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FINITE-DIFFERENCE ALGORITHMS FOR CONTROLLING MODES OF LOCAL HEAT TREATMENT OF LARGE-SIZED PRODUCTS

КІНЦЕВО-РІЗНИЦЕВІ АЛГОРИТМИ УПРАВЛІННЯ РЕЖИМАМИ МІСЦЕВОЇ ТЕРМООБРОБКИ ВЕЛИКОГАБАРИТНИХ ВИРОБІВ

The paper considers the problem of managing the modes of local heat treatment of large-sized products in order to ensure compliance with the specified temperature schedule. A number of technological schemes of the process within the framework of two-zone and three-zone models with different options for thermal insulation of the product outside the heat supply zone have been analyzed. A universal algorithm for determining the control parameters of the local heat treatment process according to a given schedule is proposed. The algorithm is based on a locally one-dimensional finite-difference method, in which direct and inverse thermal conductivity problems are sequentially solved at each time step. Algorithms for solving direct problems take into account the specifics of various thermal schemes of the process. The algorithm for solving the inverse problem is based on the use of the method of elementary heat balances and does not depend on the organization of the technological process.

The problem of organizing the process with ensuring the permissible temperature difference in a certain area around the seam is also considered. It is proposed to use buffer heating zones with the supply of heat flows of increased power. An algorithm for controlling such flows has been developed, which provides for a universal algorithm for solving a direct problem and takes into account the specifics of the mode when solving an inverse problem. All the necessary formulas are given, a flowchart of the algorithm for the implementation of the modeling process is built.

Keywords: mode control, finite-difference algorithm, local-one-dimensional method, flowchart, local heat treatment.

Обов'язковим етапом виготовлення великогабаритних металевих виробів методом електрошлакового зварювання є проведення термічної обробки для усунення залишкових напружень в зоні зварного шва. Ресурсозберігаючим варіантом такого процесу є місцева термообробка обмеженої зони виробу навколо шва. Необхідність дотримання режиму термообробки і локальний характер підведення теплоти зумовлює особливості технологічного процесу і потребує подальшого вивчення, зокрема, в напрямку формування раціональних умов зовнішнього теплообміну.

У роботі розглянуті ряд технологічних схем процесу в рамках двохзонних і трьохзонних моделей з різними варіантами теплоізоляції виробу поза зоною підведення теплоти. Запропо-

нований універсальний алгоритм визначення параметрів управління процесом місцевої термообробки за заданим графіком. В основі алгоритму лежить локально-одновимірний різницевий метод, в рамках якого на кожному часовому кроці послідовно вирішуються прямі і обернені задачі теплопровідності. Алгоритми вирішення прямих задач враховують специфіку різних теплових схем процесу. Алгоритм вирішення оберненої задачі базується на використанні методу елементарних теплових балансів і не залежить від організації технологічного процесу.

Розглянута також задача організації процесу з забезпеченням допустимого перепаду температур в певній зоні навколо шва. Запропоновано використовувати буферні зони нагріву з підведенням теплових потоків підвищеної потужності. Розроблений алгоритм управління такими потоками, який передбачає універсальний алгоритм вирішення прямої задачі і враховує специфіку режиму при вирішенні оберненої задачі. Наведені всі необхідні формули, побудована блок-схема алгоритму реалізації процесу моделювання.

Подальші дослідження планується спрямувати на поширення пропонованого алгоритму для інших задач управління технологічними процесами.

Ключові слова: управління режимами, кінцево-різницевий алгоритм, локально-одновимірний метод, блок-схема, місцева термообробка.

Problem's Formulation

Local heat treatment is an important technological process that ensures high quality and durability of welded joints in industry by relieving stresses arising as a result of welding [1]. It is of particular importance for heavy engineering, where large-sized products weighing more than 100 tons and 5—12 m long are manufactured using electroslag welding [2]. This technological process is a progressive resource-saving process that is important for the economy of any country [3]. At the same time, certain aspects of the process have not yet been sufficiently studied, which requires further research in this direction. In particular, this applies to the control of local heat treatment modes of large-sized products in order to increase the energy efficiency of the process [2].

Analysis of recent research and publications

The study of thermophysical features of local heat treatment of products of different geometries is devoted to the works of a number of domestic scientists, in particular I.N. Manusov, M.M. Bilyaev, V.A. Soroka, G.F. Alekseev and others [2]. In their works, one of the stages of heat treatment was mainly studied, namely heating to a given temperature under known conditions of heat supply to the surface of the product. The main research methods were analytical methods for solving one-dimensional linear thermal conductivity problems within the framework of two-zone models. At the same time, real heat treatment processes are significantly nonlinear and require taking into account the dependence of the thermophysical properties of the metal on temperature and the complex nature of heat supply to the surface of the product. Analytical methods are not suitable in such cases, so it is necessary to use numerical methods, in particular, finite-difference schemes for solving problems of mathematical physics. This approach is used in the works [4—8] to study the regularities of local heating of large-sized structures according to various technological schemes (Tabl. 1), taking into account the two-dimensional nature of heat transfer and nonlinearities of the process.

In all the above works, modeling algorithms based on finite-difference methods are used with the use of a modified method of elementary heat balances in some cases [9]. At the same time, the work [5] considered the full cycle of local heat treatment (heating + holding at temperature + cooling) according to a given schedule with the determination of external conditions that ensure compliance with the schedule. Thus, an attempt was made to control the local heat treatment regime. The approach used can be used for other control tasks, including the purpose of designing optimal modes. This determines the expediency of further studies of the features of local heat treatment, determination of rational conditions for the organization of the technological process.

Formulation of the study purpose

The purpose of the study is to develop finite-difference algorithms for determining external conditions, which provide rational modes of the technological process of local heat treatment of large-sized products.

Table 1. Technological schemes of local heating

| | Technological scheme | Feature |
|----------|----------------------|--|
| Scheme 1 | | Symmetrical two-zone model, [4, 5] |
| Scheme 2 | | Model with three zones, product in the zone heating lies at the bottom of the stove (adiabatic conditions), [7] |
| Scheme 3 | | Three-zone model, the product in the heating zone lies on the hearth of the stove, in the cooling zone - heat-insulated, [8] |
| Scheme 4 | | Two-zone model (heating and cooling zones) with highlighting in the heating zone of buffer zones, [6] |
| Scheme 5 | | Two-zone model (heating and thermal insulation zones) with highlighting in the heating zone of buffer zones, [8] |

Presenting main material

Heat treatment of large-sized products made by electroslag welding is carried out in stage 3 (heating + holding at temperature + cooling) according to a given schedule of changes in the temperature of the surface of the product in the weld zone. Typical heat treatment schedules are shown in Fig. 1.

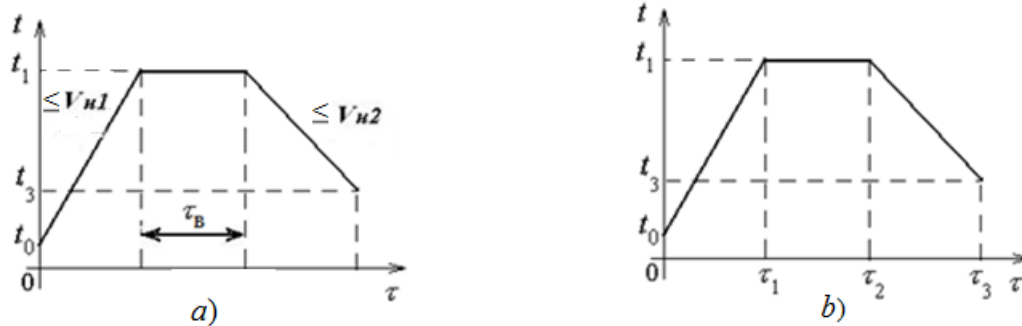


Fig. 1. Heat treatment schedules

The traditional mode of heat treatment (Fig. 1, a) sets the maximum allowable rates of heating V_{n1} and cooling V_{n2} and the time of holding at temperature τ_B . In the paper [5], an alternative mode of heat treatment is proposed (Fig. 1, b), which provides for a more stringent regulation of the temperature schedule and conducting the process at the maximum allowable rates of heating and cooling.

To implement control according to a given schedule, it is necessary to determine the law of change in the external conditions of the process. In the simplest case, a heat flux is supplied to the surface of the product, the density of which is constant on the surface, and changes over time, i.e. $q = q(\tau)$. Determination of the law of change $q(\tau)$ is one of the tasks of controlling heat treatment modes. In fact, it is about solving the inverse problem of thermal conductivity: to determine the boundary conditions that provide the required thermal state of the product. Since the method of finite differences has proven itself well for solving direct problems of local heat treatment ([4–8]), it will be natural to try to solve the inverse problem within the framework of the same approach.

The main idea of solving the problem of local heat treatment with determining the conditions that provide a given schedule is as follows. A difference grid is introduced with the selection of characteristic points in accordance with the adopted thermal scheme. For example, for scheme 1 (see Tabl. 1), it has the form shown in Fig. 2, where $N_1 = 1 + l_1 / \Delta x$, $N_2 = 1 + l_2 / \Delta x$, $M = 1 + R / \Delta y$.

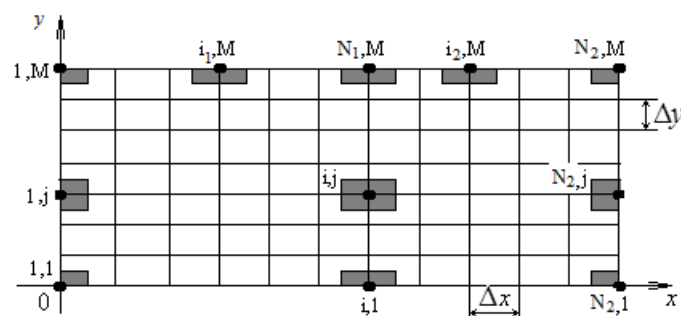


Fig. 2. Difference grid

For point (1, M), an explicit difference equation is drawn up using the method of elementary heat balances [9], from which the formula for derives $q(\tau)$. For any of the schemes given in Table 1, it will look like

$$q^n = V_H \cdot c_{v1,M} \cdot \frac{\Delta y}{2} + \frac{(\lambda_{1,M} + \lambda_{2,M}) \Delta y}{2 \Delta x^2} (t_{1,M}^n - t_{2,M}^n) + \frac{(\lambda_{1,M} + \lambda_{1,M-1})}{2 \Delta y} (t_{1,M}^n - t_{1,M-1}^n), \quad (1)$$

where $t_{i,j}^n$ is the temperature at the point (i,j) at a time $\tau_n = n\Delta\tau$; $\lambda_{i,j}$ i $c_{vi,j}$ is the coefficient of thermal conductivity and volumetric heat capacity at point (i,j), calculated by temperature values $t_{i,j}^n$;

$$V_H = \begin{cases} \frac{t_1 - t_0}{\tau_1} & , 0 \leq \tau < \tau_1 \\ 0 & , \tau_1 \leq \tau \leq \tau_2 \\ \frac{t_3 - t_1}{\tau_3 - \tau_2} & , \tau_2 < \tau \leq \tau_3 \end{cases} \quad (2)$$

Then, taking into account the value q^n , temperature calculations are performed at the next time step $t_{i,j}^{n+1}$ according to the standard procedure of the local-one-dimensional method and the transition to the next time step occurs.

A generalized flowchart of the described algorithm is shown in Fig. 3, and the detailing of one step of calculations is shown in Fig. 4.

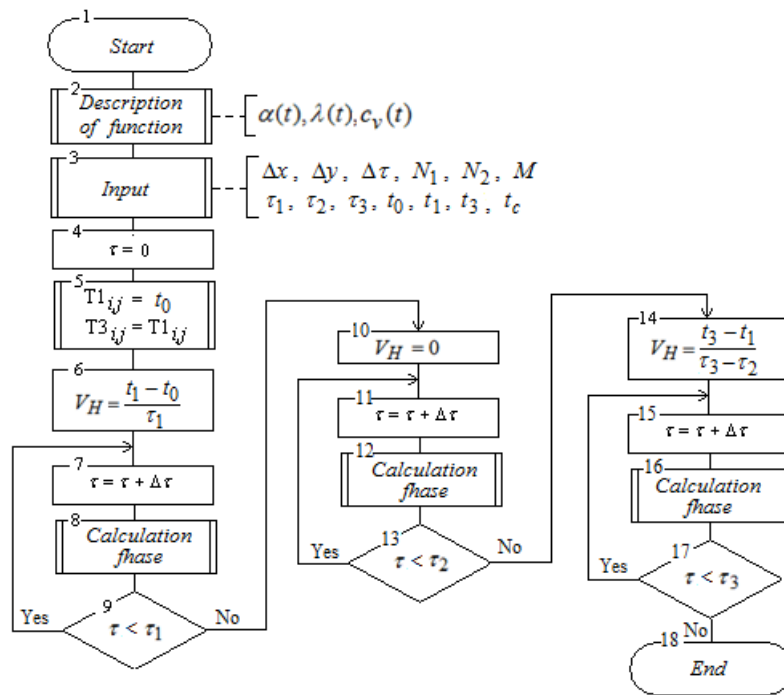


Fig. 3. Generalized flowchart of the algorithm

The algorithm uses three arrays of size $N \times M$: they store the temperatures of the design points at moments of time $\tau_n = n\Delta\tau$ (array T1); $\tau_{n+1/2} = \tau_n + \Delta\tau/2$ (array T2) and $\tau_{n+1} = \tau_n + \Delta\tau$ (array T3). When performing each step of the calculations, 3 iterations are assumed (block 2 in Fig. 4), to clarify the values of the thermophysical coefficients $\lambda_{i,j}$ i $c_{vi,j}$, as well as the values q^n of. Only after that, the values of the heat flux density q^n and the temperature of the calculated points at the corresponding time step are printed (block 6 in Fig. 4). Note that the algorithms for the implementation of blocks 4—5 (Fig. 4) depend on the thermal scheme of the process (see Tabl. 1).

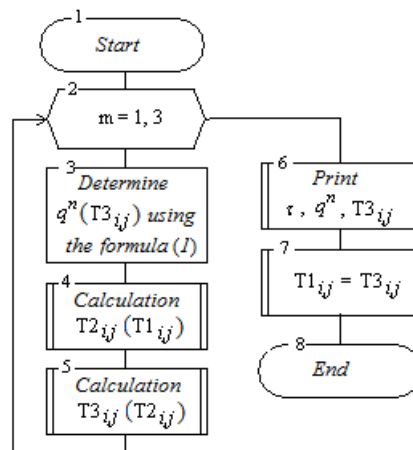


Fig. 4. Detailing the flowchart of the algorithm

Note that the records $T2_{i,j}$ ($T1_{i,j}$) and $T3_{i,j}$ ($T2_{i,j}$) mean that the T2 and T3 arrays are calculated based on the data contained in the T1 and T2 arrays, respectively. The same applies to the notation q^n ($T3_{i,j}$).

When carrying out the technological process according to schemes 4 and 5 (see Table 1), the question arises about such a heat flux power in the buffer zone $q_1(\tau)$ that provides a permissible temperature difference ΔT in the weld zone ($|x| \leq l_0$). The corresponding algorithm can be built on the basis of the approach described above, i.e. on the basis of the local-one-dimensional method and the method of elementary heat balances.

The heat flow $q_0(\tau)$ must ensure the execution of the heat treatment process according to a given schedule. Its value q_0^n at each time step is determined by the formula (1). To derive the formula for calculating the value, q_1^n it is necessary to make an explicit difference equation for the point $(i1, M)$ (see Fig. 2) at $i_1 = K = 1 + l_0 / \Delta x$. Using the method of elementary heat balances, we have

$$(q_0^n + q_1^n) \frac{\Delta x}{2} - \frac{(\lambda_{K,M} + \lambda_{K,M-1})}{2\Delta y} (t_{K,M}^n - t_{K,M-1}^n) \cdot \Delta x + \frac{(\lambda_{K-1,M} + \lambda_{K,M})}{2\Delta x} (t_{K-1,M}^n - t_{K,M}^n) \cdot \frac{\Delta y}{2} - \frac{(\lambda_{K,M} + \lambda_{K+1,M})}{2\Delta x} (t_{K,M}^n - t_{K+1,M}^n) \cdot \frac{\Delta y}{2} = c_{\nu_{K,M}} \cdot (t_{1,M}^n - \Delta T) \frac{\Delta x \Delta y}{2\Delta \tau},$$

from where

$$q_1^n = c_{\nu_{K,M}} \cdot (t_{1,M}^n - \Delta T) \frac{\Delta y}{\Delta \tau} + \frac{(\lambda_{K,M} + \lambda_{K,M-1})}{\Delta y} (t_{K,M}^n - t_{K,M-1}^n) - \frac{(\lambda_{K-1,M} + \lambda_{K,M}) \Delta y}{2\Delta x^2} (t_{K-1,M}^n - t_{K,M}^n) + \frac{(\lambda_{K,M} + \lambda_{K+1,M}) \Delta y}{2\Delta x^2} (t_{K,M}^n - t_{K+1,M}^n) - q_0^n. \quad (3)$$

Thus, the problem of controlling local heat treatment when introducing buffer zones (schemes 4, 5) can be solved using an algorithm, the generalized block diagram of which is shown in Fig. 3. In block 3 of this algorithm, it is also necessary to provide for the introduction of parameters K and ΔT , which take into account the specifics of thermal schemes 4, 5.

In addition, it is necessary to take into account that it is advisable to turn on the high power mode in the buffer zone only when the temperature difference in the weld zone exceeds the value ΔT of. To do this, it is necessary to modify the flowchart of the algorithm detailing as shown in Fig. 5.

Conclusions

A universal algorithm for determining the control parameters of the local heat treatment process according to a given schedule is proposed. The algorithm is based on a locally one-dimensional difference method, in which direct and inverse thermal conductivity problems are sequentially solved

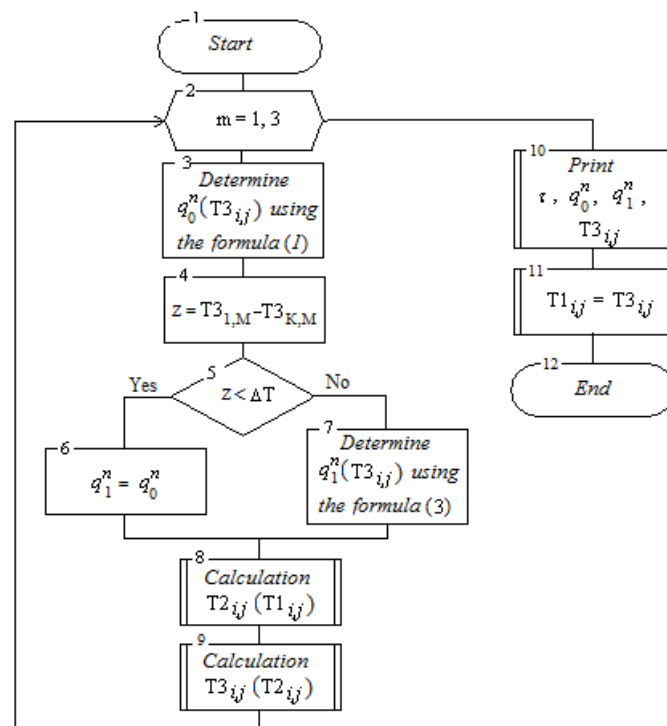


Fig. 5. Detailing the flowchart of the main algorithm for schemes 4, 5

at each time step. Algorithms for solving direct problems take into account the specifics of various thermal process schemes within the framework of two-zone and three-zone models with different options for thermal insulation of the product outside the heat supply zone. The algorithm for solving the inverse problem is based on the use of the method of elementary heat balances and does not depend on the organization of the technological process.

The problem of organizing the process with ensuring the permissible temperature difference in a certain area around the seam is also considered. It is proposed to use buffer heating zones with the supply of heat flows of increased power. An algorithm for controlling such flows has been developed, which provides for a universal algorithm for solving a direct problem and takes into account the specifics of the mode when solving an inverse problem. The necessary formulas are given, a flowchart of the algorithm for implementing the modeling process is built.

Further research is planned to be directed to the dissemination of the proposed algorithm for other tasks of technological process control.

References

- [1] KARBAZ (2025). Lokalna termichna obrobka: praktychne rishennia dlia nadiinykh zvarnykh ziednan [Local heat treatment: a practical solution for reliable welded joints]. Available at: <https://karbaz.com.ua/uk/statti-ta-publikaci/lokalna-termichna-obrobka/>
- [2] Kompaniits, R.A. (2023). Mistseva termoobrobka velykohabarytnykh vyrobiv: zastosuvannya ta problemy proektuvannya tekhnolohii [Local heat treatment of large-sized products: application and design problems of the technology]. Proceedings of the 1st International Scientific and Practical Internet Conference: *Achievements of 21st Century Scientific Community*. (pp. 223-225). Dnipro: FOP Marenichenko V.V. [in Ukrainian].
- [3] Bila, I.S., & Krosman, N.V. (2018). Rozvytok resursozberezhennia v Ukraini [Development of resource conservation in Ukraine]. *Ekonomika ta upravlinnia natsionalnym hospodarstvom - Economy and management of the national economy, Issue 21*, 53-58 [in Ukrainian].

- [4] Karimov, I.K., Karimov, H.I., Kompaniets, R.A., & Bulai, O.Iu. (2024). Kompiuterno oriientovani alhorytm modeliuвання mistsevoho nahrivu plynykh konstruktsii [Computer-oriented algorithm for modeling local heating of plate construction]. *Science and technology today*, №1(29), 689-700 [in Ukrainian].
- [5] Vernygora D., Karimov I., Karimov H., Kompaniets R. (2024). Mathematical modeling of local heat treatment of structures made by electroslag welding. *Mathematical modeling*, 1(50), 152-159.
- [6] Karimov, I.K., Vernygora, D.V., Karimov, H.I., Lisenko, A.V. (2024). Modeliuвання teplovoho stanu plynykh konstruktsii v protsesi mistsevoho nahrivu. [*Modeling of the thermal state of plate construction in the process of local heating*]. *Collection of Scientific Papers of the Dnipro State Technical University (Technical Sciences)*, № 1(44), 146-152 [in Ukrainian].
- [7] Vernygora, D.V., Karimov, I.K. (2024). Chyselnyi alhorytm modeliuвання mistsevoho nahrivu velykohabarynykh vyrobiv v hazovykh pechakh. [Numerical algorithm for modeling local heating of large-sized products in gas furnaces]. *Science and technology today*, №12(40), 1137-1148 [in Ukrainian].
- [8] Vernygora D., Karimov I. (2025). Computer modeling of rational modes of local heating of plate structures in gas furnaces. *Mathematical modeling*, 1(52), 161-167.
- [9] Vernygora, D.V. (2023). Modyfikovani metod elementarnykh teplovykh balansiv v upravlinni protsesamy mistsevoi termoobrobky [The modified method of elementary heat balances in the management of local heat treatment processes]. Proceedings of the XXXVI International science and practice conference: *Suchasni aspekty modernizatsii nauky: stan, problemy, tendentsii rozvytku - Modern aspects of the modernization of science: state, problems, development trends*. (pp. 188-191). Kyiv: HO "VADND" [in Ukrainian].

Список використаної літератури

1. Локальна термічна обробка: практичне рішення для надійних зварних з'єднань. <https://karbaz.com.ua/uk/statti-ta-publikaci/lokalna-termichna-obrobka/> (дата звернення: 15.01.2026).
2. Компанієць Р. А. Місцева термообробка великогабаритних виробів: застосування та проблеми проектування технології/ Achievements of 21st Century Scientific Community: Proceedings of the 1st International Scientific and Practical Internet Conference, September 14-15, 2023. Dnipro: FOP Marenichenko V.V., 2023. P. 223-225.
3. Біла І.С., Кросман Н.В. Розвиток ресурсозбереження в Україні. Економіка та управління національним господарством. Київ., 2018. Вип. 21 2018, С. 53-58.
4. Карімов І.К., Карімов Г.І., Компанієць Р.А., Булай О.Ю. Комп'ютерно орієнтований алгоритм моделювання місцевого нагріву плитних конструкцій. *Наука і техніка сьогодні*. 2024. №1(29). С. 689-700.
5. Vernygora D., Karimov I., Karimov H., Kompaniets R. Mathematical modeling of local heat treatment of structures made by electroslag welding. *Математичне моделювання*. 2024. №1(50). С. 152-159.
6. Карімов І.К., Вернигора Д.В., Карімов Г.І., Лисенко А.В. Моделювання теплового стану плитних конструкцій в процесі місцевого нагріву. *Збірник наукових праць Дніпровського державного технічного університету (технічні науки)*. 2024. № 1(44). С. 146-152.
7. Вернигора Д. В., Карімов І.К. Чисельний алгоритм моделювання місцевого нагріву великогабаритних виробів в газових печах. *Наука і техніка сьогодні*. 2024. №12(40). С. 1137-1148.
8. Vernygora D., Karimov I. Computer modeling of rational modes of local heating of plate structures in gas furnaces. *Математичне моделювання*. 2025. №1(52). С. 161-167.
9. Вернигора Д. В. Модифікований метод елементарних теплових балансів в управлінні процесами місцевої термообробки. Сучасні аспекти модернізації науки: стан, проблеми, тенденції розвитку: матеріали XXXVI Міжнар. наук.-практ. конф., 7 вер. 2023 р. Софія (Болгарія). К.: ГО «ВАДНД», 2023. С. 188-191.

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