DOI: 10.31319/2519-8106.1(54)2026.352418
UDC 519.1, 519.866:658.5

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MATHEMATICAL MODELING OF THE COMPREHENSIVE LOGISTICS STRATEGY OF AN ORGANIZATION

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ КОМПЛЕКСНОЇ ЛОГІСТИЧНОЇ СТРАТЕГІЇ ОРГАНІЗАЦІЇ

The study developed a methodology for building a mathematical model of a complex logistics strategy of an organization using the example of a hotel business enterprise.

Digital marketing methods in combination with adequate mathematical methods allow to determine and predict customer behavior and predict consumer choice in the event of a positive experience of receiving the desired service. According to the digital and quantum paradigms, the consumer is represented as a complex system and acts as a conceptual basis for developing an organization's logistics strategy through the effective use of data analytics and modern information technologies and systems.

To determine the mathematical model, the current problems of guest flow clustering were solved; using regression analysis approaches, the dependencies between consumer choice and the most influential factors on the behavior of hotel guests were determined; using game theory, the most successful loyalty program projects were determined; using Bayesian models, the probability of hotel booking for regular customers was predicted.

The implementation of the developed models was carried out using Excel and the MathCAD mathematical system.

***Keywords**: logistics strategy, digital marketing, information card, clustering, regression analysis, game theory, forecasting.*

У дослідженні розроблено методику побудови математичної моделі комплексної логістичної стратегії організації на прикладі підприємства готельного бізнесу. Наукове обґрунтування створення ефективної логістичної стратегії розвитку організації, оптимізація використання наявних ресурсів є умовами здобуття необхідних показників установи у прогнозний період часу.

Методи цифрового маркетингу у поєднанні з адекватними математичними методами дозволяють визначити та спрогнозувати поведінку клієнтів та передбачити споживчий вибір у разі позитивного досвіду отримання потрібної послуги. За цифровою та квантовою парадигмами споживач представляється у вигляді складної багатопараметричної системи, яка одночасно перебуває в кількох емоційних та поведінкових станах і виступає концептуальною основою для розробки логістичної стратегії організації завдяки ефективному використанню аналітики даних і сучасних інформаційних технологій та систем.

Для визначення математичної моделі розв'язано актуальні задачі кластерізації потоку гостей на основі розроблених інформаційних карток клієнтів готелю; за допомогою підходів регресійного аналізу визначено залежності між споживчим вибором та найбільш впливовими чинниками на поведінку гостей готелю; засобами теорії ігор визначено найбільш успішні для реалізації проєкти програм лояльності для певних кластерів споживачів; засобами Байєсівських моделей здійснено прогнозування ймовірності бронювання готелю для постійних клієнтів за даними їх динамічних інформаційних карток.

Реалізація розроблених моделей була здійснена за допомогою засобів ET Excel та у математичній системі MathCAD.

Запропонована математична модель може бути використана в діяльності установ готельно-ресторанного бізнесу для розуміння поведінки клієнтів, для залучення нових гостей і продовження терміну взаємодії з наявними клієнтами засобами ефективного використання клієнто-орієнтованої реклами та розроблених під особисті потреби та зацікавлення споживача програм лояльності.

***Ключові слова**: логістична стратегія, цифровий маркетинг, інформаційна картка, кластеризація, регресійний аналіз, теорія ігор, прогнозування.*

Problem's formulation

The basis of a comprehensive logistics strategy in the modern realities of the information world is its customer orientation. From the point of view of digital marketing, the consumer is considered as a dynamic multidimensional system. A modern institution must promptly determine the requests of the consumer of its services to develop its products or services. Therefore, the main direction of the logistics strategy of a hotel and restaurant business enterprise is to create and provide convenient in the modern information space and comfortable conditions for accommodation, stay and meals according to consumer needs. This will encourage an increase in demand for the organization's services and lead to a higher level of customer satisfaction after receiving them. And for this, the content developed and offered by the enterprise in the information space must be adapted to each client, based on the forecasting analysis of his online behavior [1].

With the advent of information and search systems, the understanding of the value of analytical data and their effectiveness came. The use of customer data and information about their preferences for targeted marketing made it possible to reduce advertising costs, increase profitability and increase the payback of investments and investment projects. Thanks to the Internet, it became possible to have direct contact with the consumer, maintain constant contact with customers and influence them economically and with extreme accuracy. When searching for a product or service, the Internet provides the necessary information to the consumer, and on the other hand, it helps data analysts to better understand and study consumer behavior, determine his preferences, financial capabilities, and spending patterns. Marketing paradigms and rules began to merge with science, analytical data became a mechanism for implementing new marketing opportunities [2].

Analysis of recent research and publications

The research of the issue of digital and quantum marketing and their concepts was devoted to the works of scientists R. Rajamannara, G. Kawasaki, O. Karpishchenko, Yu. Loginov, O. Romanenko, V. Ruban, I. Pedko, I. Zlatova, M. Oklander, T. Oklander, G. Vainerchuk, R. Deis, S. Godin, F. Kotler and others. Increasing the level of informatization of society contributes to an increase in the number of online buyers. And compliance with the principles of digital advertising and the search for new technologies of influence will help increase the level of efficiency of digital marketing.

The main value of the quantum paradigm is the study not only of the client's consumer behavior, but also the analysis of the consumer as a person — a holistic personality. This requires an understanding of the psychology, moral and aesthetic values of a person and the dynamics of his behavior. The attention of researchers switches from the straightforward desire to provide quality service to improving the impressions of a person and his family from service and the level of communication. This will contribute to the effective interaction of manufacturers, service providers with consumers. Ethics and universal human values become of paramount importance. The client's trust in the organization, the honesty and sincerity of its employees be-

come the main competitive advantage [3]. In view of the development of modern marketing paradigms, the so-called system of means "7P" is being formed [2], the components of which are:

1) Product — goods or services;
2) Price — optimal prices for each consumer segment;
3) Place — taking into account the location of the consumer and the offered service;
4) Promotion — advertising or SMS-sending or e-mails adapted to the needs of the consumer;
5) People;
6) Process;
7) Physical evidence — specific, material evidence.

Rapid changes in consumer behavior require business owners to use an integrated approach to revenue management, with the sales and marketing departments having the greatest influence in the decision-making process. Converging their roles and applying digital technologies is crucial for effective management of distribution channels.

The relevance of developing and implementing a comprehensive model of strategic planning at the enterprise is due to the growing dynamics of the external environment, increasing the pace and directions of its economic, political and social changes. Scientific substantiation of the creation of an effective logistics system of strategic planning of the organization's development, optimization of the use of available resources are the conditions for obtaining the necessary indicators of the institution in the forecast period of time.

Formulation of the study purpose

The purpose of the study is to develop mathematical models of a comprehensive logistics strategy of an organization. To achieve this goal, the following tasks were identified:

– to systematize mathematical methods for modeling the logistics strategy of an enterprise and the purposes of their application;

– to study the main approaches of digital and quantum marketing and their application to study factors influencing the consumer, modeling behavioral scenarios of consumers and developing technological maps of employees of the institution;

– to build mathematical models for developing a comprehensive logistics strategy of an organization using the example of a hotel business enterprise, based on the provisions of cluster and regression analysis, digital and quantum marketing approaches and game theory tools.

The proposed approaches of digital and quantum marketing in conjunction with the developed mathematical models make it possible to assess and predict the further behavior of consumers, assess the probability of attracting a new customer or maintaining the loyalty of a regular customer, and determine the types of advertising and offers that will interest a particular customer.

Presenting main material

For the description, further formalization and processing of data for further prediction of behavior, information cards of hotel customers are formed, which constitute a set of objects X_n: $X = \{x_1, \ldots, x_n\}$, which describes the information state of the system — the customer base.

Each object x is characterized by a certain list of factors $f_j(x)$, $j = \overline{1,m}$. Then the vector $(f_1(x), \ldots, f_m(x))$ serves as a characteristic description of a certain object — consumer x. And the entire system under study from the set of objects X_n: $X = \{x_1, \ldots, x_n\}$, is represented in the form of a matrix F describing the characteristics of the system of dimension $n \times m$:

$$F = \begin{pmatrix} f_1(x_1) & f_2(x_1) & \ldots & f_m(x_1) \\ f_1(x_2) & f_2(x_2) & \ldots & f_m(x_2) \\ \ldots & \ldots & \ldots & \ldots \\ f_1(x_n) & f_2(x_n) & \ldots & f_m(x_n) \end{pmatrix}.$$

Since the factors that characterize consumers can be described not only by quantitative characteristics, but also by nominal, ordinal, and binary characteristics, it is necessary to perform formalization and data recoding to describe the characteristics of the system and its further processing.

To predict the consumer behavior of a client — object x, an objective function is determined

$$z(f_j(x)) = \sum_{j=1}^{m} \alpha_j f_j(x),$$

where α_j — are the weight coefficients of the corresponding factors.

After recoding the client information card data to build consumer behavioral scenarios, the level of commitment *L(f(x))* of a given client *x* to the institution is determined. To do this, the total number of points for a given consumer is calculated. If the set of weight coefficients is represented in the form of a column matrix $\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_m \end{pmatrix}$, then the number of consumer points is determined from the product of the matrices: $L(f(x)) = \begin{pmatrix} f_1(x) & f_2(x) .. & f_m(x) \end{pmatrix} \cdot \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_m \end{pmatrix}$.

The column matrix of the number of points of all objects of the system S is determined by the formula:

$$L(f(x_i)) = F \cdot \alpha = \begin{pmatrix} f_1(x_1) & f_2(x_1) & ... & f_m(x_1) \\ f_1(x_2) & f_2(x_2) & ... & f_m(x_2) \\ ... & ... & ... & ... \\ f_1(x_n) & f_2(x_n) & ... & f_m(x_n) \end{pmatrix} \cdot \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_m \end{pmatrix}.$$

Based on the generalized marketing paradigms and the above factors of the logistics strategy, the following scheme for developing a comprehensive logistics strategy for a hotel was determined:

1. Carrying out a comprehensive analysis of the market and competitors.
2. Determining the target audience (market segmentation and identification of customer groups, their needs, interests, preferences and expectations from hotel services).
3. Developing a unique offer (unique positioning of the hotel to distinguish it among competitors and attract the target audience).
4. Setting current goals (determining specific and achievable goals aimed at fulfilling the current consumer request)
5. Determining optimal promotion channels based on digital and quantum marketing approaches (online marketing: hotel website, social networks, contextual advertising; SEO optimization: increasing the hotel's visibility in search engines; SMS — mailings, e-mails; online booking systems; loyalty programs: developing personal loyalty programs to encourage regular customers, a discount system).
6. Developing a content strategy (creating informative, meaningful and attractive content that demonstrates the advantages and attractions of the hotel and the peculiarities and value of its services).
7. Maintaining a competitive pricing policy focused on different financial capabilities of clients.
8. Developing a plan for implementing specific activities (alternative projects) and calculating the required budget.
9. Determining results (tracking key performance indicators: quality of services; meeting customer needs in housing, food, service and additional services; creating an image of the organization that inspires trust and is associated with quality and uniqueness).
10. Adjusting the strategy.

When developing a mathematical model of an organization's logistics strategy, the following mathematical methods are currently relevant and justified:

– Cluster analysis, which allows for a reasoned division of an institution's clients into clusters based on current information and the dynamics of consumer preferences and interests.

– Regression analysis, which establishes dependencies between influencing factors and con-

sumer behavior and subsequent choice.

– Digital and quantum marketing methods allow for a systematic understanding of consumer behavior in the multifactorial and dynamic environment of the digital economy and the use of an integrative approach to modeling consumer behavioral scenarios.

– Bayesian network methods allow for adaptive prediction of consumer behavior and choice and predict the probability of booking a customer number for a certain group.

– Game theory approaches are used to analyze strategic interaction between clients and organizations and make it possible to determine the probabilities of implementing certain strategies and projects.

– Machine learning tools and artificial neural network construction provide personalization of digital and quantum marketing and make it possible to predict which digital and quantum marketing methods are appropriate to use for certain groups of consumers or clusters of customers, which methods will be effective for certain loyalty programs and will be able to increase advertising profitability.

The construction and implementation of a mathematical model of the hotel's logistics strategy was carried out according to the following scheme (Fig. 1):

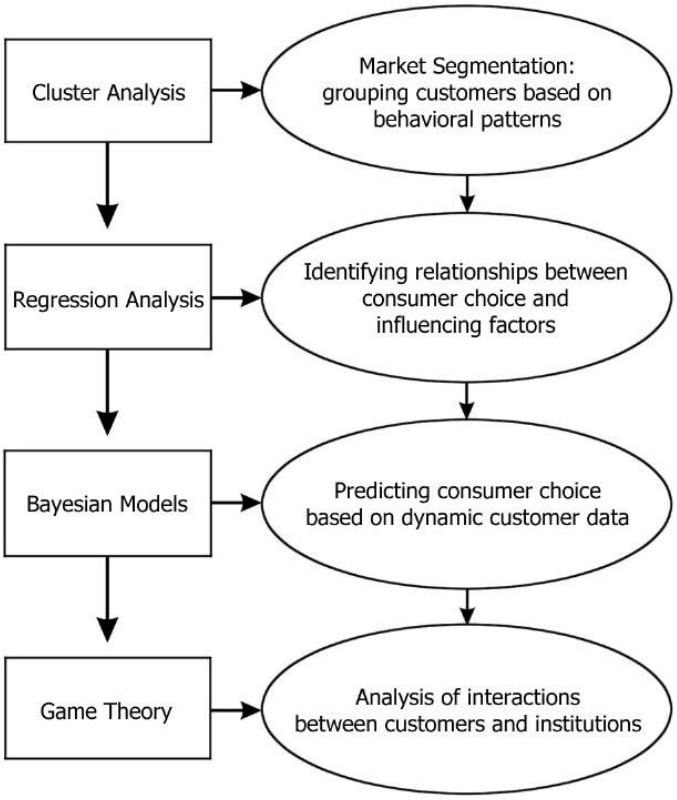

Fig. 1. Methods of mathematical modeling of a complex logistics strategy

1) Using cluster analysis tools, the hotel's customers are divided into clusters.

2) Using regression analysis tools, the dependencies between the factors influencing the customer and subsequent consumer behavior and the probability of continuing interaction with the establishment are established.

3) Using Bayesian models, the behavior and choice of the consumer are predicted and the probability of a customer booking a room in a certain group is calculated. The division into groups is carried out according to the number of points calculated based on the results of formalizing customer data.

4) Using game theory tools, the strategic interaction between customers and the organization is analyzed to determine the optimal loyalty programs to encourage customers.

When processing the hotel's customer questionnaires and based on the results of surveys in social networks and mobile messengers, it was determined that the main criterion by which the main differences in customer requests in the list of necessary services, in response to advertising offers and discounts were noted is the purpose of staying at the hotel. Based on this, the division of customers

into 6 main clusters was carried out precisely on this basis:

K1 - Hotel customers who came on a business trip;

K2 - Customers — organizers and participants of conferences, forums, seminars and other events based on the hotel;

K3 - Customers — visitors to tourist attractions;

K4 - Customers — visitors to well-known events held in the region;

K5 - Customers — visitors to this hotel chain;

K6 - Customers coming with their families for a family vacation.

But these clusters may have common elements, that is, these sets overlap. The hotel's database also includes clients belonging to several clusters [5].

After performing customer segmentation, an information card of a hotel customer x is formed, which is characterized by a list of 14 factors that serve as a characteristic description of this consumer. Tabl. 1 provides a list, designation and description of the recoding of the characteristics of certain factors.

Table 1. Structure and content of the consumer information card

Factor	Designation	Data formalization (recoding)
Availability of e-mail	X_1	0 – no, 1 – yes
Presence in social networks covered by hotel advertising	X_2	0 –no, 1 – present in 1 network, 2 – in two, etc.
Hotel visits during the current year	X_3	0 – did not visit, 1 – visited 1 time, 2 – twice, etc.
Number of hotel visits (according to information available in the hotel database)	X_4	0 – never, 1 – 1 time, 2 – 2-5 times, 3 – 5-8 times, 4 – more than 8 times
Belonging to a certain category (segment) of hotel visitors	X_5	0 – does not belong, 1 – belongs to 1 category, 2 – to 2 categories, 3 – more than 2
Use of additional hotel services	X_6	0 – does not use, 1 – use of 1 service, 2 – 2 services, etc.
Personalized offers regarding the consumer's business interests	X_7	0 – none, 1 – 1 offer, 2 – 2 offers, etc.
Personalized offers of personal interests	X_8	0 – none, 1 – 1 offer, 2 – 2 offers, etc.
Education	X_9	0 – secondary, 1 – specialized, 2 – higher
Age	X_{10}	2 – 18-25, 3 – 26-30, 4– 31-35 etc., 1 – over 65
Marital status	X_{11}	0 – single, 1 – married
Number of children	X_{12}	0 – none, 1 – 1 child, etc.
Activity on the hotel website, reaction to posts	X_{13}	0 – no activity, 1 – likes, 2 – writes reviews and comments
Number of positive reactions to offers with subsequent visits to the hotel	X_{14}	0 – none, 1 – 1, etc.

***Applying regression analysis*.** It has been determined that the most influential factors on consumer decisions, their commitment, and subsequent interaction are the reduction in the cost of a hotel room through loyalty programs and spending on customer-oriented advertising.

It is necessary to determine the impact of promotional discounts x_1, UAH and advertising costs x_2 thousand UAH on the number of customers involved in further interaction (who booked a hotel room based on advertising and loyalty programs).

To do this, we will draw up a correlation equation and determine the multiple correlation coefficient R and the coefficient of determination R^2.

Thus, it is necessary to determine a two-factor linear regression equation, which has the form:

$$\bar{y}_{x1x2} = a_0 + a_1 x_1 + a_2 x_2.$$

The coefficients of this equation are calculated using the formulas:

$$a_1 = \frac{S_{x_1 y}(S_{x_2 x_2} - S_{x_1 x_2})}{\Delta}, \qquad a_2 = \frac{S_{x_2 y}(S_{x_1 x_1} - S_{x_1 x_2})}{\Delta},$$

where $\Delta = S_{x_1 x_1} \cdot S_{x_2 x_2} - S^2_{x_1 x_2}$, $a_0 = \bar{y} - a_1 \cdot \bar{x}_1 - a_2 \cdot \bar{x}_2$.

Sample variances are determined from the formulas:

$$S_{x_1 x_1} = \sum_{i=1}^{n} (x_{1i} - \bar{x}_1)^2; \; S_{x_2 x_2} = \sum_{i=1}^{n} (x_{2i} - \bar{x}_2)^2; \; S_{x_1 x_2} = \sum_{i=1}^{n} (x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2);$$

$$S_{x_1 y} = \sum_{i=1}^{n} (x_{1i} - \bar{x}_1)(y_i - \bar{y}); \quad S_{x_2 y} = \sum_{i=1}^{n} (x_{2i} - \bar{x}_2)(y_i - \bar{y}).$$

where $\bar{x}_1$, $\bar{x}_2$, $\bar{y}$ — average sample values.

The multiple correlation coefficient R for a two-factor model is determined by the formula

$$R = R_{y(x_1 x_2)} = \sqrt{\frac{R^2_{x_1 y} + R^2_{x_2 y} - 2R_{x_1 x_2} \cdot R_{x_1 y} \cdot R_{x_2 y}}{1 - R^2_{x_1 x_2}}}.$$

Data processing and necessary calculations were performed in Excel.

As a result, we obtained a two-factor linear regression equation

$$\bar{y}_{x1x2} = -581{,}155 - 0{,}096 x_1 + 0{,}096 x_2,$$

which expresses the dependence of the number of customers attracted by loyalty programs (y) on promotional discounts (x_1) and advertising costs (x_2) in social networks, SMS — mailings, etc.

The correlation coefficient between promotional discounts on hotel bookings (x_1), UAH. and the number of consumers attracted to further cooperation (y) is equal to $r_{x1y} = 0{,}683$, which indicates a moderate relationship between these indicators. The correlation coefficient between advertising costs (x_2), thousand UAH. and the number of consumers attracted (y) is equal to $r_{x2y} = 0{,}849$, which indicates a close relationship between these factors. The multiple correlation coefficient is $R = R_{y(x_1 x_2)} = 0{,}858$ which indicates that the most significant is the simultaneous impact of both indicators of promotional discounts (x_1) and advertising costs (x_2) on the number of customers attracted by loyalty programs (y).

Bayesian Models and Probabilistic Estimation of Consumer Loyalty. To predict whether a customer will book a hotel room as a result of a promotional offer, a probabilistic estimation of consumer loyalty was performed using customer information card data to model the probability of their subsequent consumer behavior. After recoding the hotel customer information card data, a generalized information matrix $L(f(x_i))$ of consumer data is created and the total loyalty score of each customer is calculated, the value of which is equal to the sum of the elements of the corresponding data row of this customer — object x_i.

In the next step, the hotel's base customers were divided into 5 groups depending on the total loyalty score: group H1 consisted of customers with a total score in the range from 0 to 10, group H2 consisted of customers whose total score belonged to the range from 11 to 19, group H3 consisted of customers with a total score from 20 to 29, group H4 consisted of consumers with a total score from 30 to 39, group H5 consisted of consumers with more than 40 points. After the grouping was completed, a probabilistic Bayesian model was constructed.

According to the condition of the problem, event A is that a hotel base customer, having received an advertisement with a promotional offer by e-mail, in social networks or via a mobile messenger, agrees to the terms of the promotion and books a room at the hotel.

According to statistical data from the analysis of customer participation in previous promotional offers and expert assessments, it is known that event A depends on which loyalty group the customer belongs to (groups — hypotheses with which event A can occur) and the obtained conditional

probabilities *P(A/Hi).* According to the statistical probability formula $P(H_i) = \frac{n_i}{n}$, the probabilities of each of the described hypotheses $P(H_i)$ were calculated.

It is necessary to estimate the probability that a potential customer will book a hotel room based on the results of the advertising campaign and estimate the probability that this will be a representative of each of the groups, provided that the customer has booked a hotel room.

If it is known that event *A* can occur together with one of the events $H_1, H_2, ..., H_n$, which are called hypotheses and form a complete group of pairwise incompatible events, then event *A* can be represented as a combination of events

$$A = AH_1 + AH_2 + ... + AH_n.$$

The probability of event A is determined by the formula of total probability

$$P(A) = \sum_{i=1}^{n} P(H_i) \cdot P(A/H_i) = P(H_1) \cdot P(A/H_1) + P(H_2) \cdot P(A/H_2) + ... + P(H_n) \cdot P(A/H_n)$$

The conditional probability of a hypothetical event assuming that event A has already occurred is determined by Bayes' formula

$$P(H_i / A) = \frac{P(A/H_i) \cdot P(H_i)}{\sum_{i=1}^{n} P(A/H_i) \cdot P(H_i)}.$$

The results of data processing and necessary calculations are given in Tabl. 2.

Table 2. Bayesian model calculation table

Group (hypothe-sis *(Hi)*	**Number of points**	**Number of consumers** (n_i)	**Conditional probability** *P(A/Hi)*	**Probability of the hypothesis** *P(Hi)*	*P(A/Hi)* *P(Hi)*	**Probability** *P(Hi/A)*
H1	0-10	80	0,07	0,0871	0,0061	0,0101
H2	11-19	175	0,3	0,1904	0,0571	0,0949
H3	20-29	264	0,6	0,2873	0,1724	0,2864
H4	30-39	290	0,8	0,3156	0,2524	0,4195
H5	40-50	110	0,95	0,1197	0,1137	0,1890
	n	919		1	0,60174	1
Booking probability		**(total probability)**			**0,60174**	

Thus, using the formula of total probability, the probability of booking a room by a client of the hotel database $P(A) = 0.602$ was found.

Using the Bayes formula, the probabilities of booking a hotel room under promotional offers by a client of a certain group were calculated, provided that the room is booked.

Thus, sending advertising messages and letters with promotional offers is effective for hotel clients of the third, fourth and fifth groups, that is, for those who have at least 20 points according to the results of processing information cards. Therefore, it is to them that further loyalty programs should be directed, and it is for these clients that it makes sense to develop personal offers and options for providing additional services, taking into account obtaining more information about consumer data.

Selection of optimal loyalty programs using game theory. Let's consider the option of assessing the effectiveness of loyalty programs and their announcement at certain events and on online platforms using game theory approaches. The analytical department of the hotel, having studied the data of clients of the main clusters, developed business plans for loyalty programs for representatives of each cluster. Depending on external factors (political, economic situation, state of the hotel services market) and conditions of communication with consumers (advertising posts on Facebook, Instagram, SMS-sending, e-mail, etc.), 5 scenarios were created.

For each business plan *Xi, i=1, 2,..., 6* and scenarios *Yj , j=1,2, ..., 5*, the forecasted profits

(mln. UAH) were determined, which are presented in the form of a game payment matrix

$$P = \begin{pmatrix} 2,4 & 2,8 & 2,2 & 3,4 & 2,2 \\ 3,5 & 3,2 & 4,4 & 2,4 & 2,8 \\ 2,4 & 2,6 & 2,3 & 3,2 & 2,9 \\ 3,2 & 3,1 & 3,6 & 2,4 & 2,6 \\ 4,2 & 3,5 & 2,6 & 3,1 & 4,5 \\ 3,5 & 3,4 & 3,2 & 3,5 & 2,4 \end{pmatrix}.$$

It is necessary to choose the optimal variant of the loyalty program business plan or several combinations thereof. Analysis of the business plans of the developed loyalty programs according to possible scenarios of their implementation will allow to select those projects, the implementation of which will bring the greatest economic benefit even taking into account possible risks.

The problem was solved using the Mathcad symbolic mathematics system, reducing the matrix game to a linear programming problem.

To find the lower and upper prices of the game, we will compile a tabl. 3 of the game according to its payment matrix [6].

Table 3. Determination of the lower and upper prices of the game

Loyalty programs	**Program scripts**				
	Y_1	Y_2	Y_3	Y_4	Y_5
X_1	2,4	2,8	2,2	3,4	2,2
X_2	3,5	3,2	4,4	2,4	2,8
X_3	2,4	2,6	2,3	3,2	2,9
X_4	3,2	3,1	3,6	2,4	2,6
X_5	4,2	3,5	2,6	3,1	4,5
X_6	3,5	3,4	3,2	3,5	2,4

We apply the maximin strategy for player A. We determine the minimum values in each row of the game table. The lower price of the game is equal to the maximum value of these minimum values — 2.6 million UAH:

$$\alpha = \max_j \min_i a_{ij} = \max\{2,2;\ 2,4;\ 2,3;\ 2,4;\ 2,6;\ 2,4\} = 2,6$$

For player B, we apply the minimax strategy and determine the upper price of the game. In each column of the game table, we determine the maximum value. The upper price of the game is equal to the minimum value of the obtained maximum column values — 3.5 million UAH:

$$\beta = \min_i \max_j a_{ij} = \min\{4,2;\ 3,5;\ 4,4;\ 3,5;\ 4,5\} = 3,5.$$

Thus, we have obtained a mixed strategy game and, since this problem has no saddle point.

For further solving the problem using symbolic mathematics MathCAD, we will reduce the game to a linear programming problem. With this approach, the problem is reduced to finding the minimum value of the objective function: $\min F(t) = t_1 + t_2 + t_3 + t_4 + t_5 + t_6$. Subject to the following restrictions:

$$\begin{cases} 2,4t_1 + 3,5t_2 + 2,4t_3 + 3,2t_4 + 4,2t_5 + 3,5t_6 \geq 1; \\ 2,8t_1 + 3,2t_2 + 2,6t_3 + 3,1t_4 + 3,5t_5 + 3,4t_6 \geq 1; \\ 2,2t_1 + 4,4t_2 + 2,3t_3 + 3,6t_4 + 2,6t_5 + 3,2t_6 \geq 1; \\ 3,4t_1 + 2,4t_2 + 3,2t_3 + 2,4t_4 + 3,1t_5 + 3,5t_6 \geq 1; \\ 2,2t_1 + 2,8t_2 + 2,9t_3 + 2,6t_4 + 4,5t_5 + 2,4t_6 \geq 1; \end{cases}$$

where $t \geq 0 \quad (i = \overline{1,6})$.

Let’s find a solution to this problem using the symbolic mathematics system Mathcad:
We will use the matrix form of data representation and implement the solution of the problem using the Given — Minimize block. To do this, you need to:

1) Specify the initial approximations.

2) Specify the vector of variables of the objective function *T(t)* and its coefficients C (in the form of column matrices).

3) Specify the objective function *F(t)=C•T(t),* constraints for the variables and the matrix of constraint coefficients *D* and *B.*

4) Describe the **Given-Minimize block**, indicating $D \cdot T(t) \geq B, \quad T(t) \geq 0$

5) We obtain a solution vector that satisfies the constraints in the solution block and provides the smallest value of the function F(t)=0.314.

6) We set the formula and determine the price of the game *w* = 3.186 million UAH.

7) We determine the probabilities of implementing the business plans of the developed loyalty programs by the formula

$$P = \frac{t^T}{s}, \text{ where } s = \sum_{i=1}^{6} t_i .$$

Fig. 2, 3 show the steps of data entry and problem solving.

$t_1 := 0 \quad t_2 := 0 \quad t_3 := 0 \quad t_4 := 0 \quad t_5 := 0 \quad t_6 := 0$

$$T(t) := \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \end{pmatrix} \qquad C := \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$F(t) := C \cdot T(t)$

$t_1 \geq 0 \quad t_2 \geq 0 \quad t_3 \geq 0 \quad t_4 \geq 0 \quad t_5 \geq 0 \quad t_6 \geq 0$

$$D := \begin{pmatrix} 2.4 & 3.5 & 2.4 & 3.2 & 4.2 & 3.5 \\ 2.8 & 3.2 & 2.6 & 3.1 & 3.5 & 3.4 \\ 2.2 & 4.4 & 2.3 & 3.6 & 2.6 & 3.2 \\ 3.4 & 2.4 & 3.2 & 2.4 & 3.1 & 3.5 \\ 2.2 & 2.8 & 2.9 & 2.6 & 4.5 & 2.4 \end{pmatrix} \qquad B := \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Fig. 2. Data entry and problem statement in MathCAD

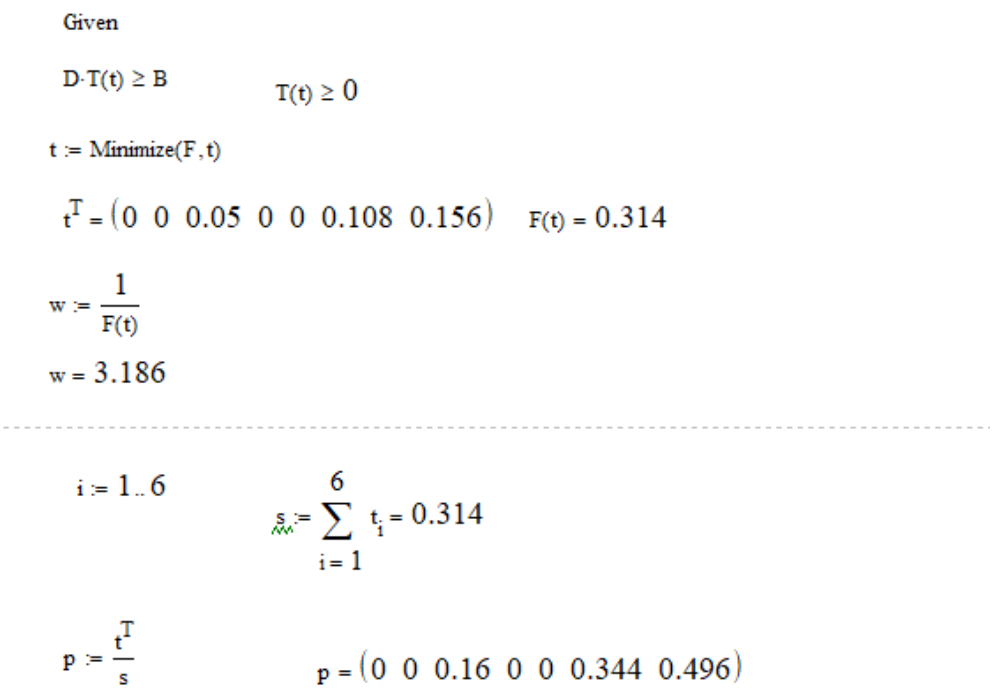

Fig. 3. Organization of the Given-Minimize block and output of results in MathCAD

The probabilities of implementing business plans for clusters K2, K5 and K6 are 0,16, 0,344 and 0,496, respectively. Thus, using consumer data from different clusters using game theory approaches, it is possible to predict the implementation of strategic development programs and select the most optimal ones.

Conclusions

During the research, the issue of studying the relevance of using mathematical models for developing logistics strategies of the organization was resolved; systematization of mathematical methods for modeling the logistics strategy of the enterprise and approaches to their application; study of the main approaches of the paradigms of digital and quantum marketing and their application for analyzing factors influencing a potential client, modeling behavioral scenarios of consumers and developing technological maps of the institution's employees; construction of mathematical models for developing a comprehensive logistics strategy of the hotel, based on the provisions of cluster and regression analysis, paradigms of digital and quantum marketing and tools of game theory. The proposed mathematical models can be used in the activities of hotel and restaurant business establishments for a deeper understanding of customer behavior, to attract new guests and extend the term of interaction with existing customers by means of effective use of customer-oriented advertising and loyalty programs developed for the personal needs and interests of the consumer.

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Надійшла до редколегії 26.11.2025
Прийнята після рецензування 10.01.2026
Опублікована 22.01.2026