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MAGLEV'S MECHANICAL SUBSYSTEM'S MOVEMENT'S ANALYTICAL DESIGN ALGORITHM

АЛГОРИТМ АНАЛІТИЧНОГО КОНСТРУЮВАННЯ РУХІВ МЕХАНІЧНОЇ ПІДСИСТЕМИ МАГЕВ ПОЇЗДА

Improving the quality of analysis and synthesis of processes occurring in the mechanical subsystem of a maglev train inevitably requires the transition to nonlinear models of their controlled dynamics. Therefore, it is urgent to develop, in a nonlinear formulation, a rational algorithm for the analytical construction of such dynamics. Currently, a number of methods have been developed that allow for a fairly high level of synthesis of the motions of systems whose models are considered linear. However, linear models, in addition to other features, have an inherent drawback: they are adequate in describing regimes of only small deviations of system states from specified levels. The current level of development of methods for designing the movements of nonlinear objects, despite the intensive development of information technologies, remains insufficient for the implementation of full automation of such design. The feasibility of developing such methods is also due to the fact that the widespread use of onboard and stationary information complexes, as well as individual microprocessors, allows, to a significant extent, to reduce the problem of the complexity of implementing nonlinear regulators and reduce the main part of the global problem of constructing movements to the synthesis of control algorithms taking into account its dominant properties, which leads to the need to use nonlinear dynamics models. Based on the above, this stage of the research is devoted to building a rational algorithm for analytically constructing the desired movements of the subsystem under consideration. This construction is based on the concept of joint use of methods of Lyapunov stability theory and terminal control. This approach was chosen because it is characterized by a fairly rigorous mathematical formulation of the main control problems related to asymptotic stability, other important qualities of motion, and the synthesis of its controller.

Keywords: *maglev train; mechanical subsystem; movement design; analytical design; algorithm.*

При побудові рухів механічної підсистеми магнітолевітуючого поїзда повинні бути задоволені вимоги забезпечення бажаних властивостей цих рухів, а також обмежень, що накладаються на них.

Підвищення якості аналізу та синтезу процесів, які протікають у підсистемі, яка розглядається, безальтернативно вимагає переходу до нелінійних моделей їхньої керованої динаміки. Тому актуальною є розробка, в нелінійній постановці, раціонального алгоритму аналітичного конструювання такої динаміки.

Наразі розвинений ряд методів, які дозволяють забезпечувати досить високий рівень синтезу рухів систем, моделі яких вважаються лінійними. Однак, лінійні моделі, крім інших особливостей, мають іманентний недолік: вони адекватні при описі режимів лише малих відхилень станів систем від рівнів, що задаються. Сучасний рівень розвитку методів конструювання рухів нелінійних об'єктів, незважаючи на інтенсивний розвиток інформаційних технологій, залишається недостатнім для здійснення повної автоматизації такого конструювання.

Доцільність розробки таких методів обумовлюється ще й тим, що повсюдне застосування бортових і стаціонарних інформаційних комплексів, а також окремих мікропроцесорів дозволяє, в значній мірі, редукувати проблему складності реалізації нелінійних регуляторів і звести основну частину глобальної проблеми побудови рухів до синтезу алгоритмів керування з урахуванням її домінуючих властивостей, що веде до необхідності використання нелінійних моделей динаміки.

Виходячи з викладеного, цей етап дослідження присвячений побудові раціонального алгоритму аналітичного конструювання бажаних рухів підсистеми, що розглядається. В основу цієї побудови покладено концепцію спільного використання методів теорії стійкості Ляпунова та термінального керування. Такий підхід обраний, оскільки йому притаманне досить строге математичне формулювання основних задач керування, пов'язаних з асимптотичною стійкістю, іншими важливими якостями руху та синтезу його регулятора.

***Ключові слова**: маглев поїзд; механічна підсистема; конструювання руху; аналітичне конструювання; алгоритм.*

Problem’s formulation

Currently, a number of methods have been developed that allow for a fairly high level of synthesis of the motions of systems whose models are considered linear. However, linear models, in addition to other features, have an inherent drawback [1]: they are adequate in describing regimes of only small deviations of system states from specified levels. Therefore, improving the quality of analysis and synthesis of processes occurring, including in MLT’s MS, inevitably requires a transition to nonlinear models of their controlled dynamics. Therefore, improving the quality of analysis and synthesis of processes occurring, including in MLT’s MS, inevitably requires a transition to nonlinear models of their controlled dynamics. In this regard, the development, in a nonlinear formulation, of a rational algorithm for the analytical construction of such dynamics is relevant.

Analysis of recent research and publications

The development of information systems, and particularly artificial intelligence systems, is gradually increasing the achievable quality of modeling analyzed processes, including nonlinear ones. Despite the noted intensive development of information technology, the ability to fully automate such modeling remains insufficient [2].

The possibility of developing such methods is also due to the fact that the use of on-board and stationary information systems, as well as microprocessors, makes it possible to reduce the main part of the problem of constructing the movements of the analyzed MS to the synthesis of control algorithms, which leads to the need to use nonlinear dynamics models [3].

Formulation of the study purpose

Based on the above, this stage of the study is dedicated to maglev’s mechanical subsystem’s motion analytical design algorithm construction, the main concept of which is the combined use of Lyapunov stability theory and terminal control methods [4].

Presenting main material

When solving the problem of designing the maglev train’s (MLT’s) mechanical subsystem’s (MS’s) desired motions, one of the central tasks is the synthesis of a controller that satisfies the performance requirements for these motions. In a fairly general case, the synthesized dynamics can be described by nonlinear differential equations of the form

$$\ddot{x}_i(t) = \Phi_i(x_i, \dot{x}_i) + \mathrm{b}_j \cdot \mathrm{U}_j(x_i, \dot{x}_i) + \mathrm{D}_k \ \forall i \in [\overline{1,n}],\ j \in [\overline{1,m}], k \in [\overline{1,r}], \quad (1)$$

where $x_i, \dot{x}_i \ \forall i \in [\overline{1,n}], n$ — MS’s calculation scheme’s phase coordinates, as well as the number of such coordinates; $\Phi_i \forall i \in [\overline{1,n}], \mathrm{b}_j \ \forall j \in [\overline{1,m}], t$ — continuous, continuously differentiable functions in their arguments, weighting coefficients of dynamics model controls, as well as the independent variable of the current time; $\mathrm{U}_j \ \forall j \in [\overline{1,m}], \mathrm{D}_k \ \forall k \in [\overline{1,r}]; m, r$ — subsystem's controls, it's perturbation and their number.

In model (1), the form of the functions $\Phi_i(x_i, \dot{x}_i)\forall i \in [\overline{1,n}]$ is known from the description of the natural dynamics of the subsystem. At the same time, the type of functions $\mathrm{U}_j(x_i, \dot{x}_i)\forall i \in [\overline{1,n}], j \in [\overline{1,m}]$ is subject to definition, from model (1). To synthesize this dynamics, it is rational to adopt the terminal principle of control by it as one of the most flexible and universal [5]. The type of admissible controls $\mathrm{U}_j\{x_i(t), \dot{x}_i(t)\}\forall i \in [\overline{1,n}], j \in [\overline{1,m}], t \in [t_s, t_f]$ can be selected from the class of piecewise-continuous functions and, then, the motion synthesis problem is reduced to determining a specific type of these functions that would ensure the transfer of the subsystem from the initial state $\{x_{is}, \dot{x}_{is}\}\forall i \in [\overline{1,n}]$ to the terminal $\{x_{if}, \dot{x}_{if}\}\forall i \in [\overline{1,n}]$, on the current segment of motion construction, state within a given time interval t .

To date, creative methods of motion synthesis have been developed, to a sufficient extent, only for systems whose dynamics can be described by linear models, which in principle do not allow modeling regimes of any significant deviations of their states from those established [6]. Meanwhile, it is obvious that in many practically important cases, to increase the efficiency of MLT's MS's motion control, a transition to nonlinear dynamics models is required, which allow implementing its qualitatively new properties. The dynamic properties of the analyzed MS can be described using various quality indicators. In turn, these indicators are inherently related to the requirements for the nature of transient processes in the subsystem, which can be fixed in various ways [7].

Any physical law can be written in two equivalent forms: as differential equations that reflect the balance of interactions that take place, or in the form of the corresponding variational principle, from which the specified equations directly follow [8]. Among the many requirements for the quality of motion of a subsystem, the most important and fundamental is the requirement of asymptotic stability of its motion. This requirement must, of course, be satisfied by means of control synthesis, the law of which, in this case, is called stabilizing [9].

However, stability, as a rule, is far from exhausting the set of requirements that are imposed on the quality of motion of the synthesized subsystem. Chronologically, the first attempt to formulate requirements for the quality of movement was a method based on setting the limit values of the primary indicators of this quality [10]. This allows for a reasonable compromise between different, to some extent conflicting requirements. This method of assessing the quality of transient processes largely corresponds to intuitive ideas about the essence of the control problem, but it can be applied mainly only for one-dimensional linear and, sometimes, nonlinear objects [11].

A different way of formalizing the requirements for the quality of transient processes, which is based on the problem of the desired differential equation by the regulated coordinate of the system [12], has wider possibilities for constructing movements.

$$F[x_i, q_r, C_i] = 0 \forall i \in [\overline{1,r}], r \in [\overline{1,m}], \quad (2)$$

where $q_r \forall r \in [\overline{1,m}]$ — impacts on the subsystem that are specified; $C_i \forall i \in [\overline{1,r}]$ — configuration options; F — a continuous function of its arguments.

These equations describe a class of motions that have properties determined by the influencing factors $q_r \forall r \in [\overline{1,m}]$ and the settings $C_i \forall i \in [\overline{1,r}]$.

Both of these methods of taking into account the requirements for the quality of movement are not effective if it needs to be endowed with extreme properties with respect to the dominant criteria. To ensure this possibility, control $\mathrm{U}_j(x_i, \dot{x}_i)\ \forall i \in [\overline{1,n}], j \in [\overline{1,m}]$, over the trajectory of motion must deliver the extremum of the corresponding functional being optimized, of the integral type — the criterion of motion quality — and the controller should be synthesized as an optimizing one. Typically, the problem of such synthesis is solved for linear objects [11]. That is, we are looking for controls $U_j(x_i, \dot{x}_i)\forall i \in [\overline{1,n}], j \in [1,m]$ by the phase coordinate $\{x_i, \dot{x}_i\}\forall i \in [\overline{1,n}]$ function that would ensure the asymptotic stability of the motion described by model (1), as well as the extremum of the quadratic quality criterion of the form [13]

$$I = 0,5 \cdot \int_0^{\infty} \left(\beta_{ik} \cdot x_i \cdot x_k \cdot \delta^i \cdot \delta^k + \alpha_j^{(2)} \cdot U_j^{(2)} \cdot \delta^j \right) \cdot dt \;\; \forall i,k \in [\overline{1,n}],\, j \in [\overline{1,m}], \tag{3}$$

where $\beta_{ik} \forall i,k \in [\overline{1,n}], \alpha_j \forall j \in [\overline{1,m}]$ — weighting factors to be selected; $\delta^i, \delta^k \forall i,k \in [\overline{1,n}]$, $\delta^j \forall j \in [\overline{1,m}]$ — unit column vectors; $\beta_{ik} \cdot x_i \cdot x_k \cdot \delta^i \cdot \delta^k \, \forall i,k \in [\overline{1,n}]$ — definitely positive quadratic form.

The expressions for the controls, that deliver the extremum of criterion (3) on the trajectory of representing subsystem's state point motion, which is described by model (1), have the form [13]

$$U_j = \alpha_j^{(-2)} \cdot \frac{\partial V}{\partial x_j} \cdot \delta^j \; \forall j \in [\overline{1,m}], \tag{4}$$

where $V(x_i) \forall i \in [\overline{1,n}]$ — is the solution of the basic functional equation

$$\frac{\partial V}{\partial x_i} \cdot f_i \cdot \delta^i - 0,5 \cdot \alpha_j^{(-2)} \cdot \left(\frac{\partial V}{\partial x_i} \cdot \delta^i \right)^{(2)} = -\,0,5 \cdot \beta_{ik} \cdot x_i \cdot x_k \cdot \delta^i \cdot \delta^k \;\; \forall i,k \in [\overline{1,n}],\, j \in [\overline{1,m}]. \tag{5}$$

Then, using criterion (3), the control synthesis problem is reduced to finding a forced solution $V(x_i) \forall i \in [\overline{1,n}]$ to the nonlinear partial differential equation (5), the solution method of which is currently unknown [14].

To eliminate the collision that thus arises, a method was proposed that involves searching for an optimizing control based on the generalized work criterion [15]

$$J = 0,5 \cdot \int_0^{\infty} \Bigg[\beta_{ik} \cdot x_i \cdot x_k \cdot \delta^i \cdot \delta^k + \alpha_j^{(2)} \cdot U_j^{(2)} \cdot \delta^j + \left(\alpha_j \cdot B_{ij} \cdot \frac{\partial V}{\partial x_i} \cdot \delta^i \right)^{(2)} \cdot \delta^j \Bigg] \cdot dt \, \forall i,k \in [\overline{1,n}],\, j \in [\overline{1,m}], \tag{6}$$

which is semi-definite and differs from criterion (3) by an additional term

$$\int_0^{\infty} \left(\alpha_j \cdot B_{ij} \cdot \frac{\partial V}{\partial x_i} \cdot \delta^i \right)^{(2)} \cdot \delta^j \cdot dt \, \forall i \in [\overline{1,n}],\, j \in [\overline{1,m}]. \tag{7}$$

The control that delivers the extremum of criterion (6) on the motion trajectories described by model (1) can be defined according to the expressions

$$U_j = -\alpha_j^{(-2)} \cdot B_{ij} \cdot \frac{\partial V}{\partial x_i} \cdot \delta^i \; \forall i \in [\overline{1,n}],\, j \in [\overline{1,m}], \tag{8}$$

where the function $V(x_i) \forall i \in [\overline{1,N}]$ is a solution to the functional equation

$$\frac{\partial V}{\partial x_i} \cdot f_i(x_i) \cdot \delta^i = -0,5 \cdot \beta_{ik} \cdot x_i \cdot x_k \cdot \delta^i \cdot \delta^k \;\; \forall i,k \in [\overline{1,n}]. \tag{9}$$

The advantage of the method of constructing optimizing controls using criterion (6) is that equation (9), unlike (5), is a linear partial differential equation. This allows finding their numerical solutions [15] and therefore constructing optimizing controls $U_j \forall j \in [\overline{1,m}]$.

Thus, to date, considerable progress in solving the problem of analytical construction of motions has been achieved for linear systems. At the same time, as noted, the MLT's MS, in order to correctly construct its necessary movements, must often be considered as significantly nonlinear, which reveals an urgent need to develop an algorithm for such construction. The results of the analysis of possible approaches to this development indicate the feasibility of dividing it into two metablocks:

– structural synthesis of subsystem's control which is guaranteed the general fundamental properties of its movements (their asymptotic stability in general, or in the desired region of the phase space; coarseness; the desired nature of transient processes, as well as other most cardinal ones);

– parametric synthesis of these control, which allows to provide the desired special properties of movements in different operational modes of the train.

The above methods of taking into account the requirements for the dynamic properties of the subsystem are certainly not independent, or, even more so, contradictory to each other. They are immanently related, as they are different attempts to formalize the requirements for the same dynamic system. Depending on the specific conditions of the subsystem design, one or another method, or some combination of them, may be used, since each of them has its own advantages and disadvantages.

When constructing effective controls for a moving object, a complex problem arises of satisfying the set of requirements imposed on the quality of this motion in steady-state and transient regimes. Such requirements can be not only diverse but also antagonistic. This can significantly complicate their reflection by a single, constant in all modes, criterion of traffic quality. Therefore, in cases where "competing", equally important requirements cannot be satisfied equally separately, there is a need to use some aggregate of them. One of the ways to construct rational and, moreover, optimizing control in such a case is to introduce a vector criterion consisting of a number of secondary criteria that the movement must simultaneously satisfy [16].

In the general case, achieving the optimum simultaneously according to several criteria is very difficult, if even possible in principle. Therefore, solving the vector optimization problem requires making a certain compromise by forming a generalizing criterion. Its formation is an informal procedure determined by the choice of a compromise scheme. This reduces the vector problem to a scalar problem and makes it essentially indeterminate [17].

The complexity of formulating and solving the problem of vector optimization of the motion of a dynamic system significantly depends on the chosen compromise scheme, that is, the principle of its scalarization [16]. As for dynamic systems, most of such schemes, depending on the optimization principle they apply, can be mainly divided into two groups:

– those in which optimization is assumed according to an integral criterion, the expression for determining the values of which can be represented by the sum of weighted secondary criteria

$$\Sigma = \min \lambda_k \cdot \Upsilon_k \cdot \delta^k \forall k \in [\overline{1,\mathrm{P}}], \quad (10)$$

where $\Upsilon_k, \lambda_k \forall k \in [\overline{1,\mathrm{P}}]$ — the private optimization criteria and scalarization weight coefficients; $\mathrm{P}, \delta^k \forall k \in [\overline{1,\mathrm{P}}]$ — the number of such criteria that are taken into account, as well as the unit column vector; Σ — summarizing optimization criterion;

– those in which optimization is assumed according to the minimax criterion:

$$\Sigma = \min \max \Upsilon_k \forall k \in [\overline{1,\mathrm{P}}]. \quad (11)$$

Other compromise schemes are usually based on various modifications of these or similar criteria.

The scalarization scheme used should be, if possible, most adequate to the current operating mode of the subsystem. In practice, for most operating conditions, it is possible to conditionally limit ourselves to considering two basic states of the MS: its small and large deviations from the required motion, which correspond to the division of the state space into internal and external areas. In the small deviation mode, when the representing state point is in the internal region, the behavior of the subsystem can be described by first approximation equations in the form of linear or linearized differential equations and optimization can be carried out according to the integral optimization criterion (10). In tense, in particular, transitional, regimes, when the imaging point is in the region of large deviations, an adequate mathematical model of motion is usually nonlinear. This circumstance significantly complicates the procedure of movement synthesis. In such modes, the minimax quality criterion of type (11) is appropriate, which allows limiting the permissible deviations of the imaging point from the required state [18]. However, a criterion of type (11) is not analytic [19]. Therefore, to construct movements in the mode of large deviations of the representing state point of the subsystem, one should use other, more convenient functionals, which, to a greater extent, have the properties of analyticity when approximating the minimax criterion in the sense of the proximity of the transient processes obtained in this case. Of the criteria used in optimal control theory, the one closest to the minimax criterion is the speed criterion [10]. However, in regimes of large deviations, the subsystem dynamics model, as noted, is significantly nonlinear, which significantly complicates the synthesis of control laws that optimize its speed [12]. In intermediate modes, as well as in a wide range of changes in the external and internal conditions of train movement, it is advisable to choose a conditionally "universal" compromise scheme that would ensure, in accordance with the mode, an approximation to both the integral and minimax criteria for optimizing movement.

Thus, there is a need to form a convenient-to-use generalized indicator of their quality — a functional — when constructing subsystem movements, including extreme ones, which: in the external region of the phase space, it would allow effectively suppressing large deviations that have arisen in the shortest possible time of the transition process; in the regimes of small deviations of the representing state point from the required one, it could be used to ensure the required quality of movements in the internal region of the phase space by maintaining the corresponding number of the first terms of the expression of the specified indicator.

The synthesis of movements using such a criterion will allow for a regulated approximation of the compromise scheme to a universal one and can be interpreted as some consistent rationalization of these movements. Its essence lies in the phased construction of movement, when the synthesis of each subsequent level is carried out taking into account the movements of the previous levels synthesized according to their own criteria. For this purpose, at each subsequent stage of synthesis, a model of the current level subsystem's dynamics is used.

The efficiency of achieving the set of goals that ensure the solution of the q-th movement problem in its i-th situation can be described by the vector

$$\{g_{ij}[u_{ij}(\bullet), w_i(\bullet)];\, u_{ij}(\bullet) \in \mathrm{U}_{ij}(\bullet), w_i(\bullet) \in \mathrm{W}(\bullet)\ \ \forall\ j \in [\overline{1,\Xi}]\}_q, \tag{12}$$

where g_{qij}, u_{qij} — is the value of the criterion spent on achieving the j-th goal when solving the q-th motion problem in its i-th situation, as well as the control necessary for this; w_i, W — the value of the disturbing influence on the subsystem, as well as the limitations on this possible value; U_{qij} — the value of the permissible control that is necessary to ensure implementation of g_{qij}; Ξ_q — the total number of movement goals being pursued.

In expression (12) and further, any function with a point in the place of arguments implies the set of its possible values for all possible values of these arguments.

Then (assuming that the vector u_{qij} is l-dimensional) to ensure the possibility of joint achievement of such goals it is necessary that

$$\mathrm{U}_{qij}(\bullet) \subseteq \mathrm{U}(\bullet) \subseteq \mathrm{E}^l\ \ \forall\ j \in [\overline{1,\Xi_q}], \tag{13}$$

where E^l — is a l-dimensional euclidean space.

The last condition formalizes the requirement of changing the criteria (12) on a single set, which limits the possible control $\mathrm{U}(\bullet)$ of the subsystem and allows transforming the set of these criteria to the form

$$g_{qij}[u_{qij}(\bullet), w_{qi}(\bullet)];\, u_{qij}(\bullet) \in \mathrm{U}(\bullet), w_{qi}(\bullet) \in \mathrm{W}(\bullet)\ \ \forall\ j \in [\overline{1,\Xi_q}]. \tag{14}$$

In this case, from solving single-criteria optimization problems of the form

$$\gamma_{qij} = \inf_{u_{qij}(\bullet)} \{g_{qij}[u_{qij}(\bullet), w_{qi}^*(\bullet)];\, u_{qij}(\bullet) \in \mathrm{U}(\bullet), w_{qi}^* \in \mathrm{W}(\bullet)\}$$

$$\forall\ j \in [\overline{1,\Xi_q}], \tag{15}$$

where $w_{qi}^*(\bullet)$ — extremely possible, when solving the q-th motion problem in its i-th situation, a disturbing influence on the subsystem private controls can be found

$$\hat{u}_{qij}(\bullet) = \{\hat{u}_{qijk}(\bullet)\ \ \forall\ k \in [\overline{1,l}]\}^{\mathrm{T}}\ \ \forall\ j \in [\overline{1,\Xi_q}], \tag{16}$$

each of which is optimal with respect to one of the criteria (14).

The first property of these criteria is their possible mutual antagonism. Therefore, the complete optimizing control of the subsystem

$$u_{qi}(\bullet) = \{\hat{u}_{qijk}(\bullet)\ \ \forall\ j \in [\overline{1,\Xi_q}]\ k \in [\overline{1,l}]\}^{\mathrm{T}} \tag{17}$$

on the set $U(\bullet)$ must occupy some intermediate position between the controls (16). To find the control (17) from the mentioned set, it is necessary to select elements that correspond to the idea of a compromise between the extreme values of the criteria (14). Such elements, as is known, form a Pareto region [20] (π -domain), within which it is impossible not to simultaneously deteriorate all the private criteria (14). Therefore, the control $[\hat{u}_{qi}]_{\pi}(\bullet)$ is Pareto-optimal according to such criteria if

$$\neg\exists\ \hat{u}_{qi}(\bullet)\ |\ g_{qij}[\hat{u}_{qi}(\bullet), w^{*}_{qi}(\bullet)] \le g_{qij}\{[\hat{u}_{qi}]_{\pi}(\bullet), w^{*}_{qi}(\bullet)\}$$
$$\forall\ j \in [\overline{1, \Xi_q}] \qquad (18)$$

and at least for one $j \in [\overline{1, \Xi_q}]$ the last inequality is strict. For convex objective functions (14) the selection of points of the π -domain (compromise domain) can be obtained [20] in the form

$$p_{qij}\{[\hat{u}_{qij}]_{\pi}(\bullet), w^{*}_{qi}(\bullet)\} = \min_{\alpha}\left\langle p_{qij\alpha}\{[\hat{u}_{qij}(\bullet), w^{*}_{qi}(\bullet)]\forall\alpha \in [\overline{1, \Xi_q}]\}\right\rangle;\ p_{qij} = \lambda_{qij}\cdot g_{qij}$$
$$\forall\ j \in [\overline{1, \Xi_q}], \qquad (19)$$

where $\lambda_{qij}\forall\, j \in [\overline{1, \Xi_q}]$ — constant for each point $[\hat{u}_{qij}]_{\pi}\ \forall\, j \in [\overline{1, \Xi_q}]$ positive coefficients $\lambda_{qij} \in [0, \infty]\ \forall\ j \in [\overline{1, \Xi_q}]$.

Algorithmically, points of the same region are selected [15] according to the expressions

$$\min_{u_{qij}}\ \max_{j}\ \lambda_{qij}\cdot g_{qij}[u_{qij}(\bullet), w^{*}_{qi}(\bullet)] = \max_{\alpha}\ \sigma_{qij\alpha}\cdot g_{qij}\{[u_{qij}]_{\pi}(\bullet), w^{*}_{qi}(\bullet)\};$$
$$u_{qij}(\bullet) \in U(\bullet)\ \ \forall\ \alpha, j \in [\overline{1, \Xi_q}], \qquad (20)$$

where $g_{qij}[u_{qij}(\bullet), w^{*}_{qi}(\bullet)] > 0;\ \sigma_{qi\alpha\alpha} = 1;\ j \ne \alpha \Rightarrow \sigma_{qij\alpha} = \lambda_{qi\alpha}\ \ \forall\alpha,\ j \in [\overline{1, \Xi_q}]$. For unimodal functions $g_{qij}[u_{qij}(\bullet), w^{*}_{qi}(\bullet)]\ \forall j \in [\overline{1, \Xi_q}]$, relations (19) allow us to select all points of the compromise region and only them [15].

Application of the described hierarchical approach to the problem of motion synthesis according to consistently applied private criteria of its quality leads to the following methodology. At the first stage, the motion is synthesized taking into account the first criterion, ensuring general, basic requirements for it (asymptotic stability in general or, at least, in the required region of the phase space; autonomy; specified time of course and nature of decay of transient processes; speed, and others), which are inherent in all subsequent motions of different levels. Then, at the second stage, synthesis is carried out using the second criterion and the corresponding level of the motion model, the dimensionality of which is, as a rule, lower than the dimensionality of the first-level model. At the same time, the characteristics of new control loops are determined. The characteristics of the contours synthesized in the previous stage remain unchanged. Then, motion synthesis is performed based on the third criterion, taking into account the lower-dimensional motion model, etc. he specified procedure allows to carry out the synthesis of the motion using a consistently applied set of private criteria of its quality. In other words, the synthesis of this motion is performed on the basis of variable, in phase space, functionals.

The developed methodology for a maglev train's mechanical subsystem's dynamic's constructing will allow significantly improving the quality of synthesized complex multi-purpose controlled movements. This possibility arises due to the implementation of an operational, compromise, harmonious adjustment of the target ratio and structure of the movement task being solved. At the same time, the generalized resources spent on this are minimized.

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