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RESULTS OF MATHEMATICAL MODELING OF IRRIGATION SYSTEM DESIGN USING THE THEORY OF OPTIMAL PARTITION OF SETS

РЕЗУЛЬТАТИ МАТЕМАТИЧНОГО МОДЕЛЮВАННЯ ПРОЄКТУВАННЯ ЗРОШУВАЛЬНИХ СИСТЕМ ЗАСОБАМИ ТЕОРІЇ ОПТИМАЛЬНОГО РОЗБИТТЯ МНОЖИН

The article develops a mathematical model and algorithm for solving the problem of optimal design of irrigation systems, which takes into account the needs of plants for water, nutrients, and disease prevention agents. The effectiveness of the developed algorithm was substantiated by the results of numerical experiments. The proposed approach to optimizing irrigation systems will contribute to increasing the economic efficiency of agricultural production, and can also be used as a basis for further research in the field of intelligent design of engineering systems.

***Keywords**: optimal partitioning of sets, design of irrigation systems.*

Робота присвячена дослідженню проблеми оптимального проєктування зрошувальних систем з урахуванням просторового розподілу потреб у воді, поживних речовинах та засобах для запобігання хворобам рослин. Актуальність представленої роботи визначається необхідністю раціонального використання природних ресурсів та експлуатації зрошувальних систем за умов різних природних, кліматичних та технологічних умов. Задача оптимального розподілу зрошуваної площі між кількома зрошувальними станціями зводиться до неперервної багатопродуктової задачі оптимального розбиття множин (ОРМ) з фіксованими центрами або центрами, які потрібно розмістити, при обмеженнях на пропускну здатність станцій. Запропоновано математичну модель, яка враховує щільність розподілу попиту на ресурси, транспортні витрати на їх доставку та зберігання, а також капітальні витрати на будівництво станцій. Для розв'язання задачі було застосовано методи теорії ОРМ, які базуються на зведенні вихідної нескінченновимірної задачі математичного програмування до двоїстої негладкої задачі з подальшим числовим розв'язанням за допомогою модифікованого r-алгоритму Н.З. Шора. Для ряду модельних задач проведено чисельні дослідження, які ілюструють вплив заданих витрат та процесу розміщення станцій на оптимальний розподіл площі зрошення. Запропонований алгоритм забезпечує виконання умов розв'язності задачі та дозволяє отримати узгоджені оптимальні потужності станції відповідно до заданих обмежень. Отримані результати підтверджують ефективність запропонованого підходу та його придатність для оптимізації складних зрошувальних систем. Застосування розробленого алгоритму сприятиме підвищенню ефективності сільськогосподарського виробництва та може бути основою для подальших досліджень у галузі інтелектуального проєктування інженерних систем.

Ключові слова: *оптимальне розбиття множин, проєктування зрошувальних систем.*

Problem's Formulation

The main function of irrigation systems is to deliver water (at a specified time and in a certain amount) from the irrigation source to the irrigated land and, by appropriately distributing it, to ensure optimal soil moisture, the amount of nutrients, and disease prevention agents in the fields for a given phase of plant growth. Depending on the source of nutrition, climatic conditions and the type of irrigated crops, irrigation is divided into regular and one-time. Regular irrigation is characterized by the ability to supply the required amount of water, nutrients or disease prevention agents to irrigated land each time the need arises.

Based on natural conditions determined by relief, groundwater levels, degree of drainage, distance to the irrigation source, soil type, as well as economic prerequisites and the type of crop to be grown, various irrigation methods are used. Namely, surface (strip irrigation, furrow irrigation, flooding of rice fields, estuarine irrigation), sprinkling, micro-irrigation, aerosol or subsurface method (pressure subsurface irrigation, adsorption).

Regardless of the types and methods of irrigation, an important issue is the optimal division of the irrigated area depending on the location of stations with water, nutrients or disease prevention agents. At the same time, the main goal of such studies is to comply with the norms of rational use of natural resources and minimize the costs of delivering the necessary resources from the station to the irrigation area.

Analysis of recent research and publications

The problems of optimal design of complex systems, in particular irrigation systems, are studied by many scientists. The results of some of them were analyzed to determine their own approaches to the study of the problem. The work [1] is interesting from the point of view of analyzing the irrigation system as a control object and determining the input, output and internal parameters of the system. The article [2] presents a hierarchical control scheme for large-scale systems, the components of which can exchange information via a data transmission network. The main goal of the management layer is to find the best compromise between management performance and communication costs by actively modifying the network topology. The performance of the proposed control scheme is tested on an irrigation canal model implemented on an accurate water systems simulator. In [3], the problem of optimizing the use of water resources for irrigation is investigated, and the impact of reuse of return flow is analyzed. Mathematical modeling of the efficiency of such irrigation and its impact on the structure of crop crops and the distribution of water resources on irrigated areas was carried out. It has been proven that the reasonable and optimized use of return flow can not only improve irrigation efficiency, avoid the impact of return flow discharge on the water quality of nearby water bodies, but also reduce water extraction costs and save water resources. The article [4] presents a methodology for optimal division of the area of crop fields to solve the problem of delimiting the agricultural machinery management zone specific to a certain area. Specificity may be related to soil or crop characteristics. The aim of the study is to develop an effective algorithm for applying an individual management strategy to optimize farmers' use of resources. In the work, genetic algorithms are used to solve the problem of delimiting a site-specific management zone. The obtained results of the algorithm are compared with the exact solution based on integer linear programming and show that the proposed strategy is practically functional. The mathematical foundations in [5] were used to create a unified approach to optimizing various types of technical systems. Their methodology for applying mathematical methods in engineering optimization contributed to the development of general decomposition principles suitable for a wide range of technological problems. A significant contribution to the development of integrated approaches to design was made in studies [6], which proposed a framework for integrating design optimization and technology. This approach demonstrates the potential of applying systems thinking to technological design, taking into account production constraints. In [7], the application of mathematical optimization theory is investigated, mainly in the theoretical application of resource allocation efficiency and the mathematical foundations of optimization procedures, which provides a basic theoretical basis for the application. In [8], the authors developed an optimization approach through a control-centric framework that integrates control theory principles with mathematical optimization methods to achieve improved system performance.

Formulation of the study purpose

The purpose of the study is to address the issue of optimizing technical design tasks, which is a key element of modern engineering activities and contributes to increasing the efficiency of implementing design solutions, especially in conditions of uncertainty, multi-vector influence of internal and external factors, as well as a dynamically changing environment. It has been established that the technological features of management and the need for intellectual support of technical design tasks determine the optimization of such processes and form unique, unrepeatable conditions for the implementation of projects. The basis of the work was to highlight the main aspects of optimization, including a detailed analysis of modern approaches, methods and tools used in this area, their strengths and weaknesses, as well as opportunities for improving design processes. The research task is to develop an algorithm for the optimal design of a certain splicing system and methodological recommendations for its application using the theory of optimal set partitioning. The main goal of the research is to obtain results that can be significant not only from a theoretical point of view, but also have practical value for further use in engineering practice.

Presenting main material

The given region must be divided into irrigation areas by each of N stations, each of which has the following resources: water, nutrients for plant development, and disease prevention agents. It is necessary to minimize the total cost of delivering resources from the station to the irrigation area, as well as minimize the costs of constructing and operating irrigation stations. The problem is reduced to a continuous multi-product problem of optimal partitioning of a set (OPS) into subsets with given centers under constraints. The results presented here are preceded by studies presented in the works [9—13].

Mathematical model. It is necessary to divide the area Ω into irrigation zones with Ω_i^j N irrigation stations (IS) separately for each type of resource so that

$$\bigcup_{i=1}^{N}\Omega_i^j=\Omega,\ j=\overline{1,M},\ mes\left(\Omega_i^j\bigcap\Omega_k^j\right)=0,\ i\neq k,\ i,k=\overline{1,N},\ j=\overline{1,M} \tag{1}$$

in order to minimize the overall cost of transporting resources to each irrigation zone:

$$F\left(\left\{\Omega_1^1,...,\Omega_N^1;...;\Omega_1^M,...,\Omega_N^M\right\}\right)=$$
$$=\sum_{i=1}^{N}\sum_{j=1}^{M}\left\{\iint_{\Omega_i^j} c^j(x,y,\tau_i)\rho^j(x,y)dxdy+\varphi_i^j\left(\iint_{\Omega_i^j}\rho^j(x,y)dxdy\right)\right\}, \quad (2)$$

where $\rho^j(x,y)$ — density with which demand for the j-th type of resource is distributed in the region Ω; (x,y) — coordinates for locating irrigation points; $\tau_1,...,\tau_N$ — accommodation points IS; $c^j(x,y,\tau_i)$ — cost of transporting the resource from IS τ_i to the area with coordinates (x,y); $\varphi_i^j(Y_i^j)$ — dependence of the cost of the j-th type of resource on the i-th capacity IS Y_i^j and is determined by the formula $Y_i^j=\iint_{\Omega_i^j}\rho^j(x,y)dxdy$.

The capacity of the i-th IS for all types of resources is determined by the total demand of irrigation points belonging to it Ω_i^j and should not exceed the existing capacities determined by the relevant restrictions:

$$\sum_{j=1}^{M}\int_{\Omega_i^j}\rho^j(x,y)dxdy\le b_i,\ i=1,...,p;\sum_{j=1}^{M}\int_{\Omega_i^j}\rho^j(x,y)dxdy=b_i,\ i=p+1,...,N. \quad (3)$$

But at the same time, the conditions for the solvability of the problem must be met:

$$S=\int_{\Omega}\sum_{j=1}^{M}\rho^j(x,y)dxdy\le\sum_{i=1}^{N}b_i,\ 0\le b_i\le S, i=1,...,N. \quad (4)$$

Therefore, the presented problem is a continuous multi-product optimal partitioning problem of the set $\Omega\in E^n$ on its disjoint subsets $\Omega_1^1,...,\Omega_N^1;...;\Omega_1^M,...,\Omega_N^M$ (which may include empty ones) with the placement of the centers of these subsets under constraints in the form of equalities and inequalities.

To study such problems, methods and algorithms for solving OPS problems were applied. Their basic concept is based on the reduction of the original infinite-dimensional mathematical programming problems with Boolean variables through the Lagrange functional to dual finite-dimensional non-smooth problems. For the numerical solution of the latter, modern modifications of N.Z. Shor's r-algorithm were applied. Numerical results of the algorithms were obtained by studying a number of model problems.

Problem A1. Let the set be given $\Omega=\{(x,y):0\le x\le 24, 0\le y\le 12\}$ plants that require three types of resources that can be provided by three IS. Transportation costs for delivery to the region (x,y) from the i-th IS to the j-th type of resources is given according to the type of resources:

$$c^j(x,y,\tau_i)=\begin{cases}\sqrt{\left(x-\tau_i^{(1)}\right)^2+\left(y-\tau_i^{(2)}\right)^2}, j=1;\\ max\left\{\left|x-\tau_i^{(1)}\right|,\left|y-\tau_i^{(2)}\right|\right\},\ j=2;\\ \left|x-\tau_i^{(1)}\right|+\left|y-\tau_i^{(2)}\right|, \quad j=3.\end{cases}$$

The demand $\rho^j(x,y)$ for resources is distributed in the region Ω with the corresponding densities given in the following analytical form:

$$\rho^j(x,y)=\frac{1}{ln((x-y)^j-110.003)}, \qquad j=1,2,3.$$

The functions $\varphi_i^j(Y_i^j)$, that describe the cost of storing the j-th type of resource at the i-th station depending on the power of the IS have the form

$$\varphi_i^j(Y_i^j)=T_i+a_i^jY_i^j, \qquad i=1,2,3, j=1,2,3,$$

$a_i^j\ge 0$ costs of maintaining a unit of resource of the j-th type at the i-th station, T_i— capital costs for the construction of the i-th station.

The capacity of the i-th station IS for all types of resources is determined by the total demand of the plantations belonging to Ω_i^j, and for the first station it must not exceed the given volumes, i.e. the following restrictions are imposed on the storage capacity:

$$0 \le \sum_{j=1}^{3} \iint_{\Omega_i^j} \rho^j(x,y)\,dxdy \le b_i, \qquad i=1, \quad b_i = 100,$$

and for IS with numbers $i = 2, 3$ it should be equal to the given volumes:

$$\sum_{j=1}^{3} \iint_{\Omega_i^j} \rho^j(x,y)\,dxdy = b_i, \qquad i = 2,3, \quad b_2 = 86, \quad b_3 = 35.$$

It is necessary to divide the set of plantations Ω into their irrigation zones by three irrigation stations for each type of resource, that is, into subsets Ω_i^j $i = 1, 2, 3, j = 1, 2, 3,$ to minimize the total cost function of storing resources and delivering them to the distribution areas:

$$F\left(\left\{\Omega_1^1,\ldots,\Omega_3^1;\Omega_1^2,\ldots,\Omega_3^2;\Omega_1^3,\ldots,\Omega_3^3\right\}\right) = \sum_{i=1}^{3}\sum_{j=1}^{3}\left\{\iint_{\Omega_i^j} c^j(x,y,\tau_i)\rho^j(x,y)dxdy + T_i + a_i^j Y_i^j\right\}.$$

It is not an exception that some of the subsets Ω_i^j, $i = 1, 2, 3,$ $j = 1, 2, 3$ will be empty.

The set Ω was covered with a mesh with nodes (i, j), $i = 1,\ldots,150,$ $j = 1,\ldots,80.$ The initial values of the dual variables are given by $\psi_i^{(0)}, i = \overline{1,3},$ initial power values: $Y_1^{j(0)} = 50,$ $Y_2^{j(0)} = 10,$ $Y_3^{j(0)} = 10,$ $j = \overline{1,3}.$ For simplicity $T_i = 0, a_i^j = 0,$ $i, j = \overline{1,3}.$

The following coordinates are given as the initial coordinates of the IS:

$$\tau^0 = \begin{pmatrix} 3.0; & 10.0; & 18.5; \\ 10.0; & 5.0; & 4.5; \end{pmatrix}.$$

The condition for completing calculations is the condition:

$$\left\| \left(Y^{(k)}, \psi^{(k)}\right) - \left(Y^{(k+1)}, \psi^{(k+1)}\right) \right\| \le \varepsilon, \quad \varepsilon = 10^{-3}.$$

After *29* iterations, the following results were obtained:

– the optimal division of the set of plantations Ω into irrigation zones by each of the three IS for three types of resources is presented in the fig. 1;

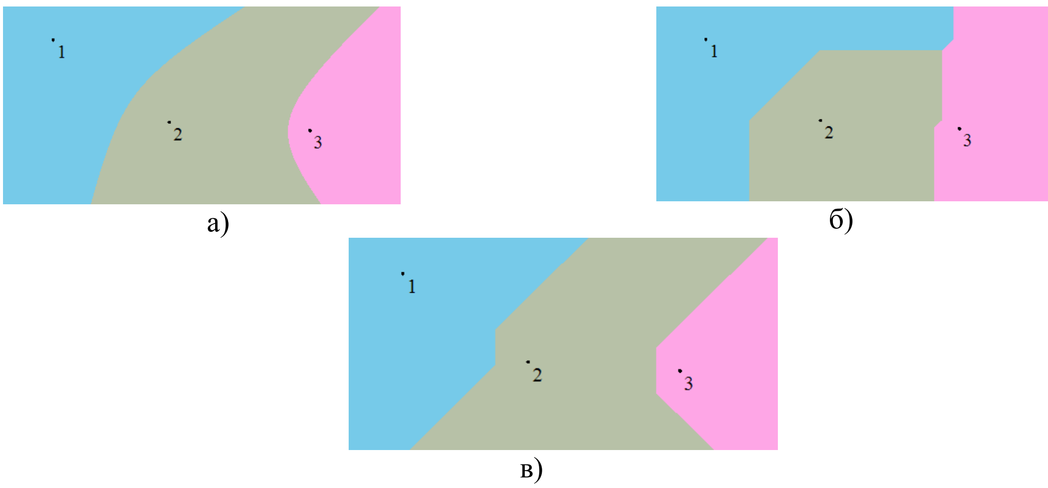

Fig. 1. Optimal distribution of the set Ω into irrigation zones by each of three irrigation stations with fixed centers for three types of resources for the model problem *A1*

- the maximum value of the functional of the dual problem $G^* \approx 998{,}4$;
- the minimum value of the functional of the direct problem $F_* \approx 996{,}0$;
- optimal capacities of each of the three locations IS:

$$Y_1 = 67{,}4;\ Y_2 = 86{,}6;\ Y_3 = 35{,}0.$$

Problem A2. In the formulation of the *model problem A1*, we specify the values of the weight coefficients for each of the three types of resources:

$$a_1^1 = 50,\ a_2^1 = 10, a_3^1 = 10;$$

$$a_1^2 = 60,\ a_2^2 = 100, a_3^2 = 20;$$

$$a_1^2 = 50,\ a_2^2 = 12, a_3^2 = 10.$$

After *51* iterations, the following results were obtained:

- the optimal division of the set of plantations Ω into irrigation zones by each of the three IS for three types of resources is presented in the fig. 2;

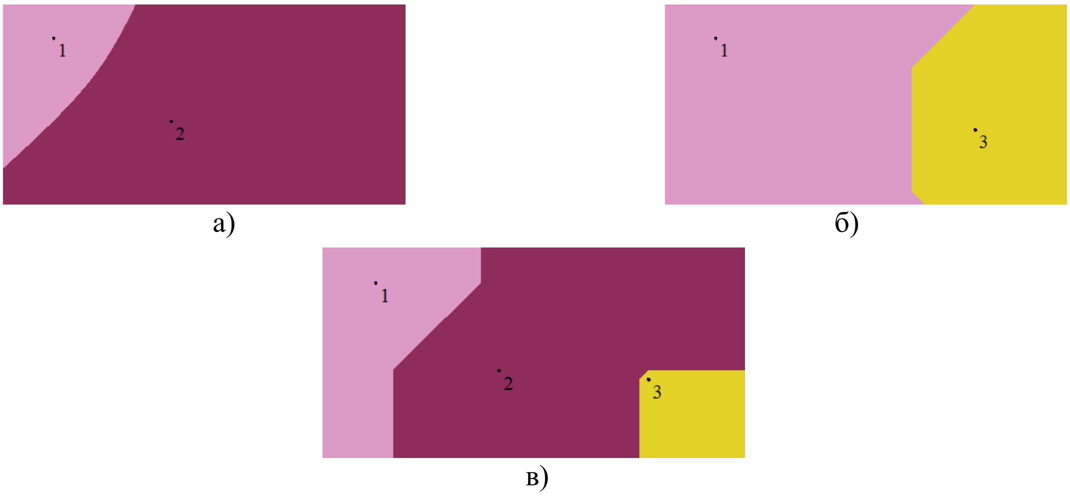

Fig. 2. Optimal distribution of the set Ω into irrigation zones by each of three irrigation stations with fixed centers for three types of resources for the model problem *A2*

- the maximum value of the functional of the dual problem $G^* \approx 6590{,}38$;
- the minimum value of the functional of the direct problem $F_* \approx 6610{,}37$;
- optimal capacities of each of the three locations IS:

$$Y_1 = 67{,}9; Y_2 = 88{,}4; Y_3 = 32{,}7.$$

Problem A3. In the formulation of the *model problem A2*, we set the condition to place irrigation stations. Then, if the initial coordinates of the irrigation station locations are $\tau_i^{(0)} = (0;0), i = 1,2,3$, then after *289* iterations we will get the following results:

- the optimal division of the set of plantations Ω into irrigation zones by each of the three IS for three types of resources is presented in the fig. 3;
- optimal coordinates for placing irrigation stations:

$$\tau_1 = (18{,}8; 6{,}6);\ \ \tau_2 = (8{,}4; 6{,}0);\ \tau_3 = (6{,}2; 6{,}0);$$

- the maximum value of the functional of the dual problem $G^* \approx 6405{,}98$;
- the minimum value of the functional of the direct problem $F_* \approx 6416{,}97$;
- optimal capacities of each of the three locations IS:

$$Y_1 = 68{,}2;\ Y_2 = 86{,}0;\ Y_3 = 34{,}7.$$

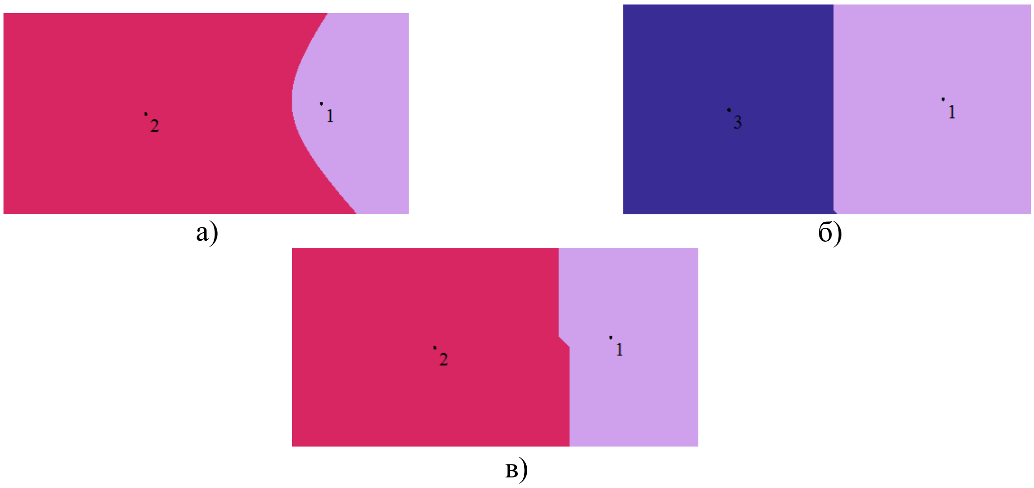

Fig. 3. Optimal distribution of the set Ω into irrigation zones by each of three irrigation stations with fixed centers for three types of resources for the model problem *A3*

The following analysis of the obtained results confirms the adequacy of the model's performance. Therefore, in problem *A1*, there are no costs for maintaining a unit of resource of each type at each station and no capital costs for their construction, while the stations themselves are already located in the given area, that is why, as a result of the simulation, the optimal values of the total cost functional are significantly smaller than in the simulation results of problem *A2*, in which these costs are assumed. In addition, the number of iterations in *A2* has increased. The process of placing irrigation stations, which was assigned in task *A3*, was determined by a significant increase in the number of required iterations. In all model problems, the solvability conditions of problem (4) are met, that is, the total optimal capacity of three irrigation stations, which is obtained by the algorithm, does not exceed the sum of the given capacities of the stations. The obtained optimal powers in each of the model problems correspond to the constraints in the form of equalities.

Conclusions

The paper presents mathematical modeling of the optimal design problems of irrigation systems using the theory of optimal set partitioning in order to optimize the functional of the total cost of storage and delivery of the relevant resources. З використанням узагальненого програмного продукту [13] було проведено численні дослідження вищезазначених проблем, які демонструють адекватність розробленої моделі та ефективність роботи алгоритму. The developed mathematical model and algorithm can be used to optimize more complex irrigation systems, which will reduce costs and increase the profitability of agricultural companies. Further research may be aimed at developing more complex models that take into account additional factors, such as the dynamics of demand for the relevant resource, limitations on the capacity of the irrigation network, and others.

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