there exists $y \in X$, that dominates x^*. Then $F_k(y) \leq F_k(x^*)$ for all k, and strictly for at least one which contradicts the optimality of x^*.

Minimax Scalarization.

Consider the problem:

$$\min_{x \in X} \max_{k} |F_k(x) - F_k^*|. \tag{20}$$

Proposition 3.

The solution obtained via minimax scalarization belongs to the set of weakly Pareto-optimal solutions.

This approach ensures a uniform approximation to the ideal point, which is particularly important in strategic management problems of transportation systems.

Theoretical Aspects of Uncertainty Consideration.

Robust Optimization

Let the problem parameters depend on an uncertainty vector $\varepsilon \in \Xi$, where $\Xi -$ is a convex and compact set.

The robust optimization problem is formulated as:

$$\min_{x \in X} \max_{\varepsilon \in \Xi} F(x, \varepsilon). \tag{21}$$

Твердження 4.

Under the convexity of $F(x, \varepsilon)$ in x and the compactness of X and Ξ, the robust optimization problem admits an optimal solution.

Stochastic Formulation.

Let $\varepsilon -$ be a random vector with a known distribution.

Consider the problem:

$$\min_{x \in X} E[(x, \varepsilon)]. \tag{22}$$

Proposition 5.

If $F(x, \varepsilon)$ is almost surely convex x then the expected value preserves convexity, and the problem possesses a global minimum.

Theoretical Interpretation from the Perspective of Sustainable Development.

The multi-criteria structure of the model allows interpreting sustainable development as a compromise between mutually conflicting criteria [3, 4, 9]. Formally, sustainable development corresponds to selecting a solution from the Pareto set:

$$x^{SD} \in X^P, \tag{23}$$

which satisfies normative or strategic constraints on environmental and socio-economic indicators.

Conclusions

This study develops a generalized methodological framework for multi-criteria mathematical modeling of transportation flows in industrial regions under uncertainty, while explicitly incorporating sustainable development objectives. The transportation system is formalized as a directed network with linear flow balance and capacity constraints, ensuring the mathematical soundness of the formulation and enabling the application of multi-criteria optimization techniques.

A vector-valued objective function has been constructed, integrating economic, environmental, and temporal criteria for transportation flow management. It is demonstrated that this formulation adequately captures the conflicting nature of transportation system objectives and allows a transition from single-criterion solutions to a systematic analysis of trade-offs within the Pareto-optimal solution space. The non-emptiness of the Pareto-optimal set is theoretically justified, and the relationship between the multi-criteria problem and its scalarized forms is established.

A key outcome is the methodological integration of uncertainty into the model through robust and stochastic formulations, which enhances the resilience of the obtained solutions to variations in system parameters. The proposed solution algorithm is of a universal nature and can serve as a theoretical foundation for the further development of strategic management tools for transportation and city-logistics systems in industrial regions within the context of sustainable development.

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